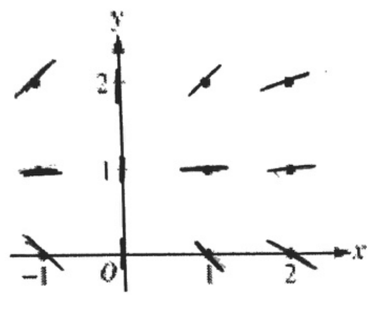
ame

Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x\to\infty} f(x)$.

b. $\frac{dy}{y-1} = \frac{dx}{x^2}$

$\int \frac{dy}{y-1} = \int x^{-2}\,dx$

$\ln|y-1| = -x^{-1} + c$ (2,0)

$\ln|0-1| = -2^{-1} + c$

$0 = -\frac{1}{2} + c \quad c = \frac{1}{2}$

$|y-1| = e^{-\frac{1}{x}+\frac{1}{2}}$

$y = 1 - e^{1/2}e^{-1/x}$

choose minus b/c of point

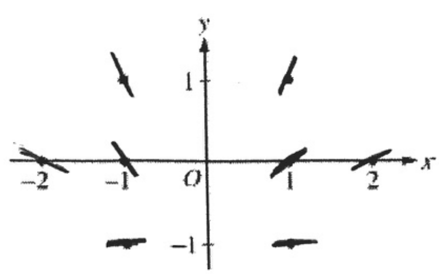
c. $\lim_{x\to\infty} 1 - e^{1/2}e^{-1/x}$

$= \lim_{x\to\infty} 1 - \frac{e^{1/2}}{e^{1/x}} = -e^{1/2} + 1$

2.

Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

$\frac{dy}{dx} = \frac{1+y}{x}$

$\int \frac{dy}{1+y} = \int \frac{dx}{x}$

$\ln|1+y| = \ln|x| + c$ (-1,1)

$\ln|2| = \ln|1| + c$

$c = \ln 2$

$e^{\ln|1+y|} = e^{\ln|x| + \ln 2}$

$|1+y| = e^{\ln|x| + \ln 2}$

$|1+y| = e^{\ln|x|} \cdot e^{\ln 2}$

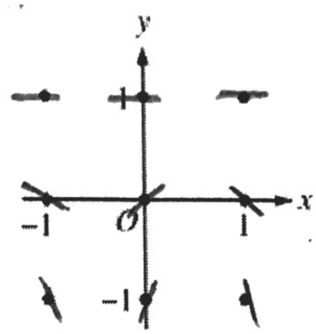
$|1+y| = 2|x|$

$y = -1 + 2|x|$

3.

Consider the differential equation $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.



(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

b. $y = 1$

c. $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$

$\int \frac{dy}{(y-1)^2} = \int \cos(\pi x)\,dx$ $\quad u = \pi x$, $du = \pi dx$, $\frac{du}{\pi} = dx$

$-(y-1)^{-1} = \frac{1}{\pi}\int \cos u\,du$

$-(y-1)^{-1} = \frac{\sin(\pi x)}{\pi} + C$

$\frac{-1}{y-1} = \frac{\sin(\pi x)}{\pi} + C$ $\quad (1,0)$

$1 = 0 + C \quad C = 1$

$\frac{-1}{y-1} = \frac{\sin(\pi x)}{\pi} + \ldots$

$\frac{-\pi}{y-1} = \sin(\pi x) + \ldots$

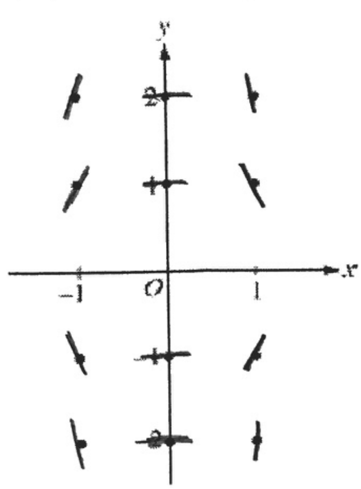
$\frac{y-1}{-\pi} = \frac{1}{\sin(\pi x) + \pi}$

$y = 1 - \frac{\pi}{\sin(\pi x) + \ldots}$

4.

Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.

b Slope @ $(1,-1) = 2$

$y + 1 = 2(x-1)$ $\quad$ when $x = 1.1$

$y + 1 = 2(1.1 - 1)$

$y + 1 = 2(.1)$

$y = -.8$

c. $\frac{dy}{dx} = \frac{-2x}{y}$

$\int y\,dy = \int -2x\,dx$

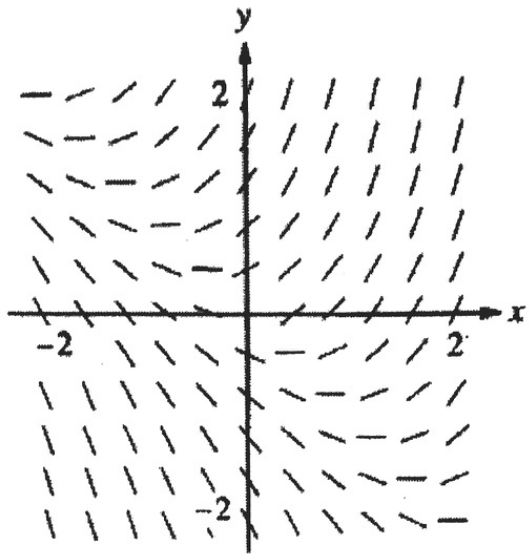
$\frac{y^2}{2} = -x^2 + C \quad (1,-1)$

$C = \frac{3}{2}$

$\frac{y^2}{2} = -x^2 + \frac{3}{2}$

$y^2 = -2x^2 + 3$

$y = -\sqrt{-2x^2 + 3}$



Shown above is a slope field for which of the following differential equations?

~~(A)~~ $\frac{dy}{dx}=1+x$ ~~(B)~~ $\frac{dy}{dx}=x^2$ (C) $\frac{dy}{dx}=x+y$ (D) $\frac{dy}{dx}=\frac{x}{y}$ ~~(E)~~ $\frac{dy}{dx}=\ln y$

8.

Which of the following is the solution to the differential equation $\frac{dy}{dx}=\frac{x^2}{y}$ with the initial condition $y(3)=-2$?

(A) $y=2e^{-9+x^3/3}$

(B) $y=-2e^{-9+x^3/3}$

(C) $y=\sqrt{\frac{2x^3}{3}}$

(D) $y=\sqrt{\frac{2x^3}{3}-14}$

(E) $y=-\sqrt{\frac{2x^3}{3}-14}$

$\int y\,dy=\int x^2\,dx$

$\frac{y^2}{2}=\frac{x^3}{3}+C$ (3, -2)

$\frac{4}{2}=\frac{27}{3}+C$

$2=9+C$

$C=-7$

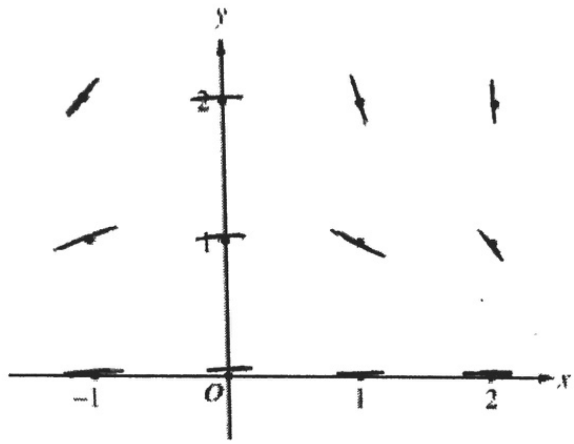
$\frac{y^2}{2}=\frac{x^3}{3}-7$

$y^2=\frac{2x^3}{3}-14$

$y=-\sqrt{\frac{2x^3}{3}-14}$

onsider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential quation with the initial condition $f(-1) = 2$.

a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) Write an equation for the line tangent to the graph of f at $x = -1$.

(c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

b. $(-1, 2)$

slope $= 2$

$y - 2 = 2(x+1)$

c. $\frac{dy}{dx} = \frac{-xy^2}{2}$

$\int \frac{dy}{y^2} = \int \frac{-x}{2}\,dx$

$-y^{-1} = \frac{-x^2}{4} + C \quad (-1, 2)$

$-\frac{1}{2} = -\frac{1}{4} + C \quad C = -\frac{1}{4}$

$-y^{-1} = -\frac{x^2}{4} - \frac{1}{4}$

$-\frac{1}{y} = -\frac{x^2 - 1}{4}$

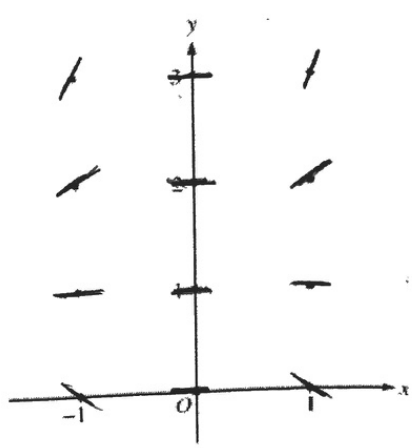
$-y = \frac{-4}{x^2 - 1}$

$y = \frac{4}{x^2 - 1}$

6.

Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

b. when $y > 1$ and $x \neq 0$

c. $\frac{dy}{dx} = x^2(y-1)$

$\int \frac{dy}{y-1} = \int x^2\,dx$

$\ln|y-1| = \frac{x^3}{3} + C \quad (0, 3)$

$\ln|2| = 0 + C$

$C = \ln|2|$

$\ln|y-1| = \frac{x^3}{3} + \ln 2$

$|y-1| = e^{x^3/3 + \ln 2}$

$|y-1| = e^{x^3/3} \cdot e^{\ln 2}$

$|y-1| = 2e^{x^3/3}$

$y = 1 + 2e^{x^3/3}$