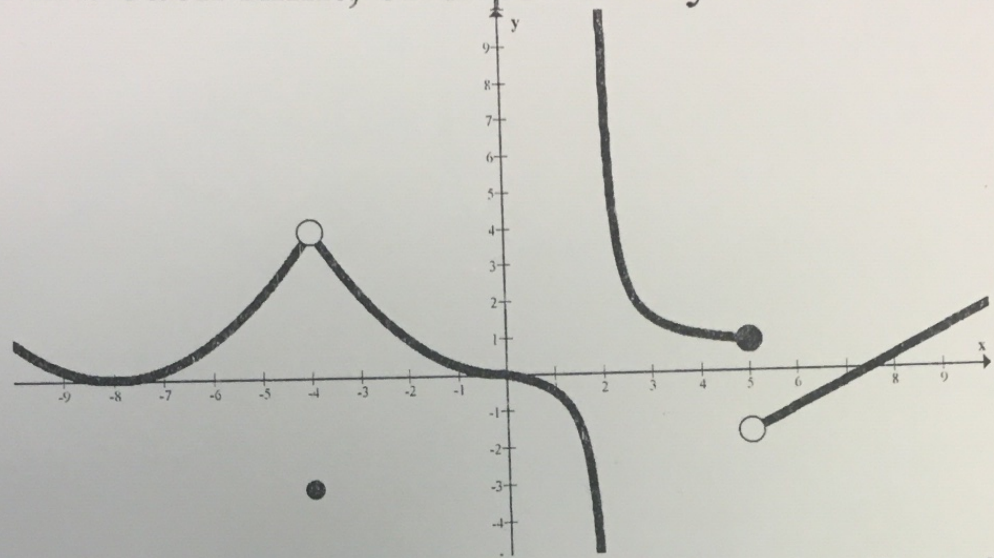
Station #1 Limits (graphs)

1. The graph of f is given.

a. Find each limit, or explain why it does not exist.



i. $\lim_{x\to 2^+} f(x)$

ii. $\lim_{x\to 5^+} f(x)$

iii. $\lim_{x\to 5} f(x)$

iv. $\lim_{x\to -4} f(x)$

v. $\lim_{x\to 0} f(x)$

vi. $\lim_{x\to 2^-} f(x)$

b. State the equations of the vertical asymptotes.

c. At what numbers is f discontinuous? Identify the type of discontinuity at each point.

1. a. i. ∞ ii. -2

iii. DNE iv. 4

v. 0 vi. $-\infty$

b. $x = 2$

c. $x = -4$; removable; $\lim_{x \to -4} f(x) \neq f(-4)$

$x = 2$; infinite $\lim_{x \to 2} f(x) = \pm\infty$

$x = 5$; jump; $\lim_{x \to 5^+} f(x) \neq \lim_{x \to 5-} f(x)$

Station #2 – Limits (algebraic)

1. Find the limit algebraically.

a. $\lim_{x\to 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x}$

b. $\lim_{v\to 2} \frac{v^2 + 2v - 8}{v^4 - 16}$

c. $\lim_{x\to 0} \frac{\sin(5x)}{2x}$

d. $\lim_{x\to -3} \frac{x^2 + 7x + 12}{x^2 - 9}$

e. $\lim_{x\to 4} \frac{x - 4}{\sqrt{x} - 2}$

f. $\lim_{t\to -3} \frac{3t + 9}{t^3 + 27}$

a. $-\frac{1}{4}$

b. ~~$\frac{1}{4}$~~ $\frac{3}{16}$

c. ~~$\frac{1}{10}$~~ $\frac{5}{2}$

d. $-\frac{1}{6}$

e. 4

f. ~~$\frac{1}{12}$~~ $\frac{1}{9}$

Station #3 – Continuity

1. Does each function have a removable discontinuity, infinite discontinuity, or jump discontinuity? And, at what value does each discontinuity occur? Justify your answer.

a. $f(x) = \dfrac{x^2 - 4}{x + 2}$

b. $f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } x \geq 0 \end{cases}$

c. $f(x) = \dfrac{3}{x - 5}$

d. $f(x) = \dfrac{x - 3}{x^2 - 9}$

a. Removable at x = -2

b. Jump discontinuity at x = 0 ($\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$)

c. Infinite discontinuity at x = 5

d. Removable discontinuity at x=3, infinite discontinuity at x= -3

Station #4 - Continuity

1. Explain why the function is discontinuous at the given number a. Then describe the type of discontinuity that exists.

a. $f(x) = \begin{cases} 1-x^2, & x<1 \\ 1/x, & x \geq 1 \end{cases}$ at a = 1

b. $f(x) = -\dfrac{1}{(x-1)^2}$ at a = 1

2. Let $f(x) = \begin{cases} \sqrt{-x}, & x<0 \\ 3-x, & 0 \leq x < 3 \\ (x-3)^2, & x > 3 \end{cases}$

a. Evaluate each limit

i. $\lim\limits_{x \to 0^+} f(x)$

ii. $\lim\limits_{x \to 0^-} f(x)$

iii. $\lim\limits_{x \to 0} f(x)$

iv. $\lim\limits_{x \to 3^-} f(x)$

v. $\lim\limits_{x \to 3^+} f(x)$

vi. $\lim\limits_{x \to 3} f(x)$

b. Does the function have any discontinuities? Justify your answer.

1. a. $\lim_{x\to 1^-} f(x) = 0$ $\lim_{x\to 1^+} f(x) = 1$

$\therefore \lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$ jump discontinuity

b. $(x - 1)^2 = 0$

infinite discontinuity at $x = 1$

2. a. i. 3 ii. 0

iii. DNE iv. 0

v. 0 vi. 0

b. jump at $x = 0$ $\left(\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)\right)$

removable at $x = 3$ (f(3) is undefined)

Station #5 – Limits (algebraic)

1. Find the limit algebraically.

a. $\lim_{x\to\infty} \frac{x-9}{\sqrt{x}+3}$

b. $\lim_{x\to 2^+} \frac{x-5}{x-2}$

c. $\lim_{t\to\infty} \frac{t-4}{t^2-3t-4}$

d. $\lim_{x\to -1^-} \frac{3x}{(x+1)^2}$

e. $\lim_{x\to 4} \frac{-3x^2+1}{x-4}$

f. $\lim_{x\to\infty} \frac{3x^2-2x+3}{2x^2+5}$

a. ∞

b. $-\infty$

c. 0

d. $-\infty$

e. Does not exist

f. 1.5

Station #6 – Limits (numeric)

1. Given that $\lim_{x \to 5} f(x) = 4$ and $\lim_{x \to 5} g(x) = 8$, use the limit laws to evaluate each limit below.

a. $\lim_{x \to 5}[-2g(x)]$

b. $\lim_{x \to 5}[f(x) + g(x)]$

c. $\lim_{x \to 5} \frac{f(x)}{g(x)}$

d. $\lim_{x \to 5} \sqrt{f(x)}$

2. Given the table below, find $\lim_{x \to 2} f(x)$

x	1.9	1.99	1.999	2.0	2.001	2.01	2.1
$f(x)$	3.700	3.970	3.997	?	4.003	4.030	4.300

1. a. -16

b. 12

c. $\frac{1}{2}$

d. 2

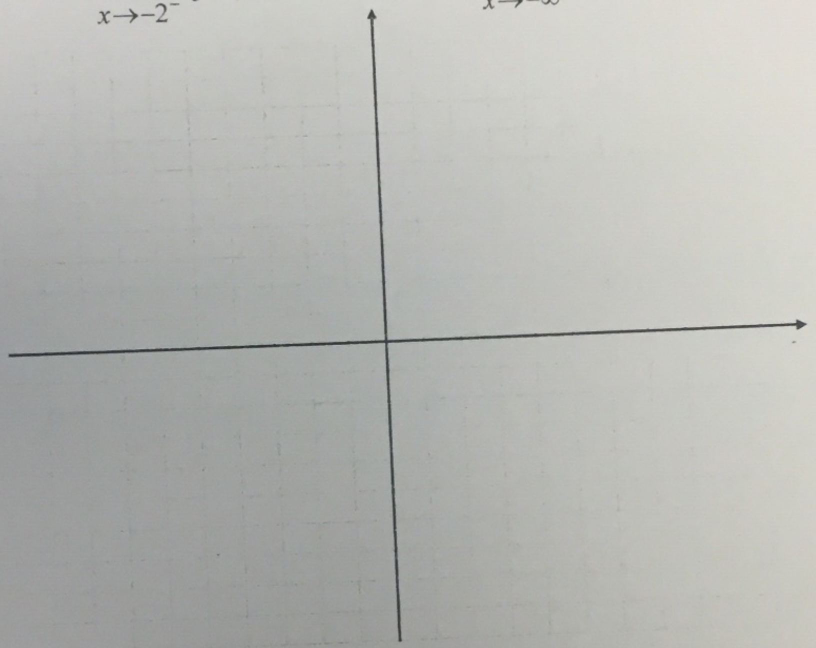
3. 4

Station #7 – Sketch a graph and IVT

1. Sketch a graph of a function $y = f(x)$ that satisfies the stated conditions. Include any asymptotes.

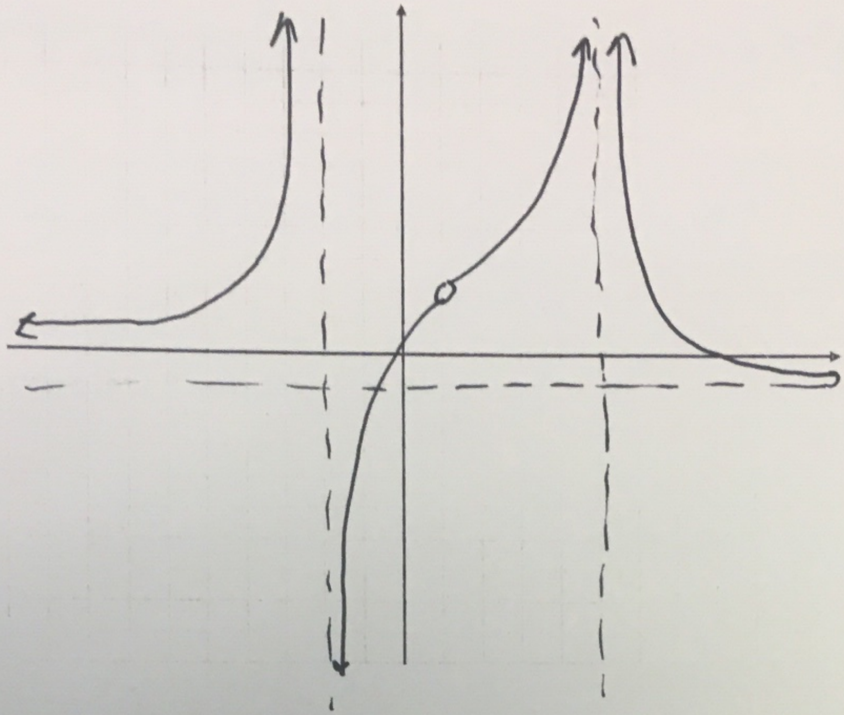
$\lim_{x\to 1} f(x) = 2$ $\lim_{x\to 5^-} f(x) = \infty$ $\lim_{x\to 5^+} f(x) = \infty$ $\lim_{x\to \infty} f(x) = -1$

$\lim_{x\to -2^+} f(x) = -\infty$ $\lim_{x\to -2^-} f(x) = \infty$ $\lim_{x\to -\infty} f(x) = 0$



2. Use the Intermediate Value Theorem to show that there is a root of the equation $x^3 + 2x^2 - 42 = 0$ on the interval (0, 3).

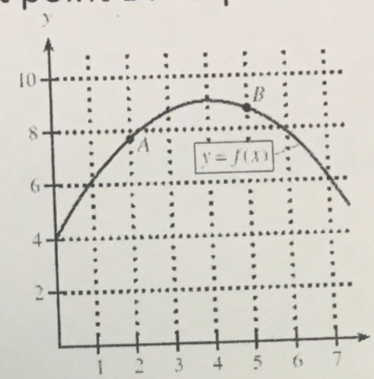
1.



2. *f(x)* is a polynomial and is therefore continuous. So, since *f*(0) $< 0 <$ *f*(3), then by the Intermediate value theorem, there exists at least one value *c* on (0, 3) such that *f*(*c*) = 0.

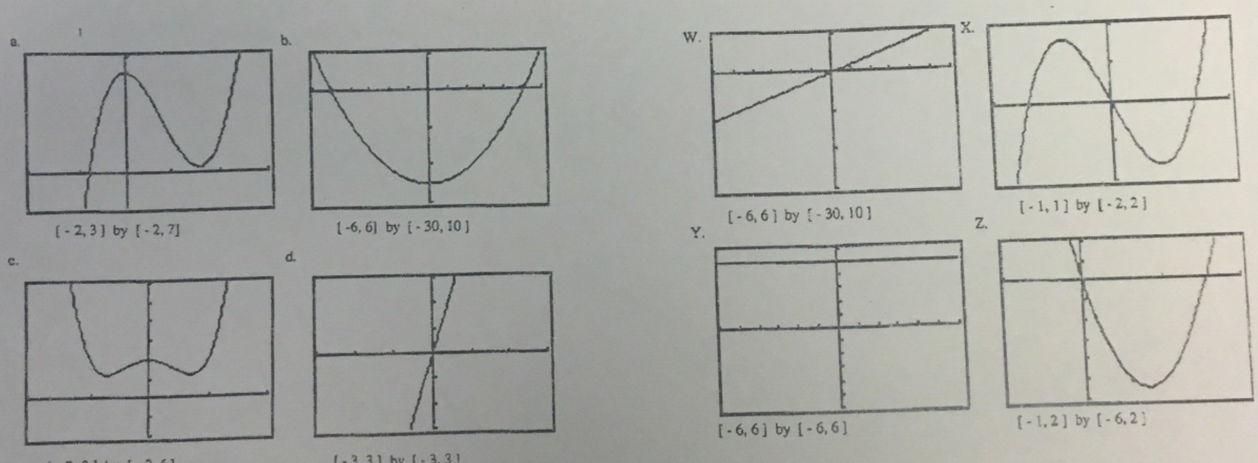
Station #8 – Derivatives

1. The graph below represents the function $y = f(x)$.
   a. Is $f'(x)$ greater at point A or at point B? Explain.

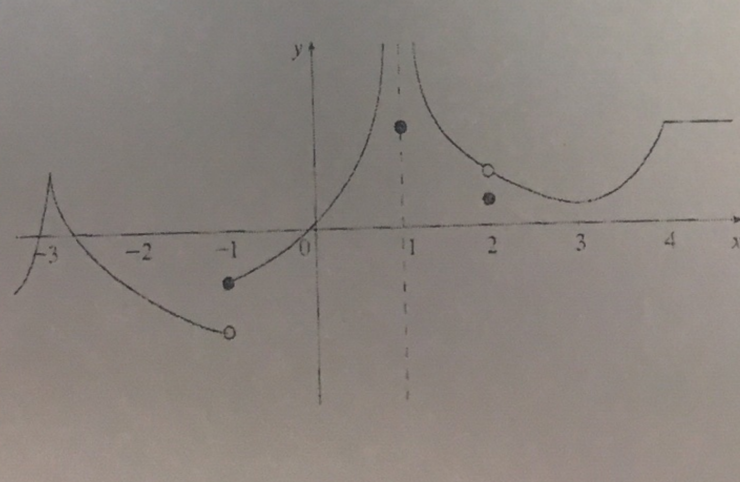


   b. Estimate $f'(x)$ at point B.

2. Match the graph of each function (a – d) with the graph of its derivative (W – Z)



3. The graph of f is given below. State the value(s) at which f is not differentiable. Justify answer.



1. a. $f'(A)$ is positive, $f'(B)$ is negative, therefore $f'(A) > f'(B)$

b. -1/2

2. a = z; b = w; c = x; d = y

3. x = -3, cusp; x = -1, discontinuity; x = 1, discontinuity; x = 2, discontinuity; x = 4, corner