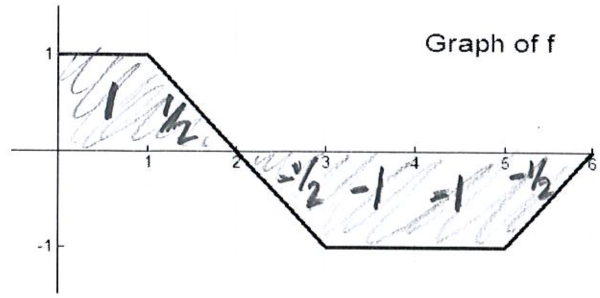
Calculus
Area & Distance

Name________________________

1. The graph of f is shown to the right.

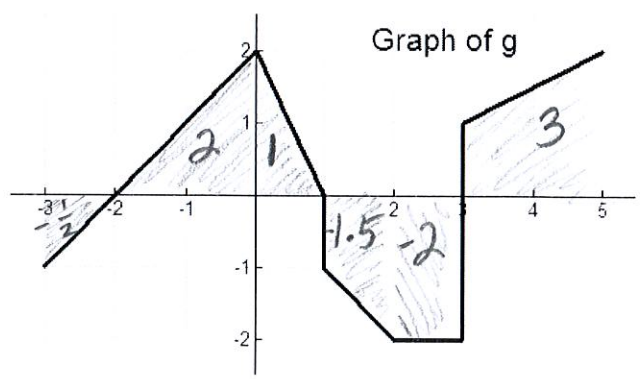


a) Find the area bound by the curve and the x-axis over the interval [0,2].
AKA $\int_0^2 f(x)dx = \frac{3}{2}$

b.) Find the area bound by the curve and the x-axis over the interval [2,6].
AKA $\int_2^6 f(x)dx = -3$

c.) Find the area bound by the curve and the x-axis over the interval [0,6].
AKA $\int_0^6 f(x)dx = -\frac{3}{2}$

2. The graph of g is shown to the right.



a.) $\int_{-3}^{-2} g(x)dx = -\frac{1}{2}$

b.) $\int_{-2}^{1} g(x)dx = 3$

c.) $\int_{1}^{3} g(x)dx = -\frac{7}{2}$

d.) $\int_{3}^{5} g(x)dx = 3$

e.) $\int_{-3}^{0} g(x)dx = \frac{3}{2}$

f.) $\int_{0}^{5} g(x)dx = \frac{1}{2}$

g.) $\int_{-3}^{5} g(x)dx = 2$

h.) $\int_{-3}^{-3} g(x)dx = 0$

i.) $\int_{3}^{1} g(x)dx$
$= -\int_1^3 g(x) = \frac{7}{2}$

3. A car comes to a stop five seconds after the driver slams on the brakes. While the brakes are on, the following velocities are recorded.

Time since brakes applied (sec)	0	1	2	3	4	5
Velocity (ft/sec)	88	60	40	25	10	0

Estimate the distance the car traveled after the brakes were applied.

a.) Using a Left-Riemann sum with five subintervals of equal length and values from the table to approximate $\int_0^5 v(t)dt$.

$w\,[\ell + \ell + \ell + \ell + \ell]$

$1[88+60+40+25+10] = \boxed{223 \text{ feet}}$

0 1 2 3 4 5
88 60 40 25 10 0

b.) Using a Right-Riemann sum with five subintervals of equal length and values from the table to approximate $\int_0^5 v(t)dt$.

$1[60+40+25+10+0] = \boxed{135 \text{ feet}}$

0 1 2 3 4 5
88 60 40 25 10 0

c.) Using a Trapezoidal sum with five subintervals of equal length and values from the table to approximate $\int_0^5 v(t)dt$. Using correct units, explain the meaning of $\int_0^5 v(t)dt$ in the context of this problem. $\frac{h}{2}[b_1+b_2]$

$\frac{1}{2}[(88+60)+(60+40)+(40+25)+(25+10)+(10+0)] = \boxed{179 \text{ feet}}$

0 1 2 3 4 5
88 60 40 25 10 0

4. Roger decides to run a marathon. Roger's friend Jeff rides behind him on a bicycle and clocks his pace every 15 minutes. Roger starts out strong, but after an hour and a half he is so exhausted that he has to stop. The data Jeff collected is summarized below.

Time spend running (min)	0	15	30	45	60	75	90
Speed (meters/minute)	12	11	10	10	8	7	0

a.) Estimate the distance that Roger ran during the first 30 minutes using a Trapezoidal sum with two subinterval of equal length and values from the table.

$\frac{15}{2}[(12+11)+(11+10)] = \boxed{330 \text{ meters}}$

0 15 30
12 11 10

b.) Estimate the total distance that Roger ran during the first 90 minutes using a Trapezoidal sum with **three** subinterval of equal length and values from the table.

$\frac{30}{2}[(12+10)+(10+8)+(8+0)] = \boxed{720 \text{ meters}}$

0 15 30 45 60 75 90
12 11 10 10 8 7 0

c.) Estimate the total distance that Roger ran during the first 90 minutes using a Midpoint sum with three subinterval of equal length and values from the table.

$30[11+10+7] = \boxed{840 \text{ meters}}$

0 15 30 45 60 75 90
12 11 10 10 8 7 0

Area & Distance

5. Coal gas is produced at a gasworks. Pollutants in the gas are removed by scrubbers, which become less and less efficient as time goes on. Measurements made at the start of each month showing the rate at which pollutants are escaping in the gas are as follows.

Time(months)	0	1	2	3	4	5	6
Rate pollutants are escaping (tons/month)	5	7	8	10	13	16	20

a.) Estimate the amount of pollutants that escape during the 6 month interval using a Left-Riemann sum with six subintervals of equal length and values from the table.

$1[5+7+8+10+13+16] = \boxed{59 \text{ tons of pollutants}}$

0 1 2 3 4 5 6
5 7 8 10 13 16 20

b.) Estimate the amount of pollutants that escape during the 6 month interval using a Right-Riemann sum with six subintervals of equal length and values from the table.

$1[7+8+10+13+16+20] = \boxed{74 \text{ tons of pollutants}}$

0 1 2 3 4 5 6
5 7 8 10 13 16 20

c.) Estimate the amount of pollutants that escape during the 6 month interval using a Trapezoidal sum with six subintervals of equal length and values from the table.

$\frac{1}{2}[(5+7)+(7+8)+(8+10)+(10+13)+(13+16)+(16+20)] = \boxed{66.5 \text{ tons of pollutants}}$

d.) Estimate the amount of pollutants that escape during the 6 month interval using the Midpoint sum with three subintervals of equal length and values from the table.

$2[7+10+16] = \boxed{66 \text{ tons of pollutants}}$

0 1 2 3 4 5 6
5 7 8 10 13 16 20

6. A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon (t=0) and 8 P.M. (t=8). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of E(t), in hundreds of entries at various times t are shown in the table below.

t (hours)	0	2	5	7	8
E(t) (hundreds of entries)	0	4	13	21	23

a.) Use a trapezoidal sum with the four subintervals given by the table to approximate the number of entries received from noon to 8 P.M.

$\frac{2}{2}[0+4] + \frac{3}{2}[4+13] + \frac{2}{2}[13+21] + \frac{1}{2}[21+23] = \boxed{85{,}500 \text{ of entries}}$

0 2 5 7 8
0 4 13 21 23

b.) Using a Left-Riemann sum with four subintervals given by the table to approximate the number of entries received from noon to 8 P.M.

$2[0] + 3[4] + 2[13] + 1[21] = \boxed{59{,}000 \text{ entries}}$

0 2 5 7 8
0 4 13 21 23

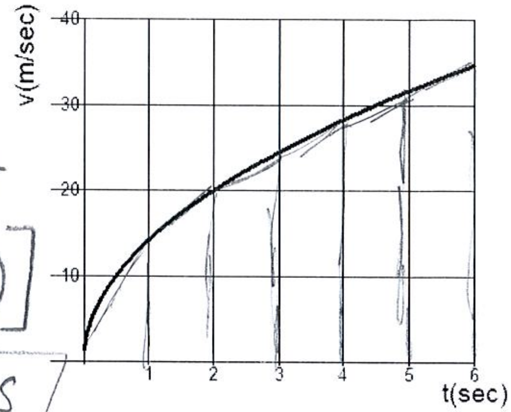
c.) Using a ~~Left~~ Right-Riemann sum with four subintervals given by the table to approximate the number of entries received from noon to 8 P.M.

0 2 5 7 8
0 4 13 21 23

$2[4] + 3[13] + 2[21] + 1[23] \quad \boxed{112{,}000 \text{ entries}}$

Area & Distance

8. The figure below shows the graph of the velocity, v, of an object (in m/sec). Estimate the total distance the object traveled between t=0 and t=6.



$Trap_6 =$

$\frac{1}{2}\left[(0+15)+(15+20)+(20+24)+(24+28)+(28+32)+(32+35)\right]$

$\boxed{136.5 \text{ meters}}$

* Answers may vary. You could use the # of rectangles / traps & whatever method you want.

9. The speed of an object is recorded in the table below.

t(seconds)	0	3	6	9	12	15	18	21	24
v(m/sec)	11	15	18	20	16	15	20	22	25

Estimate the distance traveled by the object.

a.) From t=0 to t=12 seconds using 4 left rectangles with equal subintervals.

$3[11+15+18+20]$

0	3	6	9	12
11	15	18	20	16

$\boxed{192 \text{ meters}}$

b.) From t=0 to t=24 seconds using 4 left rectangles with equal subintervals.

$6[11+18+16+20]$

$\boxed{390 \text{ meters}}$

c.) From t=0 to t=24 seconds using 4 midpoint rectangles with equal subintervals.

$6[15+20+15+22]$

$\boxed{432 \text{ meters}}$

d.) From t=0 to t=24 seconds using 2 midpoint rectangles with equal subintervals.

$12[18+20]$

$\boxed{456 \text{ meters}}$

e.) From t=0 to t=24 seconds using 8 trapezoids with equal subintervals.

$\frac{3}{2}\left[11+2(15)+2(18)+2(20)+2(16)+2(15)+2(20)+2(22)+25\right]$

$\boxed{432 \text{ meters}}$

f.) From t=0 to t=24 seconds using 4 trapezoids with equal subintervals.

$\frac{6}{2}\left[11+2(18)+2(16)+2(20)+25\right]$

$\boxed{432 \text{ meters}}$