AP Calculus

Multiple Choice Integrals

Name Vaij

Period______ Date______

1. $\int_1^2 \frac{1}{x^2}dx =$ $\int_1^2 x^{-2}\,dx = -\frac{1}{x}\Big|_1^2$

(A) $-\frac{1}{2}$

(B) $\frac{7}{24}$

(C) $\frac{1}{2}$

(D) 1

(E) $2\ln 2$

$= -\frac{1}{2} + 1 = \frac{1}{2}$

2. $\int_0^x \sin t\, dt =$

(A) $\sin x$

(B) $-\cos x$

(C) $\cos x$

(D) $\cos x - 1$

(E) $1 - \cos x$

$-\cos t\Big|_0^x$

$-\cos x + \cos 0$

$-\cos x - 1$

3. $\int_1^e \left(\frac{x^2-1}{x}\right) dx =$ $\int_1^e x - \frac{1}{x}\,dx = \frac{x^2}{2} - \ln|x|\Big|_1^e$

(A) $e - \frac{1}{e}$

(B) $e^2 - e$

(C) $\frac{e^2}{2} - e + \frac{1}{e}$

(D) $e^2 - 2$

(E) $\frac{e^2}{2} - \frac{3}{2}$

$= \frac{e^2}{2} - \ln|e| - \left(\frac{1}{2} - \ln 1\right)$

$= \frac{e^2}{2} - 1 - \frac{1}{2} + 0$

$= \frac{e^2}{2} - \frac{3}{2}$

4. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

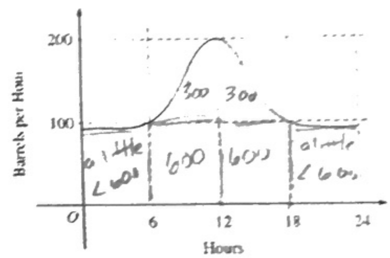
(A) 500

(B) 600

(C) 2,400

(D) 3,000

(E) ~~4,800~~



5. If $F(x) = \int_0^x \sqrt{t^3+1}\, dt$, then $F'(2) =$

(A) -3

(B) -2

(C) 2

(D) 3

(E) 18

$F'(x) = \frac{d}{dx}\int_0^x \sqrt{t^3+1}$

$F'(x) = \sqrt{x^3+1}\Big|_{x=2}$

$= \sqrt{8+1} = \sqrt{9} = 3$

6. The function f is continuous on the closed interval [2,8] and has values that are given in the table. Using the subintervals [2,5], [5,7], and [7,8], what is the trapezoidal approximation of $\int_2^8 f(x)dx$? **(Calculator)**

(A) 110

(B) 130

(C) 160

(D) 190

(E) 210

x	2	5	7	8
$f(x)$	10	30	40	20

$\int_2^8 f(x)dx \approx \frac{3}{2}(10+30) + \frac{2}{2}(30+40) + \frac{1}{2}(40+20)$

$= 1.5(40) + (70) + 30 = 160$

7. $\int_1^2 (4x^3 - 6x)\, dx =$ $x^4 - 3x^2\Big|_1^2$

(A) 2

(B) 4

(C) 6

(D) 36

(E) 42

$= 16 - 12 - (1 - 3)$

$= 4 - (-2) = 6$

8. $\int \frac{3x^2}{\sqrt{x^3+1}} dx =$

(A) $2\sqrt{x^3+1} + C$

(B) $\frac{3}{2}\sqrt{x^3+1} + C$

(C) $\sqrt{x^3+1} + C$

(D) $\ln\sqrt{x^3+1} + C$

(E) $\ln(x^3+1) + C$

$u = x^3 + 1$

$du = 3x^2\,dx$

$\int \frac{1}{\sqrt{u}}du = \int u^{-1/2}du$

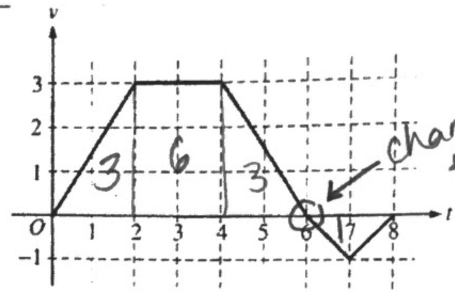
$= 2u^{1/2} + c$

$= 2\sqrt{x^3+1} + c$

AP Calculus
Multiple Choice Integrals

Name________________________
Period_________ Date____________________

Questions 9-10 refer to this graph.



change from + to −

9. A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown. At what value of t does the bug changedirection?

(A) 2
(B) 4
(C) 6
(D) 7
(E) 8

change in direction = x-intercept

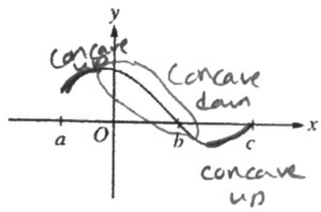
10. What is the total distance the bug traveled from $t = 0$ to $t = 8$?

(A) 14
(B) 13
(C) 11
(D) 8
(E) 6

$\int |v(t)|\, dt$

sum positive area = 3+6+3+1

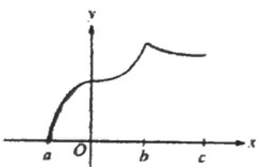
11. Let $f(x) = \int_a^x h(t)dt$, where h has the graph shown right. Which of the following could be the graph of f?



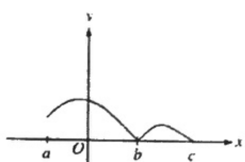
concave
concave down
concave up

derivative of f(x)
∴ + means inc, − means dec
$f(x) = \int h(t)$
$f'(x) = h(t)$
$f''(x) = h'(t)$
this narrows down to A or E

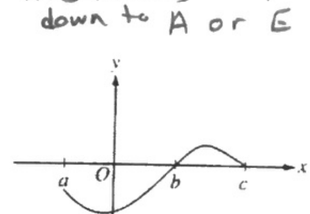
(A)



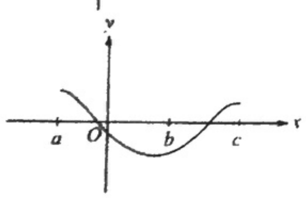
~~(B)~~



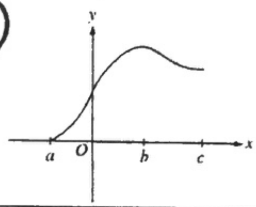
~~(C)~~



~~(D)~~



(E)



To determine answer, look at concavity of function

12. $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} =$

(A) $\frac{\pi}{3}$
(B) $\frac{\pi}{4}$
(C) $\frac{\pi}{6}$
(D) $\frac{1}{2}\ln 2$
(E) $-\ln 2$

$\int_{x=0}^{x=\sqrt{3}} \frac{dx}{\sqrt{4(1-\frac{x^2}{4})}}$

$\int \frac{dx}{2\sqrt{1-(\frac{x}{2})^2}}$

$u = \frac{x}{2}$
$du = \frac{1}{2}dx$

$\int_{u=0}^{u=\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-u^2}}\, du$

$= \sin^{-1} u \Big|_0^{\sqrt{3}/2}$

$= \sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}(0) = \frac{\pi}{3}$

13. If the definite integral $\int_0^2 e^{x^2} dx$ is first approximated by using two inscribed rectangles of equal width and then approximated by using the trapezoidal rule with $n = 2$, the difference between the two approximations is **(Calculator)**

contained under curve

(A) 53.60
(B) 30.51
(C) 27.80
(D) 26.80
(E) 12.78

x	y
0	1
1	2.718
2	54.5982

lefthand Riemann
$1(1+2.718) = 3.718$

trapezoidal rule
$\frac{1}{2}(1+2(2.718)+54.5982) = 30.5171$
$30.5171 - 3.718 = 26.7991$

P Calculus

Multiple Choice Integrals

Name______________________

Period______ Date______________________

14. $\int \sec^2 x \, dx =$

(A) $\tan x + C$
(B) $\csc^2 x + C$
(C) $\cos^2 x + C$
(D) $\frac{\sec^3 x}{3} + C$
(E) $2\sec^2 x \tan x + C$

15. If $\int_0^k (2kx - x^2)dx = 18$, then $k =$

(A) -9
(B) -3
(C) 3
(D) 9
(E) 18

$\frac{2kx^2}{2} - \frac{x^3}{3}\Big|_0^k$

$k^3 - \frac{k^3}{3} - 0 = 18$

16. If the function f has a continuous derivative on $[0, c]$, then $\int_0^c f'(x)dx =$

(A) $f(c) - f(0)$
(B) $|f(c) - f(0)|$
(C) $f(c)$
(D) $f(x) + c$
(E) $f''(c) - f''(0)$

$f(c)$

17. $\int_0^1 (3x-2)^2 dx =$

(A) $-\frac{7}{3}$
(B) $-\frac{7}{9}$
(C) $\frac{1}{9}$
(D) 1
(E) 3

$\int_0^1 9x^2 - 12x + 4 \, dx$

$= 3x^3 - 6x^2 + 4x\Big|_0^1$

$= 3 - 6 + 4 - 0 = 1$

or $u = 3x - 2$, $du = 3dx$, $\frac{du}{3} = dx$

$\frac{1}{3}\int_{u=-2}^{u=1} u^2 \, du$

$\frac{1}{3} \cdot \frac{1}{3} u^3 \Big|_{-2}^{1} = \frac{1}{9}(1) - \frac{1}{9}(-8) = \frac{9}{9} = 1$

18. $\int_2^3 \frac{x}{x^2+1} dx =$

(A) $\frac{1}{2} \ln \frac{3}{2}$
(B) $\frac{1}{2} \ln 2$
(C) $\ln 2$
(D) $2\ln 2$
(E) $\frac{1}{2} \ln 5$

$u = x^2 + 1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\frac{1}{2}\int_{u=5}^{u=10} \frac{1}{u} du$

$= \frac{1}{2} \ln|u| \Big|_5^{10}$

$= \frac{1}{2}\ln|10| - \frac{1}{2}\ln|5|$

$= \frac{1}{2}\ln\left|\frac{10}{5}\right| = \frac{1}{2}\ln 2$

19. $\int_1^2 x^{-3} dx =$

(A) $-\frac{7}{8}$
(B) $-\frac{3}{4}$
(C) $\frac{15}{64}$
(D) $\frac{3}{8}$
(E) $\frac{15}{1}$

$\frac{x^{-2}}{-2}\Big|_1^2 = \frac{1}{-2x^2}\Big|_1^2$

$= -\frac{1}{8} + \frac{1}{2}$

$= -\frac{1}{8} + \frac{4}{8} = \frac{3}{8}$

20. Which of the following is equal to ln 4 ?

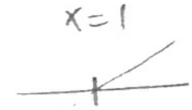
(A) $\ln 3 + \ln 1$ X
(B) $\frac{\ln 8}{\ln 2}$ X
(C) $\int_1^4 e^t \, dt = e^4 - e^1$ X
(D) $\int_1^4 \ln x \, dx$ X
(E) $\int_1^4 \frac{1}{t} dt = \ln x \Big|_1^4 = \ln 4 - \ln 1$ ✓ (0)

21. $\int_0^3 |x-1| dx =$

(A) 0
(B) $\frac{3}{2}$
(C) 2
(D) $\frac{5}{2}$
(E) 6

$x - 1 = 0$

$x = 1$



$= -\int_0^1 x - 1 \, dx + \int_1^3 x - 1 \, dx$

$-\frac{x^2}{2} + x\Big|_0^1 + \frac{x^2}{2} - x\Big|_1^3$

$-\frac{1}{2} + 1 + \frac{3^2}{2} - 3 - \left(\frac{1}{2} - 1\right) = \frac{5}{2}$

$= \frac{1}{2} + \frac{9}{2} + \frac{1}{2}$

22. $\int \tan(2x) =$

(A) $-2\ln|\cos(2x)| + C$
(B) $-\frac{1}{2}\ln|\cos(2x)| + C$
(C) $\frac{1}{2}\ln|\cos(2x)| + C$
(D) $2\ln|\cos(2x)| + C$
(E) $\frac{1}{2}\sec(2x)\tan(2x) + C$

$\int \frac{\sin 2x}{\cos 2x} du$

$u = \cos 2x$

$\frac{du}{dx} = -\sin(2x) \cdot 2$

$\frac{du}{-2} = \sin(2x)dx$

$\frac{-1}{2}\int \frac{1}{u} du$

$-\frac{1}{2}\ln|u| + c$

23. $\frac{d}{dx}\int_2^x \sqrt{1+t^2} \, dt =$

(A) $\frac{x}{\sqrt{1+x^2}}$
(B) $\sqrt{1+x^2} - 5$
(C) $\sqrt{1+x^2}$
(D) $\frac{x}{\sqrt{1+x^2}} - \frac{1}{\sqrt{5}}$
(E) $\frac{x}{2\sqrt{1+x^2}} - \frac{1}{2\sqrt{5}}$

$\sqrt{1+x^2}$

AP Calculus

Multiple Choice Integrals

Name____________

Period______ Date________

24. $\frac{d}{dx}\int_2^x \sqrt{1+t^2}\,dt =$ $\sqrt{1+x^2}$
(A) $\frac{x}{\sqrt{1+x^2}}$
(B) $\sqrt{1+x^2}-5$
(C) $\sqrt{1+x^2}$
(D) $\frac{x}{\sqrt{1+x^2}} - \frac{1}{\sqrt{5}}$
(E) $\frac{x}{2\sqrt{1+x^2}} - \frac{1}{2\sqrt{5}}$

25. $\int \frac{5}{1+x^2}\,dx =$ $5\tan^{-1}x + C$
(A) $\frac{-10x}{(1+x^2)^2} + C$
(B) $\frac{5}{2x}\ln(1+x^2) + C$
(C) $5x - \frac{5}{x} + C$
(D) $5\arctan x + C$
(E) $5\ln(1+x^2) + C$

26. $\int_0^8 \frac{dx}{\sqrt{1+x}} =$
(A) 1
(B) $\frac{3}{2}$
(C) 2
(D) 4
(E) 6

$u = 1+x$

$du = dx$

$\int_{u=1}^{u=9} \frac{1}{\sqrt{u}}\,du = \int u^{-1/2}\,du$

$= 2u^{1/2}\Big|_1^9 = 2\sqrt{9} - 2\sqrt{1}$

$= 6 - 2 = 4$

27. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x}\,dx =$
(A) $\ln\sqrt{2}$
(B) $\ln\frac{\pi}{4}$
(C) $\ln\sqrt{3}$
(D) $\ln\frac{\sqrt{3}}{2}$
(E) $\ln e$

$u = \sin x$

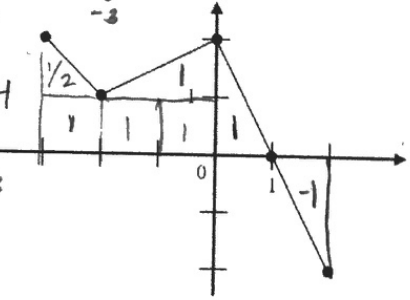
$du = \cos x$

$\int_{u=\frac{\sqrt{2}}{2}}^{u=1} \frac{1}{u}\,du = \ln|u|\Big|_{\frac{\sqrt{2}}{2}}^{1}$

$= \ln|1| - \ln\left|\frac{\sqrt{2}}{2}\right|$

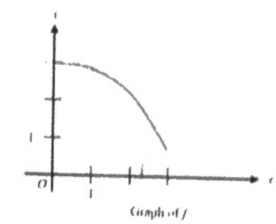
$= -\ln\left|\frac{\sqrt{2}}{2}\right| = \ln\left|\frac{\sqrt{2}}{2}\right|^{-1} = \ln\sqrt{2}$

28. The graph of the piecewise linear function f is shown in the figure. If $g(x) = \int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?
(A) $g(-3)$ $= \int_{-2}^{-3} f(t)\,dt = -\int_{-3}^{-2} f(t)\,dt = 1.5$
(B) $g(-2)$ $= 0$
(C) $g(0)$ $= \int_{-2}^{0} f(t) = 3$
(D) $g(1)$ $= \int_{-2}^{1} f(t) = 4$
(E) $g(2)$ $= \int_{-2}^{2} f(t) = 3$



Graph of f

29. The graph of function f is shown for $0 \le x \le 3$. Of the following, which has the least value?



(A) $\int_1^3 f(x)\,dx$
(B) Left Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length — over approx
(C) Right Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length — under approx
(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length
(E) Trapezoidal Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

30. Given $f(x) = \begin{cases} x+1 & \text{for } x < 0, \\ \cos\pi x & \text{for } x \ge 0, \end{cases}$ $\int_{-1}^{1} f(x)\,dx =$
(A) $\frac{1}{2} + \frac{1}{\pi}$
(B) $-\frac{1}{2}$
(C) $\frac{1}{2} - \frac{1}{\pi}$
(D) $\frac{1}{2}$
(E) $-\frac{1}{2} + \pi$

$\int_{-1}^{0} x+1\,dx + \int_{0}^{1} \cos\pi x\,dx$

$u = \pi x$, $du = \pi dx$, $\frac{du}{\pi} = dx$

$\frac{x^2}{2} + x\Big|_{-1}^{0} + \frac{1}{\pi}\int_{u=0}^{\pi} \cos u\,du$

$0 + 0 - \left(\frac{1}{2} - 1\right) + \frac{1}{\pi}\sin u\Big|_0^{\pi}$

$\frac{1}{2} + \frac{1}{\pi}(\sin\pi - \sin 0)$

$= \frac{1}{2}$

31. If $\int_{-5}^{2} f(x)\,dx = -17$ and $\int_{5}^{2} f(x)\,dx = -4$, what is the value of $\int_{-5}^{5} f(x)\,dx$?
(A) -21
(B) -13
(C) 0
(D) 13
(E) 21

$\int_5^2 = -\int_2^5$

$\int_{-5}^{2} f(x)\,dx - \int_{2}^{5} f(x)\,dx$

$-17 - (-4) = -13$