AP Calculus

Day 13 Supplement: Integration

Name____________________________

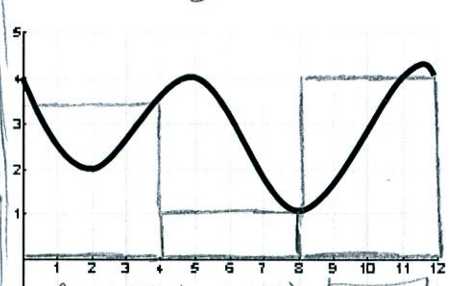
Answers Can Vary for #1

1-2 Approximate the area:

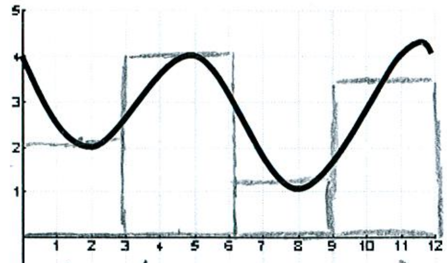
1. a. R_3 $\frac{12-0}{3}=4$

b. M_4 $\frac{12-0}{4}=3$

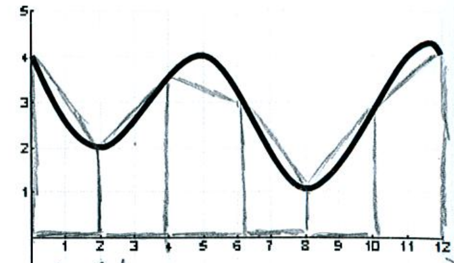
c. T_6 $\frac{12-0}{6}=2$



$A=4(3.4+1+4)=\boxed{33.6}$



$A=3(2+4+1.3+3.5)$ $=\boxed{32.4}$



$A=\frac{2}{2}(4+2+2+3.5+3.5+3+3+1+1+3+3+4)$ $\boxed{=33}$

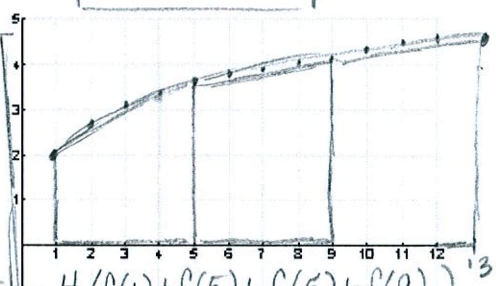
2. $f(x)=\ln(x)+2$ $[1,13]$

a. T_3 $\boxed{=44.3565}$

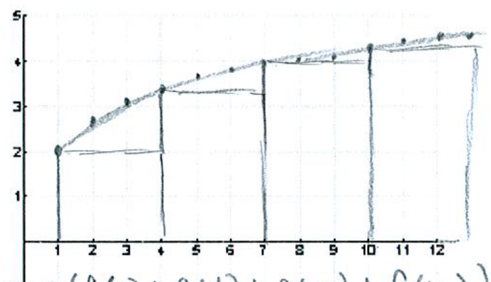
b. L_4 $=\boxed{40.9044}$

c. R_6 $=\boxed{47.6281}$

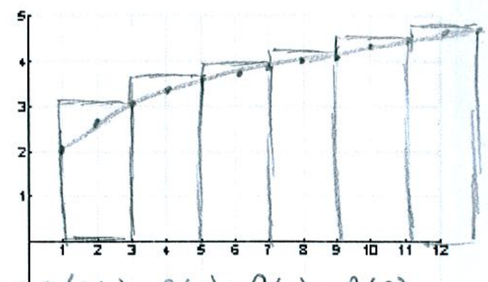
Answers Can NOT VARY for #2



$\frac{4}{2}(f(1)+f(5)+f(5)+f(9)+f(9)+f(13))$



$3(f(1)+f(4)+f(7)+f(10))$



$2(f(3)+f(5)+f(7)+f(9)+f(11)+f(13))$

3-4. Given the second derivative and the given information find the original function.

3. $f''(x)=x^{-\frac{3}{2}}$, $f'(4)=1$ *and* $f(4)=4$

$\int f''(x)=f'(x)+C$

$\int x^{-3/2}=-2x^{-1/2}+C$

$f'(x)=\frac{-2}{\sqrt{x}}+C$

$1=\frac{-2}{\sqrt{4}}+C$ $\quad 1=-1+C$ $\quad C=2$

$\int -2x^{-1/2}+2$

$-2\cdot 2x^{1/2}+2x+C$

$f(x)=-4\sqrt{x}+2x+C$

$4=-4(2)+8+C$

$\boxed{f(x)=-4\sqrt{x}+2x+4}$

4. $f''(x)=x-\cos x$, $f'(0)=2$ *and* $f(0)=-2$

$\int x-\cos x$

$f'(x)=\frac{x^2}{2}-\sin x+C$

$2=\frac{0^2}{2}-\sin(0)+C$

$C=2$

$f'(x)=\frac{1}{2}x^2-\sin x+2$

$\int f'(x)$

$\int \frac{1}{2}x^2-\sin x+2$

$f(x)=\frac{1}{2}\frac{x^3}{3}-(-\cos x)+2x+C$

$-2=\frac{1}{6}(0)^3+\cos(0)+2(0)+C$

$-3=C$

$\boxed{f(x)=\frac{1}{6}x^3+2x+\cos x-3}$

5-9. Given $\int_0^1 f(x)\,dx=3$, $\int_0^2 f(x)\,dx=1$, and $\int_1^5 f(x)\,dx=8$ answer each of the following questions:

5. $\int_0^5 f(x)\,dx=$

$\int_0^1+\int_1^5=\int_0^5$

$3+8=\boxed{11}$

6. $\int_1^2 f(x)\,dx=$

$\int_0^1+\int_1^2=\int_0^2$

$3+x=1$

$x=\boxed{-2}$

7. $\int_2^5 f(x)\,dx=$

$\int_0^2+\int_2^5=\int_0^5$

$1+x=11$

$\boxed{x=10}$

8. $\int_3^3 f(x)\,dx=$

$\boxed{0}$

9. $\int_2^0 f(x)\,dx$

$=-\int_0^2 f(x)$

$=\boxed{-1}$

10-12 Integrate each without a calculator: DO NOT INTEGRATE

10. $\int_{-3}^{3}|x^3+1|\,dx$

$\boxed{-\int_{-3}^{-1}x^3+1+\int_{-1}^{3}x^3+1}$

11. $\int_0^{2\pi}|\cos x|\,dx$

$\boxed{\int_0^{\pi/2}\cos x-\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\cos x+\int_{\frac{3\pi}{2}}^{2\pi}\cos x}$

12. $\int_{-3}^{3}|-x^2-x+6|\,dx$

$(-x+2)(x+3)$

$x=2 \quad x=-3$

$\boxed{\int_{-3}^{2}-x^2-x+6-\int_{2}^{3}-x^2-x+6}$

$\frac{d}{dx}[A(x)] = \frac{d}{dx}\int_0^x f(t)\,dt$

AP Calculus

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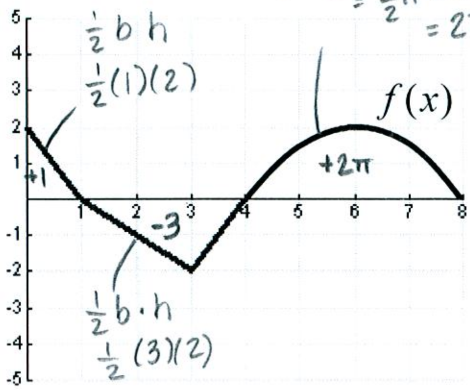
Day 13 Supplement: Integration $A'(x) = f(x)$

Answer questions 13-18 using the graph below:

$A(x) = \int_0^x f(t)\,dt$

$A = \frac{1}{2}\pi R^2 = \frac{1}{2}\pi(2)^2 = 2\pi$



13. $A(1) = \int_0^1 = \boxed{1}$

14. $A(4) = \int_0^4 = 1 - 3 = \boxed{-2}$

15. $A(8) = \int_0^8 = 1 - 3 + 2\pi = \boxed{-2+2\pi}$

16. $A'(1) = f(1) = \boxed{0}$

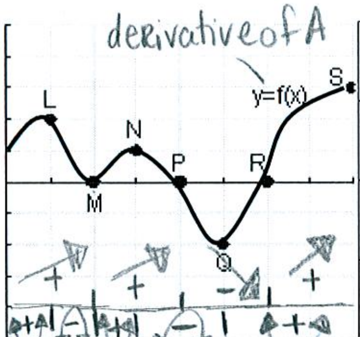
17. $A'(3) = f(3) = \boxed{-2}$

18. $A'(6) = f(6) = \boxed{2}$

Given the Graphs answer the following questions

Graph for 19-22

Let $\frac{d}{dx}[A(x)] = \frac{d}{dx}\int_0^x f(t)\,dt$ $A'(x) = f(x)$



19. Does $A(x)$ have a local minimum at M? NO

20. Where does $A(x)$ have a local minimum? R

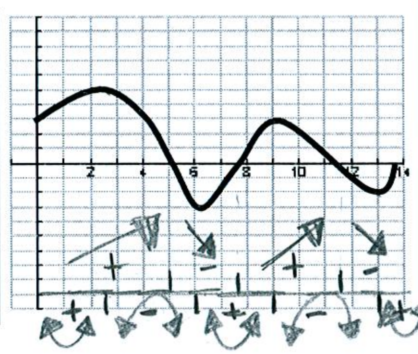
21. Where does $A(x)$ have a local maximum? P

22. Where does $A(x)$ have points of inflection?

L, M, N, and Q

Graph for 23-26

Let $\frac{d}{dx}[A(x)] = \frac{d}{dx}\int_0^x f(t)\,dt$ $A'(x) = f(x)$



23. What intervals is $A(x)$

Increasing: (0, 5) (7.5, 9.5)

Decreasing: (5, 7.5) (9.5, 14)

24. What intervals is $A(x)$

Concave Up: (0, 2.5) (5, 9) (11.5, 14)

Concave Down: (2.5, 5) (9, 11.5)

25. Where does $A(x)$ have extrema and what are they?

max: x = 5, 11.5 min: x = 7.5

26. Where does $A(x)$ have points of inflection?

Evaluate each: 27-30

27. $\frac{d}{dx}\int_0^x \sin^{-1} t^3\,dt$

$\boxed{\sin^{-1}(x)^3}$

28. $\frac{d}{dx}\int_7^{x^2} (3t^3 + t)\,dt$

$2x[3(x^2)^3 + x^2]$

$2x[3x^6 + x^2]$

$\boxed{6x^7 + 2x^3}$

29. $\frac{d}{dx}\int_x^{2\pi} \tan t\,dt$

$-\int_{2\pi}^x \tan t\,dt$

$\boxed{-\tan x}$

30. $\frac{d}{dx}\int_0^{2x} \sqrt{2t^2 - 1}\,dt$

$2[\sqrt{2(2x)^2 - 1}]$

$= \boxed{2\sqrt{8x^2 - 1}}$

31. A population of insects increases at a rate of $300t - 20t + .32t^2$ insects per day/ Find the insect population after 5 days if you start out with 10 insects at $t = 0$. $10 + \int_0^5 300t - 20t + .32t^2 = 10 + 3504 = \boxed{3514 \text{ bugs}}$

Limits

32. Find the total displacement and total distance traveled. Draw a diagram. $v(t) = 20 - 8t$. [0, 10]

displacement $= \int_0^{10} 20 - 8t = -200$ distance $= \int_0^{10} |20 - 8t| = 250$

33. Suppose a particle travels along a linear path at a velocity of $v(t) = 1 - \sin(\pi t)$, in meters per second, on the interval $0 \le t \le 1$. If the particle starts at a position 4 meters from the origin, find the position function.