AP Calculus

Name_____ Pd.____

Derivatives of Logarithmic Functions

1-10: Differentiate the function.

1. $f(x) = x\ln x - x$

$f'(x) = x\cdot\frac{1}{x} + \ln x\cdot 1 - 1$

$= 1 + \ln x - 1$

$= \boxed{\ln x}$

2. $f(x) = \sin(\ln x)$

$f'(x) = \cos(\ln x)\cdot\frac{1}{x}$

$= \boxed{\frac{\cos(\ln x)}{x}}$

3. $f(x) = \ln(\sin^2 x) = \ln(\sin x)^2$

$f'(x) = \frac{1}{(\sin x)^2}\cdot 2\sin x\cdot\cos x$

$= \frac{2\sin x\cos x}{(\sin x)^2} = \frac{2\cos x}{\sin x} = \boxed{2\cot x}$

or $\ln(\sin x)^2 = 2\ln\sin x$

so $f'(x) = \frac{2}{\sin x}\cdot\cos x = \frac{2\cos x}{\sin x} = 2\cot x$

4. $f(x) = \ln\frac{1}{x} = \ln(x^{-1})$

$f'(x) = \frac{1}{x^{-1}}\cdot -x^{-2}$

$= x\cdot\frac{-1}{x^2} = \boxed{\frac{-1}{x}}$

or $y = \ln(x^{-1}) = -\ln(x)$

$y' = -\frac{1}{x}$

5. $y = \frac{1}{\ln x} = (\ln x)^{-1}$

$y' = -(\ln x)^{-2}\cdot\frac{1}{x}$

$= \frac{-1}{(\ln x)^2}\cdot\frac{1}{x} = \boxed{\frac{-1}{x(\ln x)^2}}$

6. $f(x) = \log_{10}(x^3+1)$

$f'(x) = \frac{1}{(x^3+1)\ln 10}\cdot 3x^2$

$= \boxed{\frac{3x^2}{(x^3+1)\ln 10}}$

7. $f(x) = \log_5(xe^x)$

$f'(x) = \frac{1}{xe^x\ln 5}(xe^x + e^x\cdot 1)$

$= \frac{xe^x + e^x}{xe^x\ln 5} = \frac{e^x(x+1)}{xe^x\ln 5}$

$= \boxed{\frac{x+1}{x\ln 5}}$

or $y = \log_5(xe^x)$

$y = \log_5 x + \log_5 e^x$

$y' = \frac{1}{x\ln 5} + \frac{1}{e^x\ln 5}\cdot e^x$

$= \frac{1}{x\ln 5} + \frac{1}{\ln 5}\left(\frac{x}{x}\right)$

$= \frac{1+x}{x\ln 5}$

8. $f(x) = \sin x\ln(5x)$

$f'(x) = \sin x\cdot\frac{1}{5x}\cdot 5 + \ln(5x)\cdot\cos x$

$= \boxed{\frac{\sin x}{x} + \cos x\ln(5x)}$

9. $g(x) = \ln\left(x\sqrt{x^2-1}\right)$ expand

$g(x) = \ln x + \ln(x^2-1)^{1/2}$

$= \ln x + \frac{1}{2}\ln(x^2-1)$

$g'(x) = \frac{1}{x} + \frac{1}{2}\cdot\frac{1}{x^2-1}\cdot 2x$

$= \boxed{\frac{1}{x} + \frac{x}{x^2-1}}$

10. $g(r) = r^2\ln(2r+1)$

$g'(r) = r^2\cdot\frac{1}{2r+1}\cdot 2 + \ln(2r+1)\cdot 2r$

$= \boxed{\frac{2r^2}{2r+1} + 2r\ln(2r+1)}$

11-12: Find an equation of the tangent line to the curve at the given point.

11. $y = \ln(x^2 - 3x + 1), \quad (3,0)$

$$y' = \frac{1}{x^2-3x+1} \cdot 2x-3 = \frac{2x-3}{x^2-3x+1}\Big|_{x=3}$$

$$= \frac{6-3}{9-9+1} = 3$$

$$y - 0 = 3(x-3)$$

12. $y = x^2 \ln x, \quad (1,0)$

$$y' = x^2 \cdot \frac{1}{x} + \ln x \cdot 2x$$

$$= x + 2x\ln x\Big|_{x=1}$$

$$= 1 + 2\ln(1) = 1 + 0 = 1$$

$$y - 0 = 1(x-1)$$

13-18: Use logarithmic differentiation to fine the derivative of the function.

13. $y = (x+2)^2 (x^4+4)^4$

14. $y = \sqrt{\dfrac{x-1}{x^4+1}}$

15. $y = x^x$

16. $y = x^{\cos x}$

17. $y = x^{\sin x}$

18. $y = \sqrt{x}^x$