AP Calculus

Integration by U-Substitution

Name Key

Integration Day 8

Evaluate the indefinite integral.

1. $\int x\sin(x^2)\,dx$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$= \int \sin u \cdot \frac{du}{2}$

$\frac{1}{2}\int \sin u\,du$

$-\frac{1}{2}\cos u + C = \boxed{-\frac{1}{2}\cos(x^2) + C}$

2. $\int x^2 e^{x^3}\,dx$

$u = x^3$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$\int e^u \cdot \frac{du}{3}$

$= \frac{1}{3}\int e^u\,du = \frac{1}{3}e^u + C = \boxed{\frac{1}{3}e^{x^3} + C}$

3. $\int (1-2x)^9\,dx$

$u = 1-2x$

$du = -2\,dx$

$\frac{du}{-2} = dx$

$= \int u^9 \cdot \frac{du}{-2}$

$-\frac{1}{2}\int u^9\,du = -\frac{1}{2}\cdot\frac{u^{10}}{10} + C = \frac{-u^{10}}{20} + C$

$= \boxed{-\frac{(1-2x)^{10}}{20} + C}$

4. $\int (3t+2)^{2.4}\,dt$

$u = 3t+2$

$du = 3\,dt$

$\frac{du}{3} = dt$

$\int u^{2.4} \cdot \frac{du}{3}$

$= \frac{1}{3}\int u^{2.4}\,du = \frac{1}{3}\frac{u^{3.4}}{3.4} + C$

$= \frac{u^{3.4}}{10.2} + C = \boxed{\frac{(3t+2)^{3.4}}{10.2} + C}$

5. $\int (x+1)\sqrt{2x+x^2}\,dx$

$u = 2x + x^2$

$du = (2+2x)\,dx$

$\frac{du}{2} = (1+x)\,dx$

$= \int \sqrt{u} \cdot \frac{du}{2}$

$= \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$

$= \boxed{\frac{1}{3}(2x+x^2)^{3/2} + C}$

6. $\int \sec^2 2\theta\,d\theta$

$u = 2\theta$

$du = 2\,d\theta$

$\frac{du}{2} = d\theta$

$\int \sec^2 u \cdot \frac{du}{2}$

$= \frac{1}{2}\int \sec^2 u\,du$

$= \frac{1}{2}\tan u + C = \boxed{\frac{1}{2}\tan(2\theta) + C}$

7. $\int \sin \pi t\,dt$

$u = \pi t$

$du = \pi\,dt$

$\frac{du}{\pi} = dt$

$\int \sin u \cdot \frac{du}{\pi}$

$= \frac{1}{\pi}\int \sin u\,du$

$= -\frac{1}{\pi}\cos u + C = \boxed{-\frac{1}{\pi}\cos(\pi t) + C}$

8. $\int e^x \cos(e^x)\,dx$

$u = e^x$

$du = e^x\,dx$

$\int \cos u\,du$

$= \sin u + C = \boxed{\sin(e^x) + C}$

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9. $\int \cos^4\theta \sin\theta \, d\theta$

$u = \cos\theta$

$du = -\sin\theta \, d\theta$

$-du = \sin\theta \, d\theta$

$= \int (\cos\theta)^4 \sin\theta \, d\theta$

$= \int u^4 (-du) = -\int u^4 du = -\frac{u^5}{5} + C$

$= \boxed{-\frac{(\cos\theta)^5}{5} + C}$

10. $\int \sec^2\theta \tan^3\theta \, d\theta$

$u = \tan\theta$

$du = \sec^2\theta \, d\theta$

$\int u^3 du = \frac{u^4}{4} + C = \boxed{\frac{(\tan\theta)^4}{4} + C}$

11. $\int \sqrt{x} \sin\left(1 + x^{\frac{3}{2}}\right) dx$

$u = 1 + x^{3/2}$

$du = \frac{3}{2} x^{1/2} dx$

$\frac{2}{3} du = x^{1/2} dx$

$= \int x^{1/2} \sin(1 + x^{3/2}) dx$

$= \int \sin u \cdot \frac{2}{3} du$

$= \frac{2}{3} \int \sin u \, du = -\frac{2}{3} \cos u + C$

$= \boxed{-\frac{2}{3} \cos(1 + x^{3/2}) + C}$

12. $\int e^x \sqrt{1 + e^x} \, dx$

$u = 1 + e^x$

$du = e^x dx$

$\int \sqrt{u} \, du = \int u^{1/2} du$

$= \frac{2}{3} u^{3/2} + C$

$= \boxed{\frac{2}{3} (1 + e^x)^{3/2} + C}$

13. $\int (x^2 + 1)(x^3 + 3x)^4 dx$

$u = x^3 + 3x$

$du = (3x^2 + 3) dx$

$\frac{du}{3} = (x^2 + 1) dx$

$= \int u^4 \cdot \frac{du}{3}$

$= \frac{1}{3} \int u^4 du = \frac{1}{3} \cdot \frac{u^5}{5} + C = \frac{u^5}{15} + C$

$= \boxed{\frac{(x^3 + 3x)^5}{15} + C}$

14. $\int e^{\cos t} \sin t \, dt$

$u = \cos t$

$du = -\sin t \, dt$

$-du = \sin t \, dt$

$\int e^u (-du)$

$= -\int e^u du = -e^u + C = \boxed{-e^{\cos t} + C}$

15. $\int 5^t \sin(5^t) dt$

$u = 5^t$

$du = 5^t \ln 5 \, dt$

$\frac{du}{\ln 5} = 5^t dt$

$\int \sin u \cdot \frac{du}{\ln 5}$

$= \frac{1}{\ln 5} \int \sin u \, du$

$= \frac{1}{\ln 5} (-\cos u) + C = \boxed{\frac{-\cos(5^t)}{\ln 5} + C}$

16. $\int x^2 \sqrt{x^3 + 1} \, dx$

$u = x^3 + 1$

$du = 3x^2 dx$

$\frac{du}{3} = x^2 dx$

$\int \sqrt{u} \cdot \frac{du}{3}$

$= \frac{1}{3} \int u^{1/2} du = \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C$

$= \boxed{\frac{2}{9} (x^3 + 1)^{3/2} + C}$