AP Calculus

Review Derivatives (1)

Name________________ Pd.___

1-24: Calculate y':

1. $y=(x^2+x^3)^4$ $y'=4(x^2+x^3)^3[2x+3x^2]$ $y'=4(2x+3x^2)(x^2+x^3)^3$	2. $y=\frac{1}{\sqrt{x}}-\frac{1}{\sqrt[5]{x^3}}$ $y=x^{-1/2}-x^{-3/5}$ $y'=-\frac{1}{2}x^{-3/2}+\frac{3}{5}x^{-8/5}$ $y'=\frac{-1}{2x^{3/2}}+\frac{3}{5x^{8/5}}$	3. $y=\frac{x^2-x-2}{\sqrt{x}}$ $y=\frac{x^2}{x^{1/2}}-\frac{x}{x^{1/2}}-\frac{2}{x^{1/2}}$ $y=x^{3/2}-x^{1/2}-2x^{-1/2}$ $y'=\frac{3}{2}x^{1/2}-\frac{1}{2}x^{-1/2}+x^{-3/2}$ $y'=\frac{3x^{1/2}}{2}-\frac{1}{2x^{1/2}}+\frac{1}{x^{3/2}}$
4. $y=\frac{\tan x}{1+\cos x}$ $y'=\frac{(1+\cos x)\sec^2 x-\tan x(-\sin x)}{(1+\cos x)^2}$ $y'=\frac{\sec^2 x(1+\cos x)+\sin x\tan x}{(1+\cos x)^2}$ Can keep going ☺ look at last page if interested	5. $y=x^2\sin(\pi x)$ $y'=x^2\cos(\pi x)[\pi]+\sin(\pi x)[2x]$ $y'=\pi x^2\cos(\pi x)+2x\sin(\pi x)$	6. $y=\frac{t^4-1}{t^4+1}$ $y'=\frac{(t^4+1)(4t^3)-(t^4-1)(4t^3)}{(t^4+1)^2}$ $y'=\frac{\cancel{4t^7}+4t^3-\cancel{4t^7}+4t^3}{(t^4+1)^2}$ $y'=\frac{8t^3}{(t^4+1)^2}$
7. $y=\ln(x\ln x)$ $\frac{1}{x\ln x}\frac{d}{dx}[x\ln x]$ $\frac{1}{x\ln x}\left[x\left(\frac{1}{x}\right)+\ln x(1)\right]$ $\frac{1}{x\ln x}[1+\ln x]$ $\frac{1+\ln x}{x\ln x}$	8. $y=\sqrt{x}\cos\sqrt{x}=x^{1/2}\cos x^{1/2}$ $y'=x^{1/2}(-\sin x^{1/2})\frac{d}{dx}[x^{1/2}]+\cos x^{1/2}[\frac{1}{2}x^{-1/2}]$ $y'=-\sqrt{x}\sin\sqrt{x}(\frac{1}{2}x^{-1/2})+\cos\sqrt{x}(\frac{1}{2\sqrt{x}})$ $y'=\frac{-\sqrt{x}\sin\sqrt{x}}{2\sqrt{x}}+\frac{\cos\sqrt{x}}{2\sqrt{x}}$ $=\frac{-\sqrt{x}\sin\sqrt{x}+\cos\sqrt{x}}{2\sqrt{x}}$	9. $y=\frac{e^{\frac{1}{x}}}{x^2}$ $\frac{x^2e^{\frac{1}{x}}\cdot\frac{d}{dx}[x^{-1}]-e^{\frac{1}{x}}[2x]}{x^4}$ $\frac{x^2e^{1/x}(-x^{-2})-2xe^{\frac{1}{x}}}{x^4}$ $\frac{\cancel{x^2}e^{1/x}(-\frac{1}{\cancel{x^2}})-2xe^{1/x}}{x^4}$ $\frac{-e^{1/x}-2xe^{1/x}}{x^4}=\frac{-e^{1/x}(1+2x)}{x^4}$

10. $y = \ln \sec x$

$\frac{1}{\sec x} \frac{d}{dx}[\sec x]$

$\frac{1}{\cancel{\sec x}}[\cancel{\sec x}\tan x]$

$= \tan x$

11. $y = \cot(\csc x)$

$-\csc^2(\csc x)\frac{d}{dx}[\csc x]$

$-\csc^2(\csc x)[-\csc x\cot x]$

$\csc^2(\csc x)\csc x\cot x$

12. $y = e^{x\sec x}$

$e^{x\sec x}\frac{d}{dx}[x\sec x]$

$e^{x\sec x}[x\sec x\tan x + \sec x(1)]$

$e^{x\sec x}[x\sec x\tan x + \sec x]$

$e^{x\sec x}\cdot\sec x[x\tan x + 1]$

13. $y = 3^{x\ln x}$

$y' = 3^{x\ln x}\frac{d}{dx}[x\ln x]$

$= 3^{x\ln x}[x(\frac{1}{x}) + \ln x(1)]$

$= 3^{x\ln x}[1 + \ln x]$

14. $y = \sec(1+x^2)$

$\sec(1+x^2)\tan(1+x^2)[2x]$

$2x\sec(1+x^2)\tan(1+x^2)$

15. $y = \log_5(1+2x)$

$y' = \frac{1}{(1+2x)\ln 5}[2]$

$= \frac{2}{(1+2x)\ln 5}$

16. $y = (\cos x)^x$

$\ln y = x \cdot \ln\cos x$

$\frac{1}{y}[y'] = x\left(\frac{1}{\cos x}\right)(-\sin x) + \ln\cos x(1)$

$\frac{1}{y}[y'] = [-x\tan x + \ln\cos x] \cdot y$

$y' = [-x\tan x + \ln\cos x]\cos x^x$

17. $y = \frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}$

$\ln y = 4\ln(x^2+1) - 3\ln(2x+1) - 5\ln(3x-1)$

$\frac{1}{y}[y'] = 4\left(\frac{1}{x^2+1}\right)(2x) - 3\left(\frac{1}{2x+1}\right)(2) - 5\left(\frac{1}{3x-1}\right)(3)$

$\frac{1}{y}[y'] = \frac{8x}{x^2+1} - \frac{6}{2x+1} - \frac{15}{3x-1}$

$y' = \left[\frac{8x}{x^2+1} - \frac{6}{2x+1} - \frac{15}{3x-1}\right]\left[\frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}\right]$

18. $y = e^{\cos x} + \cos(e^x)$

$e^{\cos x}(-\sin x) + (-\sin e^x)e^x$

$-\sin x e^{\cos x} - \sin e^x \cdot e^x$

19. $y = 10^{\tan \pi\theta}$

$y' = 10^{\tan\pi\theta} \frac{d}{dx}[\tan\pi\theta]$

$= 10^{\tan\pi\theta} \sec^2(\pi\theta)[\pi]$

$= \pi 10^{\tan\pi\theta} \sec^2(\pi\theta)$

20. $y = \cot(3x^2+5)$

$-\csc^2(3x^2+5)[6x]$

$-6x\csc^2(3x^2+5)$

21. $y = \sqrt{t\ln(t^4)} = (t\ln(t^4))^{1/2}$

$\frac{1}{2}[t\ln(t^4)]^{-1/2}\{t(\frac{1}{t^4})(4t^3) + \ln(t^4)(1)\}$

$\frac{1}{2}[t\ln(t^4)]^{-1/2}[4 + \ln(t^4)]$

$\frac{4 + \ln(t^4)}{2\sqrt{t\ln(t^4)}}$

22. $y = \tan^2(\sin\theta) = [\tan(\sin\theta)]^2$

$2[\tan(\sin\theta)]^1 \frac{d}{d\theta}[\tan(\sin\theta)]$

$2\tan(\sin\theta)\sec^2(\sin\theta)\frac{d}{d\theta}[\sin\theta]$

$2\tan(\sin\theta)\sec^2(\sin\theta)\cos\theta$

23. $y = (3x+2)^5(5-x)^8$

$(3x+2)^5(8)(5-x)^7(-1) + (5-x)^8(5)(3x+2)^4(3)$

$-8(3x+2)^5(5-x)^7 + 15(5-x)^8(3x+2)^4$

$(3x+2)^4(5-x)^7[-8(3x+2) + 15(5-x)]$

$(3x+2)^4(5-x)^7[-24x - 16 + 75 - 15x]$

$(3x+2)^4(5-x)^7[-39x + 59]$

24. $y = \left(\frac{2x-5}{x+4}\right)^8$

$8\left(\frac{2x-5}{x+4}\right)^7\left[\frac{(x+4)(2)-(2x-5)(1)}{(x+4)^2}\right]$

$\frac{8(2x-5)^7[\cancel{2x} + 8 - \cancel{2x} + 5]}{(x+4)^7(x+4)^2}$

$\frac{8(2x-5)^7(13)}{(x+4)^9} = \frac{104(2x-5)^7}{(x+4)^9}$

25-27: Find an equation of the tangent to the curve at the given point.

25. $y = 4\sin^2 x, \left(\frac{\pi}{6}, 1\right)$

$(\frac{\pi}{2}, \frac{1}{2})$ $y = 4[\sin x]^2$

$y' = 8\sin x\cos x$

$y'(\frac{\pi}{6}) = 8^2(\frac{1}{2})(\frac{\sqrt{3}}{2}) = 2\sqrt{3}$

$y - 1 = 2\sqrt{3}(x - \frac{\pi}{6})$

26. $y = \frac{x^2-1}{x^2+1}$, $(0,-1)$

$y' = \frac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2}$

$y' = \frac{\cancel{2x^3} + 2x - \cancel{2x^3} + 2x}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$

$y'(0) = \frac{4(0)}{(0^2+1)^2} = 0$

$y - -1 = 0(x - 0)$ $\boxed{y = -1}$

27. $y = \sqrt{1+4\sin x}$, $(0,1)$

$y = (1+4\sin x)^{1/2}$

$y' = \frac{1}{2}(1+4\sin x)^{-1/2} 4\cos x$

$y' = \frac{4\cos x}{2\sqrt{1+4\sin x}}$

$y'(0) = \frac{4\cos(0)}{2\sqrt{1+4\sin(0)}} = \frac{4(1)}{2\sqrt{1}} = 2$

28-29: Suppose that h(x)=f(x)g(x) and F(x)=f(g(x)), where f(2)=3, g(2)=5, g'(2)=4, f'(2)= -2, and f'(5)=11.

28. Find $h'(2)$

$h(x) = f(x)g(x)$

$h'(x) = f(x)g'(x) + g(x)f'(x)$

$h'(2) = f(2)g'(2) + g(2)f'(2)$

$= (3)(4) + (5)(-2)$

$= 12 - 10$

$= 2$

29. Find $F'(2)$

$F(x) = f(g(x))$

$F'(x) = f'(g(x)) \cdot g'(x)$

$F'(2) = f'[g(2)] \cdot g'(2)$

$F'(2) = f'[5][4]$

$(11)(4)$

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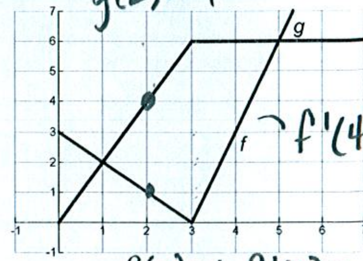
30-32: Suppose f and g are functions whose graphs are shown, let $P(x)=f(x)g(x)$, $Q(x)=f(x)/g(x)$, and $C(x)=f(g(x))$.

$g'(2)=2$

$g(2)=4$



$f'(4)=3$

$f(2)=1 \quad f'(2)=-1$

$$Q(x)=\frac{f(x)}{g(x)}$$

30. Find $P'(2)$

$P(x)=f(x)g(x)$

$P'(x)=f(x)g'(x)+g(x)f'(x)$

$P'(2)=f(2)g'(2)+g(2)f'(2)$

$P'(2)=(1)(2)+(4)(-1)$

$P'(2)=2-4$

$P'(2)=-2$

31. Find $Q'(2)$

$$Q'(x)=\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$$

$$Q'(2)=\frac{g(2)f'(2)-f(2)g'(2)}{[g(2)]^2}$$

$$Q'(2)=\frac{4(-1)-(1)(2)}{4^2}$$

$$Q'(2)=\frac{-6}{16}=-\frac{3}{8}$$

32. Find $C'(2)$

$C(x)=f(g(x))$

$C'(x)=f'(g(x))\cdot g'(x)$

$C'(2)=f'(g(2))\cdot g'(2)$

$C'(2)=f'(4)\cdot[2]$

$C'(2)=[3][2]$

$C'(2)=6$

33-40: Find f' in terms of g'

33. $f(x)=x^2g(x)$

$x^2g'(x)+g(x)[2x]$

$x^2g'(x)+2xg(x)$

34. $f(x)=g(x^2)$

$g'(x^2)[2x]$

$2xg'(x^2)$

35. $f(x)=[g(x)]^2$

$2[g(x)]g'(x)$

$2g(x)g'(x)$

36. $f(x)=g(g(x))$

$g'[g(x)]\cdot g'(x)$

37. $f(x)=g(e^x)$

$g'[e^x]\cdot e^x$

38. $f(x)=e^{g(x)}$

$e^{g(x)}\cdot g'(x)$

39. $f(x)=\ln|g(x)|$

$\frac{1}{g(x)}\cdot g'(x)$

$\frac{g'(x)}{g(x)}$

40. $f(x)=g(\ln x)$

$g'[\ln x]\cdot\frac{1}{x}$

$\frac{g'[\ln x]}{x}$