AP Calculus

Product & Quotient Rules

1-10: Differentiate

Name________________ Pd.___

1. $f(x)=(x^2+2x)e^x$

$f'(x)=(x^2+2x)e^x+e^x(2x+2)$

$f'(x)=e^x(x^2+2x+2x+2)$

$f'(x)=e^x(x^2+4x+2)$

2. $g(x)=\sqrt{x}e^{x} = x^{1/2}\cdot e^x$

$g'(x)=x^{1/2}e^x+e^x\left(\frac{1}{2}x^{-1/2}\right)$

$g'(x)=e^x\left(\frac{2x}{2x^{1/2}}+\frac{1}{2x^{1/2}}\right)$

$g'(x)=e^x\left(\frac{2x+1}{2x^{1/2}}\right)$

3. $g(x)=\dfrac{1+2x}{3-4x}$

$g'(x)=\dfrac{(3-4x)(2)-(1+2x)(-4)}{(3-4x)^2}$

$g'(x)=\dfrac{6\cancel{-8x}+4\cancel{+8x}}{(3-4x)^2}=\dfrac{10}{(3-4x)^2}$

4. $G(x)=\dfrac{x^2-2}{2x+1}$

$G'(x)=\dfrac{(2x+1)(2x)-(x^2-2)(2)}{(2x+1)^2}$

$G'(x)=\dfrac{4x^2+2x-2x^2+4}{(2x+1)^2}=\dfrac{2x^2+2x+4}{(2x+1)^2}$

5. $H(u)=\left(u-\sqrt{u}\right)\left(u+\sqrt{u}\right)$ conjugates

$H(u)=u^2-u$

$H'(u)=2u-1$

6. $F(y)=\left(\dfrac{1}{y^2}-\dfrac{3}{y^4}\right)\left(y+5y^3\right)=(y^{-2}-3y^{-4})(y+5y^3)$

$F(y)=y^{-1}+5y-3y^{-3}-15y^{-1}$

$F(y)=-3y^{-3}-14y^{-1}+5y$

$F'(y)=9y^{-4}+14y^{-2}+5$

7. $y=\dfrac{t^2+2}{t^4-3t^2+1}$ $=\dfrac{-2t^5-8t^3+14t}{(t^4-3t^2+1)^2}$

$y'=\dfrac{(t^4-3t^2+1)(2t)-(t^2+2)(4t^3-6t)}{(t^4-3t^2+1)^2}$

$=\dfrac{2t^5\cancel{-6t^3}+2t-4t^5\cancel{+6t^3}-8t^3+12t}{(t^4-3t^2+1)^2}$

8. $y=e^p\left(p+p\sqrt{p}\right)=e^p(p+p\cdot p^{1/2})$

$y=e^p(p+p^{3/2})$

$y'=e^p(1+\frac{3}{2}p^{+1/2})+(p+p^{3/2})e^p$

$y'=e^p(p^{3/2}+p+\frac{3}{2}p^{1/2}+1)$

9. $g(t)=\dfrac{t-\sqrt{t}}{t^{\frac{1}{3}}}=\dfrac{t^1}{t^{1/3}}-\dfrac{t^{1/2}}{t^{1/3}}=\dfrac{t^{3/3}}{t^{1/3}}-\dfrac{t^{3/6}}{t^{2/6}}$

$g(t)=t^{2/3}-t^{1/6}$

$g'(t)=\frac{2}{3}t^{-1/3}-\frac{1}{6}t^{-5/6}$

10. $y=\dfrac{e^x}{1-e^x}$ $y'=\dfrac{(1-e^x)e^x-e^x(-e^x)}{(1-e^x)^2}$

$=\dfrac{e^x(1\cancel{-e^x}\cancel{+e^x})}{(1-e^x)^2}$

$=\dfrac{e^x}{(1-e^x)^2}$

11-12: Find $f'(x)$ and $f''(x)$.

11. $f(x) = x^{\frac{5}{2}} e^x$

$f'(x) = x^{5/2} e^x + e^x \left(\frac{5}{2} x^{3/2}\right)$

$f'(x) = e^x \left(x^{5/2} + \frac{5}{2} x^{3/2}\right)$

$f''(x) = e^x \left(\frac{5}{2} x^{3/2} + \frac{15}{4} x^{1/2}\right) + \left(x^{5/2} + \frac{5}{2} x^{3/2}\right) e^x$

$f''(x) = e^x \left(\frac{5}{2} x^{3/2} + \frac{15}{4} x^{1/2} + x^{5/2} + \frac{5}{2} x^{3/2}\right)$

$f''(x) = e^x \left(x^{5/2} + \frac{10}{2} x^{3/2} + \frac{15}{4} x^{1/2}\right)$

12. $f(x) = \dfrac{2}{x-3}$ $\quad f'(x) = \dfrac{(x-3)(0) - 2(1)}{(x-3)^2}$

$f'(x) = \dfrac{-2}{(x-3)^2} = \dfrac{-2}{x^2 - 6x + 9}$

$f''(x) = \dfrac{(x^2-6x+9)(0) - (-2)(2x-6)}{(x^2-6x+9)^2}$

$f''(x) = \dfrac{4(2x-6)}{(x^2-6x+9)^2}$

13. If $f(x) = \dfrac{x^2}{1+x}$, find $f''(1)$

$f'(x) = \dfrac{(1+x)(2x) - x^2(1)}{(1+x)^2}$

$f'(x) = \dfrac{2x + 2x^2 - x^2}{(1+x)^2}$

$f'(x) = \dfrac{x^2 + 2x}{1 + 2x + x^2}$

$f''(x) = \dfrac{(1+2x+x^2)(2x+2) - (x^2+2x)(2+2x)}{(1+2x+x^2)^2}$

$f''(1) = \dfrac{(1+2+1)(2+2) - (1+2)(2+2)}{(1+2+1)^2}$

$f''(1) = \dfrac{(4)(4) - (3)(4)}{(4)^2} = \dfrac{16-12}{16} = \dfrac{4}{16} = \dfrac{1}{4}$

14-17: Suppose that $f(2) = -3$, $g(2) = 4$, $f'(2) = -2$, and $g'(2) = 7$. Find $h'(2)$ for each.

14. $h(x) = f(x)g(x)$

$h'(x) = f(x)g'(x) + g(x)f'(x)$

$h'(2) = f(2)g'(2) + g(2)f'(2)$

$(-3)(7) + (4)(-2)$

$-21 - 8 = \boxed{-29}$

15. $h(x) = \dfrac{f(x)}{g(x)}$ $\quad h'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

$h'(2) = \dfrac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2}$

$\dfrac{4(-2) - (-3)(7)}{4^2} = \dfrac{-8+21}{16} = \boxed{\dfrac{13}{16}}$

16. $h(x) = \dfrac{g(x)}{f(x)}$

$h'(x) = \dfrac{f(x)g'(x) - g(x)f'(x)}{[f(x)]^2}$

$h'(2) = \dfrac{f(2)g'(2) - g(2)f'(2)}{[f(2)]^2}$

$h'(2) = \dfrac{-3(7) - (4)(-2)}{(-3)^2} = \dfrac{-21+8}{9} = \boxed{\dfrac{-13}{9}}$

17. $h(x) = \dfrac{g(x)}{1+f(x)}$

$h'(x) = \dfrac{[1+f(x)]g'(x) - g(x)[f'(x)]}{[1+f(x)]^2}$

$h'(2) = \dfrac{[1+f(2)]g'(2) - g(2)[f'(2)]}{[1+f(2)]^2}$

$h'(2) = \dfrac{[1-3](7) - (4)(-2)}{[1+-3]^2} = \dfrac{(-2)(7)+8}{(-2)^2} = \dfrac{-6}{4} = \boxed{-\dfrac{3}{2}}$

18. If $g(x) = xf(x)$, where $f(3) = 4$ and $f'(3) = -2$, find an equation of the tangent line to the graph of g at the point where $x = 3$.

$g(x) = x f(x)$

$g'(x) = x f'(x) + f(x)(1)$

$g'(3) = 3f'(3) + f(3)$

$g'(3) = 3(-2) + 4$

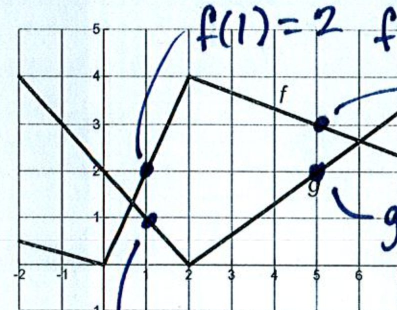
$g'(3) = -2$

Point: $(3, 4)$

Slope: $g'(3) = -2$

$y - 4 = -2(x - 3)$

19. If f and g are the functions whose graphs are shown, let $u(x) = f(x)g(x)$ and $w(x) = \dfrac{f(x)}{g(x)}$.



$f(1) = 2$ $f'(1) = 2$

$f(5) = 3$ $f'(5) = -\frac{1}{3}$

$g(5) = 2$

$g'(5) = 2/3$

$g(1) = 1$

$g'(1) = \text{slope} = -1$

A.) Find $u'(1)$. $u(x) = f(x) \cdot g(x)$

$u'(x) = f(x)g'(x) + g(x)f'(x)$

$u'(1) = f(1)g'(1) + g(1)f'(1)$

$(2)(-1) + (1)(2)$

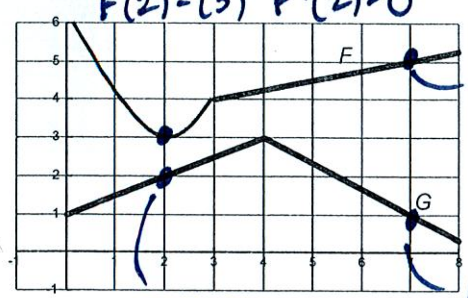
$-2 + 2 = \boxed{0}$

B.) Find $w'(5)$.

$w(x) = \dfrac{f(x)}{g(x)}$ $\quad w'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

$w'(5) = \dfrac{g(5)f'(5) - f(5)g'(5)}{[g(5)]^2} = \dfrac{2(-\frac{1}{3}) - 3(\frac{2}{3})}{(2)^2} = \dfrac{-\frac{2}{3} - \frac{6}{3}}{4} = -\frac{8}{3} \cdot \frac{1}{4} = \boxed{-\frac{2}{3}}$

20. Let $P(x) = F(x)G(x)$ and $Q(x) = \dfrac{F(x)}{G(x)}$, where F and G are the functions whose graphs are shown.



$F(2) = (3)$ $F'(2) = 0$

$F(7) = 5$

$F'(7) = \frac{1}{4}$

$G(2) = 2$

$G'(2) = 2$

$G(7) = 1$

$G'(7) = -\frac{2}{3}$

A.) Find $P'(2)$.

$P(x) = F(x)G(x)$

$P'(x) = F(x)G'(x) + G(x)F'(x)$

$P'(2) = F(2)G'(2) + G(2)F'(2)$

$P'(2) = 3(2) + (2)(0) = \boxed{6}$

B.) Find $Q'(7)$.

$Q(x) = \dfrac{G(x)F'(x) - F(x)G'(x)}{[G(x)]^2}$

$Q(7) = \dfrac{G(7)F'(7) - F(7)G'(7)}{[G(7)]^2} = \dfrac{(1)(\frac{1}{4}) - 5(-\frac{2}{3})}{[1]^2} = \frac{1}{4} + \frac{10}{3}$

$= \frac{3}{12} + \frac{40}{12} = \frac{43}{12}$