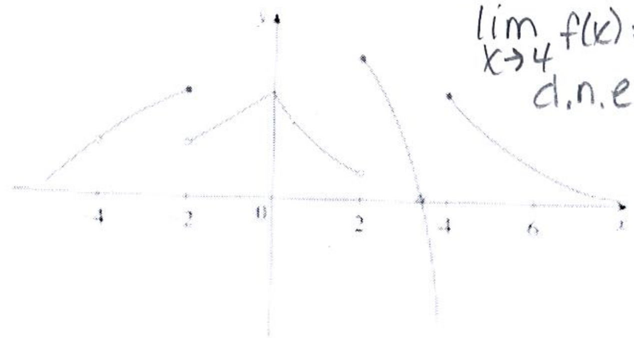
AP Calculus AB
Continuity

Name_______________________
Limits, Cont., & R.O.C Day 8

1.



$x=4$ b.c. $\lim_{x\to4} f(x) =$ d.n.e.

A.) From the graph of f, state the numbers at which f is discontinuous and explain why.

$x=-2$ b.c. $\lim_{x\to-2} f(x) =$ does not exist

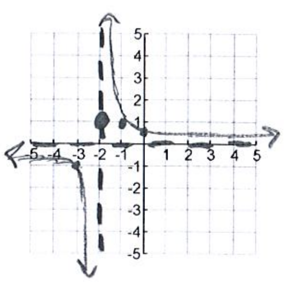
$x=2$ b.c. $\lim_{x\to2} f(x) =$ dne

B.) For each of the numbers stated in part (A), determine whether f is continuous from the right, or from the left, or neither.

2-4: Explain why the function is discontinuous at the given number a. Sketch the graph of the function.

2. $f(x) = \begin{cases} \frac{1}{x+2} & if\ x \neq 2 \\ 1 & if\ x = 2 \end{cases}$

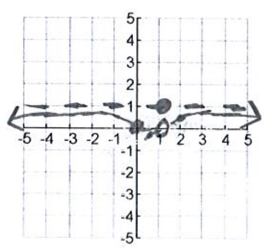
At $a = 2$.



Discontinuous because $\lim_{x\to2} f(x) =$ does not exist

3. $f(x) = \begin{cases} \frac{x^2-x}{x^2+1} & if\ x \neq 1 \\ 1 & if\ x = 1 \end{cases}$

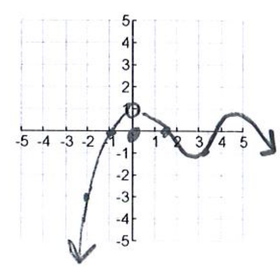
At $a = 1$.



Discontinuous at $x=1$ because $\lim_{x\to1} f(x) \neq f(1)$

4. $f(x) = \begin{cases} \cos x & if\ x < 0 \\ 0 & if\ x = 0 \\ 1 - x^2 & if\ x > 0 \end{cases}$

At $a = 0$.



Discontinuous because $\lim_{x\to0} f(x) \neq f(0)$

5-6: How would you "remove the discontinuity" of f? In other words, how would you define $f(2)$ in order to make f continuous at 2?

5. $f(x) = \frac{x^2-x-2}{x-2} = \frac{\cancel{(x-2)}(x+1)}{\cancel{x-2}}$

$\lim_{x\to2} f(x) = 2+1 = 3$

let $f(2) = 3$

So the $\lim_{x\to2} f(x)$ would $= f(2)$

6. $f(x) = \frac{x^3-8}{x^2-4} = \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+2)}$

$\lim_{x\to2} f(x) = \frac{4+4+4}{4} = 3$

let $f(2) = 3$

So the $\lim_{x\to2} f(x) = f(2)$

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7-8: Find the numbers at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither? Sketch the graph of f.

7.

$$f(x) = \begin{cases} 1 + x^2 & if \quad x \le 0 \\ 2 - x & if \quad 0 < x \le 2 \\ (x-2)^2 & if \quad x > 2 \end{cases}$$

$\lim_{x \to 0^+} f(x) = 2 - 0 = 2$

$\lim_{x \to 0^-} f(x) = 1 + 0^2 = 1$

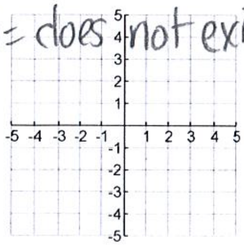
Discontinuous at $x = 0$ because $\lim_{x \to 0} f(x) =$ does not exist

$\lim_{x \to 2^+} (2-2)^2 = 0$

$\lim_{x \to 2^-} 2 - 2 = 0$

$f(2) = 2 - 2 = 0$

Continuous $(-\infty, 0) \cup (0, \infty)$



8.

$$f(x) = \begin{cases} x + 2 & if \quad x < 0 \\ e^x & if \quad 0 \le x \le 1 \\ 2 - x & if \quad x > 1 \end{cases}$$

$\lim_{x \to 0^+} f(x) = e^0 = 1$

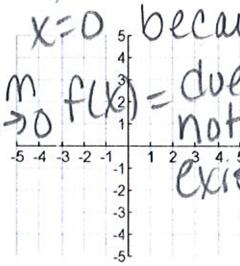
$\lim_{x \to 0^-} f(x) = 0 + 2 = 0$

Discontinuous at $x = 0$ because $\lim_{x \to 0} f(x) =$ does not exist

$\lim_{x \to 1^+} f(x) = 2 - 1 = 1$

$\lim_{x \to 1^-} f(x) = e^1$

Discontinuous at $x = 1$ because $\lim_{x \to 1} f(x) =$ does not exist



Continuous $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

9. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x & if \quad x < 2 \\ x^3 - cx & if \quad x \ge 2 \end{cases}$$

$c(2)^2 + 2(2) = (2)^3 - c(2)$

$4c + \cancel{4} = 8 - \cancel{2c}$

$+2c \qquad -4$

$6c = 4$

$c = 2/3$

10. Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & if \quad x < 2 \\ ax^2 - bx + 3 & if \quad 2 \le x < 3 \\ 2x - a + b & if \quad x \ge 3 \end{cases}$$

$\dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$

$$f(x) = \begin{cases} x + 2 & (-\infty, 2) \\ ax^2 - bx + 3 & [2, 3) \\ 2x - a + b & [3, \infty) \end{cases}$$

At $x = 2$

$2 + 2 = a(2)^2 - b(2) + 3$

$\cancel{4} = 4a - 2b + 3$

$-\cancel{4} \qquad -4$

At $x = 3$

$a(3)^2 - b(3) + 3 = 2(3) - a + b$

$9a - 3b + 3 = \cancel{6} - \cancel{a} + \cancel{b}$

$+a \; -b \; -6$

$-2(4a - 2b - 1 = 0)$

$10a - \cancel{4b} - 3 = 0$

$-8a + \cancel{4b} + 2 = 0$

$2a - 1 = 0$

$\boxed{a = 1/2}$

$4(\frac{1}{2}) - 2b - 1 = 0$

$2 - 2b - 1 = 0$

$-2b + 1 = 0$

$2b = 1$

$\boxed{b = 1/2}$