AP Calculus
Review #2 Curve Sketching

Name______________________ Pd.___
Curve Sketching Day 8

For problems 1-3, justify whether the Mean Value Theorem applies over the indicated interval. If it does apply, find the value(s) of c guaranteed in the interval (a,b).

1. $f(x)=x^{\frac{2}{3}}$ $[-1,1]$

$f'(x)=\frac{2}{3}x^{-1/3}=\frac{2}{3\sqrt[3]{x}}$ x=0 is not differentiable

So f(x) is <u>not</u> differentiable on (-1,1)

So MVT does not apply

2. $f(x)=\frac{x+1}{x}$ $\left[\frac{1}{2},2\right]$ $f'(x)=\frac{x(1)-(x+1)(1)}{x^2}=\frac{x-x-1}{x^2}=\frac{-1}{x^2}=\frac{\frac{3}{2}-3}{2-\frac{1}{2}}$

f(x) is continuous $[\frac{1}{2},2]$

f(x) is differentiable on $(\frac{1}{2},2)$

∴ MVT applies

$f(2)=\frac{3}{2}$

$f(\frac{1}{2})=\frac{\frac{1}{2}+1}{\frac{1}{2}}=\frac{3}{2}\cdot\frac{2}{1}=3$

$\frac{-1}{x^2}=\frac{-\frac{3}{2}}{\frac{3}{2}}$

$\frac{-1}{x^2}=-1$

$-x^2=-1$ $x^2=1$

$x=\pm1$

$\boxed{x=1}$

3. $f(x)=\sin x$ $[0.\pi]$

$f'(x)=\cos x$

f(x) is continuous $[0,\pi]$

f(x) is differentiable $(0,\pi)$

∴ MVT applies

$\cos x=\frac{\sin\pi-\sin(0)}{\pi-0}$

$\cos x=\frac{0}{\pi}$

$\cos x=0$

$\boxed{x=\frac{\pi}{2}}$

Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.

4. $f(x)=x^3-x^2-6x+2,$ $[0,3]$

$f'(x)=3x^2-2x-6$

f(x) is continuous $[0,3]$

f(x) is differentiable $(0,3)$

∴ Rolle's Thm applies

$f'(x)=0$

$3x^2-2x-6=0$

$(3x\quad)(x\quad)$

$y_1=3x^2-2x-6$ $x=-1.12$ $\boxed{x=1.786}$

5. $f(x)=\cos(2x),$ $\left[\frac{\pi}{8},\frac{7\pi}{8}\right]$

$f'(x)=-2\sin(2x)$

f(x) is continuous $[\frac{\pi}{8},\frac{7\pi}{8}]$

f(x) is differentiable $(\frac{\pi}{8},\frac{7\pi}{8})$

$f'(x)=0$

$-2\sin(2x)=0$

$\boxed{x=\frac{\pi}{2}}$

6. Consider the function f, whose formula and derivatives are given by

$$f(x)=\frac{x^2-4}{x+1},\quad f'(x)=\frac{x^2+2x+4}{(x+1)^2},\quad f''(x)=\frac{-6}{(x+1)^3}$$

a. Find and describe all of the vertical and horizontal asymptotes of this function, if any. Justify.

$f(x)=\dfrac{x^2-4}{x+1}$

$\boxed{\text{VA: } x=-1}$

$\boxed{\text{HA: none}}$

b. Find all of the roots of this function, if any.

$f(x)=\dfrac{x^2-4}{x+1}$

x int: $x^2-4=0$

$x^2=4$

$x=\pm 2$

$\boxed{(2,0)\quad(-2,0)}$

y int: $\dfrac{0^2-4}{0+1}=-4$

$\boxed{(0,-4)}$

c. Find and classify all of the local extrema of this function, if any. Show justification.

$f'(x)=\dfrac{x^2+2x+4}{(x+1)^2}$ → no crit# ; $(x+1)^2$ → $x=-1$

+ VA −1 + f'

$\boxed{\text{no extrema}}$

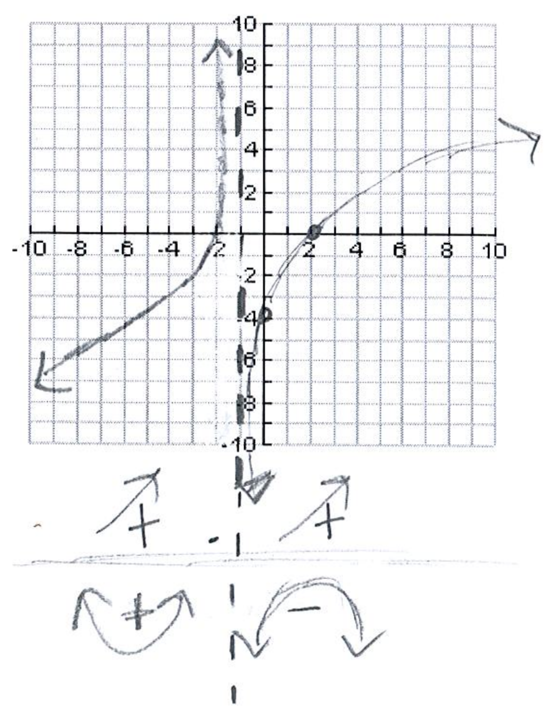
d. Find all of the inflection points of this function, if any. Show justification.

$f''(x)=\dfrac{-6}{(x+1)^3}$ → no possible POI ; $(x+1)^3$ → $x=-1$

(−) −1 VA (+) f''

$\boxed{\text{no POI}}$

e. Sketch the function and include all the features above.



7. $f(x) = 2x^3 - 5x^2 + 4x + 10$

a. f'(x)= $6x^2 - 10x + 4$

b. critical points

c. Intervals increasing: $(-\infty, -\frac{1}{3}) \cup (2, \infty)$

Intervals decreasing: $(-\frac{1}{3}, 2)$

d. f''(x)= $12x - 10$

e. Points of Inflection $x = 5/6$

f. Intervals of concave up $(5/6, \infty)$

Intervals of concave down $(-\infty, 5/6)$

g. Sign Chart

max

$-\infty$ + $-\frac{1}{3}$ − 2 + ∞

min

− $5/6$ +

h. At what x-value(s) does the graph have Any minimums or maximums? Justify your answer with the first and second derivatives tests.

$$0 = 6x^2 - 10x + 4$$
$$\frac{0}{2} = \frac{6x^2-10x+4}{2}$$
$$0 = 3x^2 - 5x + 2$$
$$0 = (3x+1)(x-2)$$
$$x = -\frac{1}{3} \quad x = 2$$

$$f'(-1) = (-)(-) = +$$
$$f'(0) = (+)(-) = -$$
$$f'(3) = (+)(+) = +$$

$$0 = 12x - 10$$
$$12x = 10$$
$$x = \frac{5}{6}$$

$$f''(1) = +$$
$$f''(0) = -$$

1st

$x = -\frac{1}{3}$ is a maximum b.c. it is a critical # where $f'(x)$ changes from pos to neg.

$x = 2$ is a minimum b.c it is a critical # where f' changes from neg to pos

2nd

$x = -\frac{1}{3}$ is a max b.c it is a crit # & $f''(-\frac{1}{3}) = $ neg

$x = 2$ is a min b.c it is a crit # & $f''(2) = $ pos

8. $f(x = \cos x + x, \qquad [0, 2\pi]$

a. f'(x)= $-\sin x + 1$

b. critical points $x = \pi/2$

c. Intervals increasing: $(0, 2\pi)$

Intervals decreasing: none

d. f''(x) = $-\cos x$

e. Points of Inflection $\pi/2$, $3\pi/2$

f. Intervals of concave up $(\pi/2, 3\pi/2)$

Intervals of concave down $(0, \pi/2)(3\pi/2, 2\pi)$

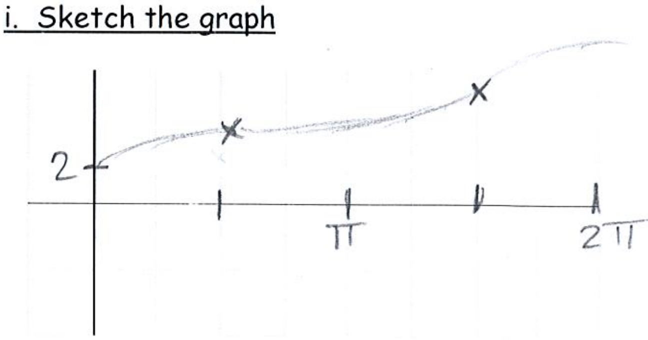
g. Sign Chart

0 + $\pi/2$ + 2π

− $\pi/2$ + $3\pi/2$ −

h. At what x-value(s) does the graph have Any minimums or maximums? Justify your answer with the first and second derivatives tests.

i. Sketch the graph



Find each limit.

1. $\lim_{x\to 1}\frac{\ln x}{x-1}$ $\frac{0}{0}$

$\lim_{x\to 1}\frac{1/x}{1}$

$\lim_{x\to 1}\frac{1}{x}=\frac{1}{1}=\boxed{1}$

2. $\lim_{x\to\infty}\frac{x^2+x}{\ln x}$ $\frac{\infty}{\infty}$

$\lim_{x\to\infty}\frac{2x+1}{\cancel{\frac{1}{x}}\cdot\cancel{\frac{x}{1}}}\cdot\frac{x}{1}$

$\lim_{x\to\infty}2x^2+x=\infty+\infty=\boxed{\infty}$

3. $\lim_{x\to -1}\frac{x^2-x}{x+1}$ — VA at $x=-1$

$\lim_{x\to -1^+}\frac{(-.999)^2-(-.999)}{-.999+1}=\frac{+}{+}=+\infty$

$\lim_{x\to -1^-}\frac{(-1.001)^2-(-1.001)}{-1.001+1}=\frac{+}{-}=-\infty$

$\lim_{x\to -1}\frac{x^2-x}{x+1}=\boxed{dne}$

4. $\lim_{x\to 0^+}\sqrt{x}\ln x$ $0(-\infty)$

$\lim_{x\to 0^+}\frac{\ln x}{x^{-1/2}}$ $\frac{-\infty}{\infty}$

$\lim_{x\to 0^+}\frac{1/x}{-\frac{1}{2}x^{-3/2}}$

$\lim_{x\to 0^+}\frac{\frac{1}{x}}{\frac{-1}{2x^{3/2}}}\cdot\frac{2x^{3/2}}{-1}$

$\lim_{x\to 0^+}\frac{2x^{1/2}}{-1}$

$\frac{2\sqrt{0}}{-1}$

$=\boxed{0}$

5. $\lim_{x\to\infty}\frac{e^x}{x^3}$ $\frac{\infty}{\infty}$

$\lim_{x\to\infty}\frac{e^x}{3x^2}$ $\frac{\infty}{\infty}$

$\lim_{x\to\infty}\frac{e^x}{6x}$ $\frac{\infty}{\infty}$

$\lim_{x\to\infty}\frac{e^x}{6}=\frac{\infty}{6}=\boxed{\infty}$

6. $\lim_{x\to 0^+}\left(1-x^2\right)^{1/x^2}$ 1^0

$\ln y=\lim_{x\to 0^+}\ln(1-x^2)^{1/x^2}$

$\ln y=\lim_{x\to 0^+}\frac{1}{x^2}\ln(1-x^2)$

$\ln y=\lim_{x\to 0^+}\frac{\ln(1-x^2)}{x^2}$ $\frac{0}{0}$

$\ln y=\lim_{x\to 0^+}\frac{\frac{1}{1-x^2}\cdot(-\cancel{2x})}{\frac{\cancel{2x}}{1}}\cdot\frac{\frac{1}{2x}}{\frac{1}{2x}}$

$\ln y=\lim_{x\to 0^+}\frac{-1}{1-x^2}$ $\ln y=\frac{-1}{1-0}$

$e^{\ln y}=e^{-1}$

$\boxed{y=e^{-1}}$