AP Calculus

Differential Equations

1-7: Solve the differential equation.

Name________________________________

Application of Integration Day 7

1. $\frac{dy}{dx} = xy^2$

$dy = xy^2 dx$

$\frac{1}{y^2} dy = x dx$

$\int y^{-2} dy = \int x\, dx$

$\frac{y^{-1}}{-1} = \frac{x^2}{2} + C$

$\frac{-1}{y} = \frac{\frac{1}{2}x^2 + C}{1}$

$-1 = y(\frac{1}{2}x^2 + C)$

$y = \frac{-1}{\frac{1}{2}x^2 + C}$

2. $\frac{dy}{dx} = xe^{-y}$

$(\frac{1}{e^{-y}}) dy = xe^{-y} dx (\frac{1}{e^{-y}})$

$\int e^y dy = \int x dx$

$e^y = \frac{x^2}{2} + C$

$\ln e^y = \ln \frac{1}{2}x^2 + C$

$y = \ln(\frac{1}{2}x^2 + C)$

3. $xy^2y' = x + 1$

$xy^2 \frac{dy}{dx} = x + 1$

$\frac{xy^2 dy}{x} = \frac{(x+1)dx}{x}$

$y^2 dy = \frac{x}{x} + \frac{1}{x} dx$

$\int y^2 dy = \int 1 + \frac{1}{x} dx$

$\frac{y^3}{3} = x + \ln|x| + C$

$y^3 = 3x + 3\ln|x| + C$

$y = \sqrt[3]{3x + 3\ln|x| + C}$

4. $(y^2 + xy^2)y' = 1$

$y^2(1+x)\frac{dy}{dx} = 1$

$y^2(1+x)dy = dx$

$\int y^2 dy = \int \frac{1}{1+x} dx$

$u = 1+x$

$du = dx$

$\int y^2 dy = \int \frac{1}{u} du$

$\frac{y^3}{3} = \ln|1+x| + C$

$y^3 = 3\ln|1+x| + C$

$y = \sqrt[3]{3\ln|1+x| + C}$

5. $(y + \sin y)y' = x + x^3$

$(y + \sin y)\frac{dy}{dx} = x + x^3$

$\int (y + \sin y) dy = \int (x + x^3) dx$

$4\left[\frac{y^2}{2} - \cos y = \frac{x^2}{2} + \frac{x^4}{4} + C\right]$

$2y^2 - 4\cos y = 2x^2 + x^4 + C$

6. $\frac{dp}{dt} = t^2p - p + t^2 - 1$

$\frac{dp}{dt} = p(t^2-1) + 1(t^2-1)$

$\frac{dP}{dt} = (P+1)(t^2-1)$

$dP = (P+1)(t^2-1)dt$

$\int \frac{1}{P+1} dP = \int t^2 - 1\, dt$

$u = p+1 \quad du = dp$

$\int \frac{1}{u} du = \int t^2 + 1 dt$

$e^{\ln|p+1|} = e^{\frac{t^3}{3} + t + C}$

$|P+1| = e^{\frac{1}{3}t^3 + t} e^C$

$p = Ce^{\frac{1}{3}t^3 + t} - 1$

7. $\frac{dz}{dt} + e^{t+z} = 0$

$\frac{dz}{dt} = -e^{t+z}$

$\frac{dz}{dt} = -e^t \cdot e^z$

$dz = -e^t e^z dt$

$\frac{1}{e^z} dz = -e^t dt$

$\int e^{-z} dz = \int -e^t dt$

$\frac{e^{-z}}{-1} = -e^t + C$

$\ln e^{-z} = \ln(e^t + C)$

$-z = \ln(e^t + C)$

$z = -\ln(e^t + C)$

8-11: Find the solution of the differential equation that satisfies the given initial condition.

8. $\frac{dy}{dx} = \frac{x}{y}$, $y(0) = -3$

$y\,dy = x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + C$

$\frac{9}{2} = \frac{0}{2} + C$

$C = \frac{9}{2}$

$\frac{y^2}{2} = \frac{x^2}{2} + \frac{9}{2}$

$y^2 = x^2 + 9$

$y = \pm\sqrt{x^2+9}$

$-3 = \pm\sqrt{0^2+9}$

$\boxed{y = -\sqrt{x^2+9}}$

9. $\frac{dy}{dx} = \frac{\ln x}{xy}$, $y(1) = 2$

$\int y\,dy = \int \frac{\ln x}{x}\,dx$ $\quad u = \ln x$, $du = \frac{1}{x}dx$

$\int y\,dy = \int u\,du$

$\frac{y^2}{2} = \frac{u^2}{2} + C$

$\frac{1}{2}y^2 = \frac{1}{2}(\ln x) + C$

$\frac{1}{2}(2)^2 = \frac{1}{2}(\ln(1)) + C$

$2 = C$

$2\left(\frac{1}{2}y^2 = \frac{1}{2}\ln x + 2\right)$

$y^2 = \ln x + 4$

$y = \pm\sqrt{\ln x + 4}$

$2 = \pm\sqrt{\ln(1)+4}$

$\boxed{y = +\sqrt{\ln x + 4}}$

10. $\frac{du}{dt} = \frac{2t + \sec^2 t}{2u}$, $u(0) = -5$

$\int 2u\,du = \int 2t + \sec^2 t\,dt$

$\frac{2u^2}{2} = \frac{2t^2}{2} + \tan t + C$

$u^2 = t^2 + \tan t + C$

$25 = 0 + \tan(0) + C$

$C = 25$

$u^2 = t^2 + \tan t + 25$

$u = \pm\sqrt{t^2 + \tan t + 25}$

$-5 = \pm\sqrt{0^2 + \tan(0) + 25}$

$\boxed{u = -\sqrt{t^2 + \tan t + 25}}$

11. $\frac{dP}{dt} = \sqrt{Pt}$, $P(1) = 2$

$\frac{dP}{dt} = P^{1/2}t^{1/2}$

$\frac{1}{P^{1/2}}\,dP = t^{1/2}\,dt$

$\int P^{-1/2}\,dP = \int t^{1/2}\,dt$

$2P^{1/2} = \frac{2}{3}t^{3/2} + C$

$2\sqrt{2} = \frac{2}{3}(\sqrt{1})^3 + C$

$2\sqrt{2} = \frac{2}{3} + C$

$C = 2\sqrt{2} - \frac{2}{3}$

$\frac{1}{2}(2P^{1/2}) = \left(\frac{2}{3}t^{3/2} + 2\sqrt{2} - \frac{2}{3}\right)\frac{1}{2}$

$P^{1/2} = \frac{1}{3}t^{3/2} + \sqrt{2} - \frac{1}{3}$

$\boxed{P = \left(\frac{1}{3}t^{3/2} + \sqrt{2} - \frac{1}{3}\right)^2}$

12. Find an equation of the curve that passes through the point $(0,1)$ and whose slope at (x,y) is xy.

Slope = xy $\quad \frac{dy}{dx} = xy$

$dy = xy\,dx$

$\int \frac{1}{y}\,dy = \int x\,dx$

$\ln|y| = \frac{x^2}{2} + C \Rightarrow e^{\ln|y|} = e^{\frac{1}{2}x^2}$

$\ln|1| = \frac{0^2}{2} + C$

$C = 0$

$|y| = e^{\frac{1}{2}x^2}$

$y = \pm e^{\frac{1}{2}x^2}$

$y = \pm e^{\frac{1}{2}x^2}$

$1 = \pm e^{\frac{1}{2}(0)}$

$\boxed{y = e^{\frac{1}{2}x^2}}$

13. A.) Solve the differential equation $y' = 2x\sqrt{1-y^2}$.

B.) Solve the initial-value problem $y' = 2x\sqrt{1-y^2}$, $y(0) = 0$, and graph the solution.

C.) Does the initial-value problem $y' = 2x\sqrt{1-y^2}$, $y(0) = 2$, have a solution? Explain.

A.

$\frac{dy}{dx} = 2x\sqrt{1-y^2}$

$dy = 2x\sqrt{1-y^2}\,dx$

$\int \frac{1}{\sqrt{1-y^2}}\,dy = \int 2x\,dx$

$\sin^{-1} y = \frac{2x^2}{2} + C$

$\sin \sin^{-1} y = \sin(x^2 + C)$

$y = \sin(x^2 + C)$

B.

$\sin^{-1} y = x^2 + C$

$\sin^{-1}(0) = 0^2 + C$

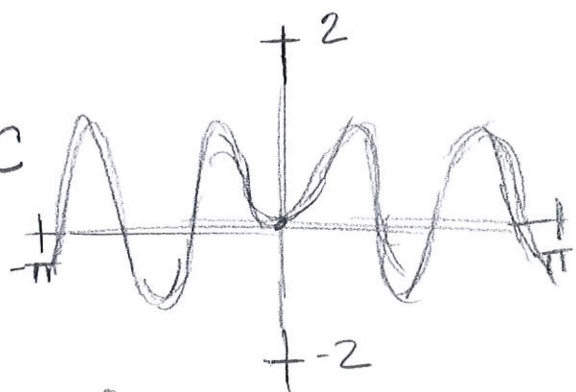
$0 = 0 + C$

$C = 0$

$\sin^{-1} y = x^2$

$\sin(\sin^{-1} y) = \sin(x^2)$

$y = \sin(x^2)$



C. No! $\sin^{-1}(2) = 0^2 + C$

↑

sin x only has answers between ±1.

2 is not in the domain