AP Calculus

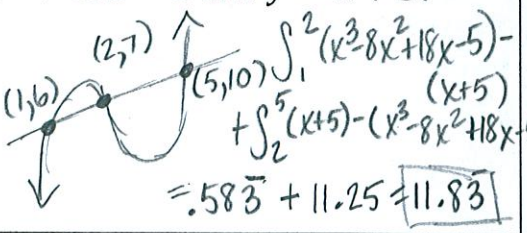
Area, Volume, & Avg. Value: Multiple Choice

Name_____

Date_____ Pd._____ Day____

1. What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$? **(Calculator)**

(A) 10.667
(B) 11.833
(C) 14.583
(D) 21.333
(E) 32



$\int_1^2 (x^3-8x^2+18x-5) - (x+5) + \int_2^5 (x+5) - (x^3-8x^2+18x-5)$

$= .583 + 11.25 = 11.83$

2. What is the average value of $y = \frac{cosx}{x^2+x+2}$ on the closed interval $[-1,3]$? **(Calculator)**

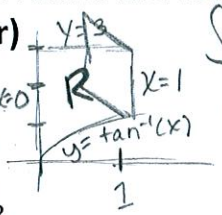
(A) −0.085
(B) 0.090
(C) 0.183
(D) 0.244
(E) 0.732

$\frac{1}{3-1}\int_{-1}^{3} \frac{\cos x}{(x^2+x+2)}$

$\frac{1}{4}[.73180422] = .18295105$

3. The base of a solid is the region in the first quadrant bounded by the y −axis, the graph of $y = tan^{-1}x$, the horizontal line $y = 3$, and the vertical line $x = 1$. For this solid, each cross section perpendicular to the $x-axis$ is a square. What is the volume of the solid? **(Calculator)**

(A) 2.561
(B) 6.612
(C) 8.046
(D) 8.755
(E) 20.773



$\int_{x_1}^{x_2} \text{Area Square}\, dx \quad dx$

$\int_0^1 (3-\tan^{-1}x)^2 dx$

6.612333

4. If $0 \le k < \frac{\pi}{2}$ and the area under the curve $y = cosx$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$ **(Calculator)**

(A) 1.471
(B) 1.414
(C) 1.277
(D) 1.120
(E) 0.436

$\int_k^{\pi/2} \cos x\, dx = .1$

$\sin x\Big|_k^{\pi/2} = .1$

$\sin\frac{\pi}{2} - \sin k = .1$

$1 - \sin k = .1$

$-\sin k = -.9$

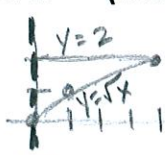
$\sin k = .9$

$k = \sin^{-1}(.9)$

$k = 1.119769515$

5. If the region enclosed by the y −axis, the line $y = 2$, and the curve $y = \sqrt{x}$ is revolved about the y −axis, the volume of the solid generated is **(Non-Calculator)**

(A) $\frac{32\pi}{5}$
(B) $\frac{16\pi}{3}$
(C) $\frac{16\pi}{5}$
(D) $\frac{8\pi}{3}$
(E) π



$y = \sqrt{x}$, $x = y^2$

$\pi\int_0^2 (y^2-0)^2 dy$

$\pi\int_0^2 y^4\, dy$

$\pi\left[\frac{y^5}{5}\Big|_0^2\right] = \pi\left[\frac{32}{5} - \frac{0}{5}\right]$

$= \frac{32\pi}{5}$

6. The average value of $cosx$ on the interval $[-3,5]$ is **(Non-Calculator)**

(A) $\frac{sin5 - sin3}{8}$
(B) $\frac{sin5 - sin3}{2}$
(C) $\frac{sin3 - sin5}{2}$
(D) $\frac{sin3 + sin5}{2}$
(E) $\frac{sin3 + sin5}{8}$

$\frac{1}{5-3}\int_{-3}^{5} \cos x\, dx$

$\frac{1}{8}[\sin x\Big|_{-3}^{5}$

$\frac{1}{8}[\sin 5 - \sin(-3)]$

$\frac{1}{8}(\sin(5) - -\sin(3))$

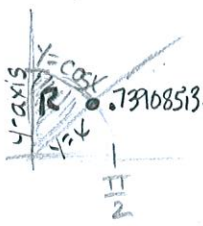
$\frac{\sin 5 + \sin 3}{8}$

sinx : even function

sin(−x) = −sinx

7. What is the area of the region in the first quadrant enclosed by the graphs of $y = cosx$, $y = x$, and the y −axis? **(Calculator)**

(A) 0.127
(B) 0.385
(C) 0.400
(D) 0.600
(E) 0.947

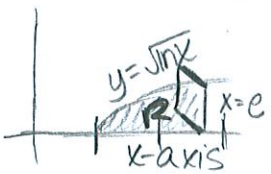


$\int_0^{.7390851} (\cos x - x)\, dx$

$= .40048861$

8. The base of a solid S is the region enclosed by the graph of $y = \sqrt{lnx}$, the line $x = e$, and the x −axis. If the cross sections of S perpendicular to the x −axis are squares, then the volume of S is **(Non-Calculator)**

(A) $\frac{1}{2}$
(B) $\frac{2}{3}$
(C) 1
(D) 2
(E) $\frac{1}{3}(e^3 - 1)$



$\int_1^e (\sqrt{\ln x} - 0)^2\, dx$

$= 1$

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9. The area of the region enclosed by the curve $= \frac{1}{x-1}$, the x −axis, and the lines $x = 3$ and $x = 4$ is **(Non-Calculator)**

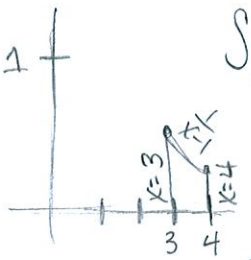
(A) $\frac{5}{36}$

(B) $ln\frac{2}{3}$

(C) $ln\frac{4}{3}$

(D) $ln\frac{3}{2}$

(E) $ln6$



$\int_3^4 \left(\frac{1}{x-1} - 0\right)^2 dx$

$\int_3^4 \frac{1}{(x-1)^2} dx$

$u = x-1 \quad u(3)=2$

$du = dx \quad u(4)=3$

$\int_2^3 \frac{1}{u} du$

$\ln|u|]_2^3 \quad \ln(3) - \ln(2) = \boxed{\ln\left(\frac{2}{3}\right)}$

10. The region enclosed by the x −axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the x −axis. What is the volume of the solid generated? **(Non-Calculator)**

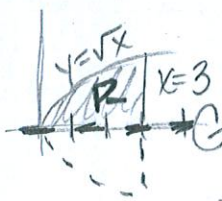
(A) 3π

(B) $2\sqrt{3}\pi$

(C) $\frac{9}{2}\pi$

(D) 9π

(E) $\frac{36\sqrt{3}}{5}\pi$



$\pi\int_0^3 (\sqrt{x} - 0)^2 dx$

$\pi\int_0^3 x\, dx$

$\pi\left[\frac{x^2}{3}\right|_0^3$

$\pi\left[\frac{9}{3} - \frac{0}{3}\right] = \boxed{3\pi}$

$\pi\left[3 - \frac{1}{27}(27) - \left[-3 - \frac{1}{27}(-3)^3\right]\right] = \pi[3-1+3-1] = \boxed{4\pi}$

$\left.\frac{x^3}{-3}\right|_{-3}^{3}$

11. What is the average value of y for the part of the curve $y = 3x - x^2$ which is in the first quadrant ? **(Non-Calculator)**

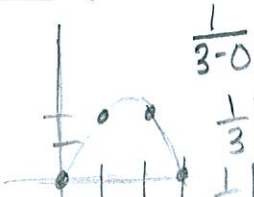
(A) -6

(B) -2

(C) $\frac{3}{2}$

(D) $\frac{9}{4}$

(E) $\frac{9}{2}$



$\frac{1}{3-0}\int_0^3 3x - x^2 dx$

$\frac{1}{3}\left[\frac{3x^2}{2} - \frac{x^3}{3}\right|_0^3$

$\frac{1}{3}\left[\frac{27}{2} - \frac{27}{3}\right]$

$\frac{1}{3}\left[\frac{27\cdot 3}{2\cdot 3} - \frac{27\cdot 2}{3\cdot 2}\right] = \frac{1}{3}\left[\frac{81}{6} - \frac{54}{6}\right] = \frac{1}{3}\left[\frac{27}{6}\right]$

$\frac{3}{2} = \frac{27}{18}$

12. The volume of the solid obtained by revolving the region enclosed by the ellipse $x^2 + 9y^2 = 9$ about the x −axis is **(Non-Calculator)**

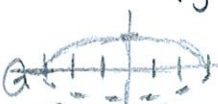
(A) 2π

(B) 4π

(C) 6π

(D) 9π

(E) 12π



$9y^2 = 9 - x^2$

$\sqrt{y^2} = \sqrt{\frac{9-x^2}{9}}$

$y = \sqrt{\frac{9-x^2}{9}}$

$\pi\int_{-3}^{3} \left(\sqrt{\frac{9-x^2}{9}} - 0\right)^2 dx$

$\pi\int_{-3}^{3} \frac{9-x^2}{9} dx = \pi\int_{-3}^{3} 1 - \frac{1}{9}x^2 = \pi\left[x - \frac{1}{9}\frac{x^3}{3}\right|_{-3}^{3}$

13. The area of the region in the first quadrant that is enclosed by the graphs of $y = x^3 + 8$ and $y = x + 8$ is **(Non-Calculator)**

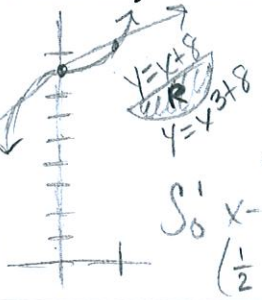
(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) 1

(E) $\frac{65}{4}$



$\int_0^1 (x+8) - (x^3+8) dx$

$\int_0^1 x + 8 - x^3 - 8\, dx$

$\int_0^1 x - x^3 dx = \frac{x^2}{2} - \frac{x^4}{4}\Big|_0^1$

$\left(\frac{1}{2} - \frac{1}{4}\right) - (0) = \boxed{\frac{1}{4}}$

14. The average value of $f(x) = x^2\sqrt{x^3+1}$ on the closed interval $[0,2]$ is **(Non-Calculator)**

(A) $\frac{26}{9}$

(B) $\frac{13}{3}$

(C) $\frac{26}{3}$

(D) 13

(E) 26

$\frac{1}{2-0}\int_0^2 x^2\sqrt{x^3+1}\, dx$

$u = x^3+1 \quad du = 3x^2 dx \quad u(0)=1 \quad u(2)=9$

$\frac{1}{6}\int_1^9 u^{1/2} du$

$\frac{1}{36}\frac{2}{3}u^{3/2}\Big|_1^9 = \frac{1}{9}(\sqrt{9})^3 - \frac{1}{9}(\sqrt{1})^3$

$\frac{27}{9} - \frac{1}{9} = \boxed{\frac{26}{9}}$

15. The area of the region bounded by the lines $x = 0$, $x = 2$, and $y = 0$ and the curve $y = e^{\frac{x}{2}}$ is **(Non-Calculator)**

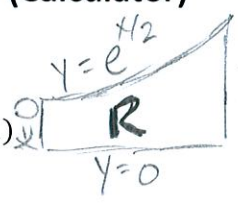
(A) $\frac{e-1}{2}$

(B) $e-1$

(C) $2(e-1)$

(D) $2e-1$

(E) $2e$



$2\int_0^2 \left(e^{x/2} - 0\right) dx$

$u = \frac{1}{2}x \quad u(0)=0 \quad u(2)=1$

$du = \frac{1}{2}dx$

$2\int_0^1 e^u du$

$2e^u\Big|_0^1$

$2e^1 - 2e^0$

$\boxed{2e-2}$

16. The area of the region enclosed by the graphs of $y = x$ and $y = x^2 - 3x + 3$ is **(Calculator)**

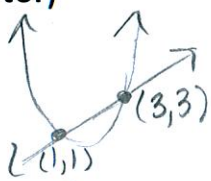
(A) $\frac{2}{3}$

(B) 1

(C) $\frac{4}{3}$

(D) 2

(E) $\frac{14}{3}$



$\int_1^3 x - (x^2 - 3x + 3) dx$

1.333333

$= \boxed{\frac{4}{3}}$