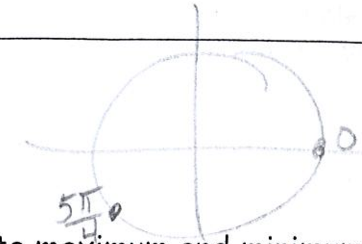
AP Calculus
Review for Application of Derivatives Test

Name________________________



Answer the following. With a calculator

1. Suppose $y = e^{\sin x}$ on the interval $\left[0, \frac{5\pi}{4}\right]$. Identify all absolute maximum and minimum values. Show all work that leads to your answer and give exact answers, not decimal approximations.

$y' = e^{\sin x} \cdot \cos x$

$0 = e^{\sin x} \cdot \cos x$

$\cancel{e^{\sin x}} = 0 \qquad \cos x = 0$

$x = \frac{\pi}{2}$

	x	$y = e^{\sin x}$	
	0	1	$e^{\sin(0)} = e^0$
(Abs max)	$\frac{\pi}{2}$	$e \approx 2.7$	$e^{\sin\frac{\pi}{2}} = e^1 = e$
(Abs min)	$\frac{5\pi}{4}$	$.493$	$e^{\sin(\frac{5\pi}{4})} = e^{-\frac{\sqrt{2}}{2}}$

2. A coffee filter has the shape of an inverted circular cone with equal base radius and height. Water drains out of the filter at a rate of 10 cm^3/min. At what rate is the height of the water changing when the height of the water is 8 cm? $r = h$

$\left(\text{Hint: } V_{\text{cone}} = \frac{1}{3}\pi r^2 h\right)$

Know: $\frac{dV}{dt} = -10\frac{\text{cm}^3}{\text{min}}$

Find: $\frac{dh}{dt} = \_\_\_\_$

When: $h = 8\text{ cm}$

Eqn: $V = \frac{1}{3}\pi R^2 h$

$V = \frac{1}{3}\pi (h)^2 h$

$V = \frac{\pi}{3}h^3$

Der: $\frac{dV}{dt} = \frac{\pi}{\cancel{3}}(\cancel{3}h^2)\frac{dh}{dt}$

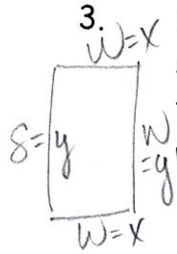
$\frac{dV}{dt} = \pi h^2 \frac{dh}{dt}$

Sub: $-10\frac{\text{cm}^3}{\text{min}} = \pi(8\text{cm})^2\frac{dh}{dt}$

$\left(\frac{1}{64\pi\text{cm}^2}\right)\frac{-10\text{cm}^3}{\text{min}} = 64\pi\text{cm}^2\frac{dh}{dt}\left(\frac{1}{64\pi\text{cm}^2}\right)$

$\frac{dh}{dt} = \frac{-5}{32\pi}\frac{\text{cm}}{\text{min}}$

3. A gardener wants to build a wall around a 150 ft^2 plot of land. One of the sides is to be a stone wall that will cost \$50 per foot, and the other three sides are to be made of wood that will cost \$10 per foot. What is the minimum cost of the fence (to the nearest whole-number dollar amount)?



$\text{Cost} = 50(y) + 10(2x+y)$

$\text{Cost} = 50y + 20x + 10y$

$\text{Cost} = 60y + 20x$

$\text{Cost} = 60\left(\frac{150}{x}\right) + 20x$

$A = 150$

$xy = 150$

$y = \frac{150}{x}$

$\text{Cost} = 9000x^{-1} + 20x$

$\text{Cost}' = 9000x^{-2} + 20$

$0 = \frac{9000}{x^2} + 20$

$\frac{9000}{x^2} = 20$

$20x^2 = 9000$

$x^2 = 450$

$x = 15\sqrt{2} \approx 21.213$

21.213

$\nearrow + \mid - \searrow$

$c'(20) \quad c'(24)$

$\boxed{C = 848.53}$

AP Calculus Name______________________

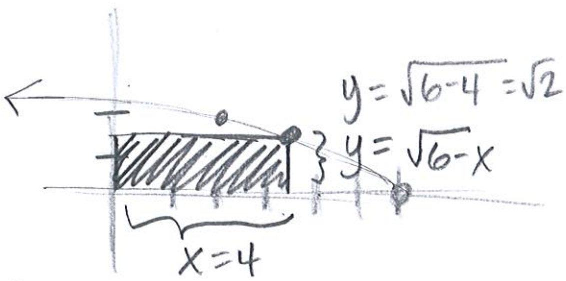
Review for Application of Derivatives Test

$\nearrow$ 4 $\searrow$

$+$ | $-$

$A'(3)$ $A'(5)$

4. Find the dimensions of the rectangle with largest area that can be inscribed in the region bounded by the curve $y=\sqrt{6-x}$ in the first quadrant.



$A = \text{length} \cdot \text{width}$

$A = x \cdot \sqrt{6-x}$

$A' = x\,\frac{1}{2}(6-x)^{-1/2}(-1) + \sqrt{6-x}\,(1)$

$0 = \dfrac{-x}{2\sqrt{6-x}} + \dfrac{\sqrt{6-x}\,(2\sqrt{6-x})}{2\sqrt{6-x}}$

$0 = \dfrac{-x+2(6-x)}{2\sqrt{6-x}}$

$0 = -x+12-2x$

$0 = -3x+12$

$3x = 12$

$x = 4$

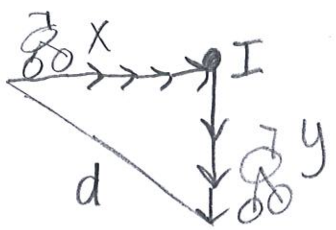
$\boxed{\text{dimensions } \ell \cdot w \quad 4 \cdot \sqrt{2}}$

5. A bicyclist is traveling east towards an intersection at the rate of 9 miles per hour. A second bicyclist is traveling south away from the intersection at the rate of 10 miles per hour. What is the rate of change between the bicycles when the first bicycle is 4 miles east of the intersection and the second bicycle is 3 miles south of the intersection?

$x^2+y^2=d^2$

$16+9=d^2$

$d=5$



Eqn: $x^2+y^2=d^2$

D: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2d\frac{dd}{dt}$

$x\frac{dx}{dt} + y\frac{dy}{dt} = d\frac{dd}{dt}$

K: $\frac{dx}{dt} = -9\,\frac{mi}{h}$

$\frac{dy}{dt} = 10\,\frac{mi}{h}$

F: dd/dt

W: $x=4$ & $y=3$ then $d=5$

$(4)(-9)+(3)(10) = 5\frac{dd}{dt}$

$-36+30 = 5\frac{dd}{dt}$

$-6 = 5\frac{dd}{dt}$

$\boxed{\frac{dd}{dt} = \frac{-6}{5}\,\frac{mi}{hR}}$

6. Find the point on the parabola $y=\sqrt{x}$ that is closest to the point (3, 0).

$(3,0)\ (x,\sqrt{x})$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d=\sqrt{(x-3)^2+(\sqrt{x}-0)^2}$

$d=\sqrt{x^2-6x+9+x}$

$d=(x^2-5x+9)^{1/2}$

$d' = \frac{1}{2}(x^2-5x+9)^{-1/2}(2x-5)$

$0 = \dfrac{2x-5}{2\sqrt{x^2-5x+9}}$

$2x-5=0$

$2x=5$

$x=\frac{5}{2}$

$\searrow$ $-$ $5/2$ $+$ $\nearrow$

$d'(1)$ $d'(3)$

$\boxed{\left(\frac{5}{2}, \sqrt{\frac{5}{2}}\right)}$

AP Calculus
Review for Application of Derivatives Test

Name_______________________

7. A spherical balloon is inflate with gas at the rate of 500 cubic centimeters per minute. How fast is the radius of the balloon increasing at the instant the radius is (a.) 30 centimeters. (b.) 60 centimeters.

K: $\frac{dV}{dt} = 500 \frac{cm^3}{min}$

F: $\frac{dr}{dt} =$ ___

W: $R = 30cm$ & $R = 60cm$

Eqn: $V = \frac{4}{3}\pi R^3$

D: $\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi(\cancel{3}R^2)\frac{dR}{dt}$

$\frac{dV}{dt} = 4\pi R^2 \frac{dR}{dt}$

S: $500 = 4\pi(30)^2 \frac{dR}{dt}$

$\frac{dR}{dt} = \frac{500}{3600\pi} = \boxed{\frac{5}{36\pi}\frac{cm}{min}}$

$500 = 4\pi(60)^2 \frac{dR}{dt}$

$\frac{dR}{dt} = \frac{500}{14400\pi} = \boxed{\frac{5}{144\pi}\frac{cm}{min}}$

8. A circular oil slick is being formed in such a way that the radius of the slick is increasing at a constant rate of 12 ft/hr. What will be the rate of area increase when the slick has radius 300 ft?

K: $\frac{dR}{dt} = 12 ft/hR$

F: $\frac{dA}{dt} =$ ___

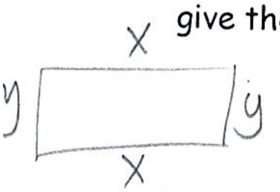
W: $R = 300 ft$

Eqn: $A = \pi R^2$

$\frac{dA}{dt} = 2\pi R \frac{dR}{dt}$

Sub: $\frac{dA}{dt} = 2\pi(300)(12) = \boxed{7200\pi \frac{ft^2}{hR}}$

9. A rectangular area is to be enclosed with 320 ft of fence. What dimensions of the rectangle give the maximum area? $P = 320$



$320 = 2x + 2y$

$320 - 2x = 2y$

$160 - x = y$

$A = x \cdot y$

$A = x(160 - x)$

$A = 160x - x^2$

$A' = 160 - 2x$

$0 = 160 - 2x$

$2x = 160$

$\boxed{x = 80}$

$\nearrow$ 80 $\searrow$

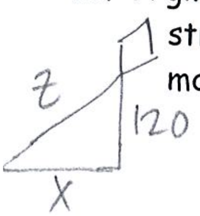
A'(10) A'(100)

$y = 160 - x$

$y = 160 - 80$

$\boxed{y = 80}$

10. A girl is flying a kite. The kite is moving horizontally at a height of 120 ft when 250 ft of string is out and the rate of increase in string length is 2 ft/s. How fast is the kite moving in the horizontal direction for these conditions?



Know: $\frac{dz}{dt} = 2\frac{ft}{sec}$

Find: $\frac{dx}{dt} =$ ___

When: $z = 250$

$x^2 + 120^2 = 250^2$

$x = \sqrt{250^2 - 120^2} = 219.317$

Eqn: $x^2 + 120^2 = z^2$

D: $\frac{2x\frac{dx}{dt}}{2x} = \frac{\cancel{2}z\frac{dz}{dt}}{2x}$

$\frac{dx}{dt} = \frac{z}{x}\frac{dz}{dt}$

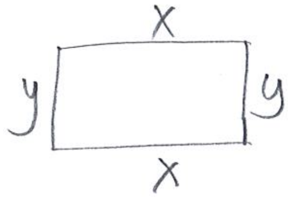
S: $\frac{dx}{dt} = \frac{250}{219.317}(2)$

$\boxed{\frac{dx}{dt} = 2.28 \frac{ft}{sec}}$

AP Calculus
Review for Application of Derivatives Test

Name_______________________________

11. A rectangular pen will be built using 100 feet of fencing. What dimensions will maximize the area?



$2x+2y=100$
$2y=100-2x$
$y=50-x$

$A=x\cdot y$
$A=x(50-x)$
$A=50x-x^2$
$A'=50-2x$

$0=50-2x$
$2x=50$
$x=25$

$\nearrow$ 25 $\searrow$
$A'(10)$ $A'(30)$

$\boxed{x=25 \quad y=25}$

12. Approximate using linearization.

A. $\ln(1.02)$ $f(x)=\ln x$ $a=1$

Point $(1,0)$ $f(1)=\ln(1)=0$ $f(x)=\ln x$ $f'(x)=\frac{1}{x}$ $f'(1)=\frac{1}{1}=1$
Slope 1 $f'(1)=\underline{1}$ $y-0=1(x-1)$ $y=1(x-1)$
$y(1.02)=1(1.02-1)=\boxed{.02}$

B. $(1.89)^3$ $f(x)=x^3$ $a=2$ $f(x)=x^3$

Point $(2,8)$ $f(2)=2^3=8$ $f'(x)=3x^2$ $y-8=12(x-2)$
Slope $f'(2)=\underline{12}$ $f'(2)=3(2)^2$ $y=12(x-2)+8$
$f'(2)=12$ $y=12(1.89-2)+8$ $=\boxed{6.68}$

C. $\sqrt[3]{7.999}$ $f(x)=(x)^{1/3}$ $a=8$ $f(x)=x^{1/3}$ $f'(8)=\frac{1}{3(2)^2}=\frac{1}{12}$

Point$(8,2)$ $f(8)=\sqrt[3]{8}=2$ $f'(x)=\frac{1}{3}x^{-2/3}$ $y-2=\frac{1}{12}(x-8)$
Slope $f'(8)=\frac{1}{12}$ $f'(x)=\frac{1}{3(\sqrt[3]{x})^2}$ $y=\frac{1}{12}(x-8)+2$
$y(7.999)=\frac{1}{12}(7.999-8)+2=\boxed{1.9999166667}$

D.) $\cos(89^\circ)$ $f(x)=\cos x$ $a=90^\circ$

Point$(90^\circ,0)$ $f(90^\circ)=\cos 90^\circ=0$ $f'(x)=-\sin x$ $y-0=-1(x-90^\circ)$
Slope $m=-1$ $f'(90^\circ)=-\sin(90^\circ)=-1$ $y=-(x-90^\circ)$
$y(89^\circ)=-(89^\circ-90^\circ)$
$=-(-1^\circ)=1^\circ\left(\frac{\pi}{180^\circ}\right)=\boxed{\frac{\pi}{180}}$

E.) $\sqrt{50}$ $f(x)=x^{1/2}$ $a=49$ $f'(x)=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$

Point$(49,7)$ $f(49)=\sqrt{49}=7$ $f'(49)=\frac{1}{2\sqrt{49}}=\frac{1}{14}$ $y(50)=\frac{1}{14}(50-49)+7$
Slope $f'(49)=\frac{1}{14}$ $=7\frac{1}{14}$
$y-7=\frac{1}{14}(x-49)$ $y=\frac{1}{14}(x-49)+7$ $\approx\boxed{7.071}$