AP Calculus

Optimization

Name________________________Pd.___

Day 6 Application of Derivatives

1. Find two numbers whose difference is 100 and whose product is a minimum.

$x - y = 100$

$y = x - 100$

$x \cdot y = \text{Product}$

$x(x-100) = \text{Product}$

$x^2 - 100x = \text{Product}$

$2x - 100 = \text{Product}'$

$2x - 100 = 0$

$2x = 100$

$x = 50$

2. Find two positive numbers whose product is 100 and whose sum is a minimum.

$x \cdot y = 100$

$y = \frac{100}{x}$

$x + y = \text{sum}$

$x + 100x^{-1} = \text{sum}$

$1 - 100x^{-2} = \text{sum}'$

$0 = 1 - \frac{100}{x^2}$

$\frac{100}{x^2} = 1$

$x^2 = 100$ $\boxed{x = 10 \quad y = 10}$

3. The sum of two positive numbers is 16. What is the smallest possible value of the sum of their squares?

$x + y = 16$

$y = 16 - x$

$x^2 + y^2 = \text{sum}$

$x^2 + (16-x)^2 = \text{sum}$

$2x + 2(16-x)^1(-1) = \text{sum}'$

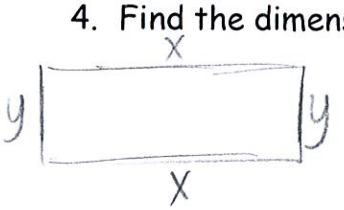
$2x - 32 + 2x = 0$

$4x = 32$

$\boxed{x = 8}$

$\boxed{y = 8}$

4. Find the dimensions of a rectangle with perimeter 100 m whose area is as large as possible.



$2x + 2y = 100$

$2y = 100 - 2x$

$y = 50 - x$

$x \cdot y = \text{Area}$

$x(50-x) = \text{Area}$

$50x - x^2 = \text{Area}$

$50 - 2x = \text{Area}'$

$50 = 2x$

$x = 25$

$y = 25$

25

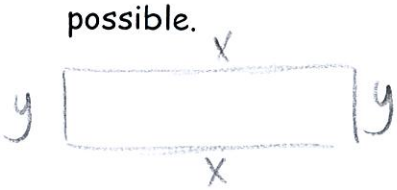
$+ \mid -$

$A' = 50 - 2x$

$A'(24) = +$

$A'(26) = -$

5. Find the dimensions of a rectangle with area 1000 m^3 whose perimeter is as small as possible.



$1000 = xy$

$y = \frac{1000}{x} = 1000x^{-1}$

$2x + 2y = \text{Perimeter}$

$2x + 2(1000x^{-1}) = \text{Perimeter}$

$2 - 2000x^{-2} = \text{Perimeter}'$

$2 = \frac{2000}{x^2}$

$2x^2 = 2000$

$x^2 = 1000$ $x =$

$C'(1)$ $\qquad$ $C(1501)$

$- \searrow \quad 1500 \quad \nearrow +$

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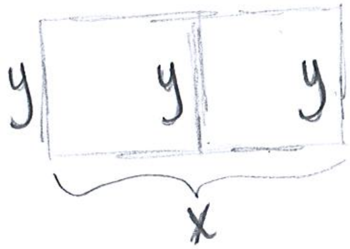
6. A farmer wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the cost of the fence?



$A = 1{,}500{,}000 = \cancel{x}y$

$\dfrac{1{,}500{,}000}{x} = y$

$Cost = 2x + 3y$

$Cost = 2x + 3(1{,}500{,}000)x^{-1}$

$Cost = 2x + 4{,}500{,}000x^{-1}$

$Cost' = 2 - 4{,}500{,}000x^{-2}$

$0 = 2 - 4{,}500{,}000x^{-2}$

$\dfrac{4{,}500{,}000}{x^2} = 2$

$2x^2 = 4{,}500{,}000$

$\sqrt{x^2} = \sqrt{2{,}250{,}000}$

$\boxed{x = 1500 \quad y = 1000}$

7. Find the point on the line $y = 2x + 3$ that is closest to the origin.

$\boxed{1}$ $(0,0)$ $\boxed{2}$ $(x, 2x+3)$

$d = \sqrt{(x-0)^2 + (2x+3-0)^2}$

$d = \sqrt{x^2 + 4x^2 + 12x + 9}$

$d = \sqrt{5x^2 + 12x + 9}$

$d = (5x^2 + 12x + 9)^{1/2}$

$d' = \frac{1}{2}(5x^2 + 12x + 9)^{-1/2}(10x + 12)$

$0 = \dfrac{10x + 12}{2\sqrt{5x^2 + 12x + 9}}$

$10x + 12 = 0$

$10x = -12$

$x = -6/5$

$\searrow \; -6/5 \; \nearrow +$

$y = 2x + 3$

$y(-6/5) = 2\left(-\frac{6}{5}\right) + 3$

$y = -\frac{12}{5} + \frac{15}{5}$

$y = 3/5$

$\boxed{\left(-\frac{6}{5}, \frac{3}{5}\right)}$

8. Find the point on the curve $y = \sqrt{x}$ that is closest to the point (3,0). $(x, \sqrt{x})$ OR (y^2, y)

$\boxed{1}$ $(0,0)$ $\boxed{2}$ $(x, \sqrt{x})$

$d = \sqrt{(x-0)^2 + (\sqrt{x} - 0)^2}$

$d = (x^2 + x)^{1/2}$

$d' = \frac{1}{2}(x^2 + x)^{-1/2}(2x + 1)$

$0 = \dfrac{2x + 1}{2\sqrt{x^2 - x}}$

$0 = 2x + 1$

$2x = -1$

$x = -\frac{1}{2}$

$\searrow \; -1/2 \; + \nearrow$

$\boxed{\left(\frac{1}{2}, \sqrt{1/2}\right)}$

9. Find the points on the ellipse $4x^2 + y^2 = 4$ that are furthest away from the point (1,0).

2 points $(1,0)$ $(x, \pm\sqrt{4 - 4x^2})$ $\quad y = \pm\sqrt{4 - 4x^2}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(x-1)^2 + (\pm\sqrt{4 - 4x^2} - 0)^2}$

$d = \sqrt{x^2 - 2x + 1 + 4 - 4x^2}$

$d = (-3x^2 - 2x + 5)^{1/2}$

$d' = \frac{1}{2}(-3x^2 - 2x + 5)^{-1/2}(-6x - 2)$

$0 = \dfrac{-6x - 2}{2\sqrt{-3x^2 - 2x + 5}}$

$0 = -6x - 2$

$6x = -2$

$x = -1/3$

$-1/3$