AP Calculus — Name________________________________

Slope Fields — Application of Integration Day 6

1. Show that $y=\frac{2}{3}e^{x}+e^{-2x}$ is a solution of the differential equation $y'+2y=2e^{x}$.

$y'=\frac{2}{3}e^{x}+e^{-2x}(-2)$

$y'=\frac{2}{3}e^{x}-2e^{-2x}$

$y'+2y=2e^{x}$

$\frac{2}{3}e^{x}-2e^{-2x}+2\left(\frac{2}{3}e^{x}+e^{-2x}\right)=2e^{x}$

$\frac{2}{3}e^{x}\cancel{-2e^{-2x}}+\frac{4}{3}e^{x}+\cancel{2e^{-2x}}=2e^{x}$

$\frac{6}{3}e^{x}=2e^{x}$

$2e^{x}=2e^{x}$:)

2. Verify that $y=-t\cos t-t$ is a solution of the initial-value problem $t\frac{dy}{dt}=y+t^{2}\sin t \quad y(\pi)=0$

$\frac{d}{dt}\left[y=-t\cos t-t\right]$

$\frac{dy}{dt}=-t(-\sin t)+\cos t(-1)-1$

$\frac{dy}{dt}=t\sin t-\cos t-1$

$t\frac{dy}{dt}=y+t^{2}\sin t$

$t[t\sin t-\cos t-1]=-t\cos t-t+t^{2}\sin t$

$\cancel{t^{2}\sin t}-t\cos t\cancel{-t}=-t\cos t\cancel{-t}+\cancel{t^{2}\sin t}$

$0=0$:)

3. Which of the following functions are solutions of the differential equation $y''+y=\sin x$?

A. $y=\sin x$

B. $y=\cos x$

C. $y=\frac{1}{2}x\sin x$

D. $y=-\frac{1}{2}x\cos x$

A. $y=\sin x$

$y'=\cos x$

$y''=-\sin x$

$y''+y=\sin x$

$-\sin x+\sin x=\sin x$

$0=\sin x$

$\boxed{\text{False}}$

B. $y=\cos x$

$y'=-\sin x$

$y''=-\cos x$

$y''+y=\sin x$

$-\cos x+\cos x=\sin x$

$0=\sin x$

$\boxed{\text{False}}$

C. $y=\frac{1}{2}x\sin x$

$y'=\frac{1}{2}x\cos x+\sin x\left(\frac{1}{2}\right)$

$y'=\frac{1}{2}x\cos x+\frac{1}{2}\sin x$

$y''=\frac{1}{2}x(-\sin x)+\cos x\left(\frac{1}{2}\right)+\frac{1}{2}\cos x$

$y''=-\frac{1}{2}x\sin x+\frac{1}{2}\cos x+\frac{1}{2}\cos x$

$y''=-\frac{1}{2}x\sin x+\cos x$

$y''+y=\sin x$

$\cancel{-\frac{1}{2}x\sin x}+\cos x+\cancel{\frac{1}{2}x\sin x}=\sin x$

$\cos x=\sin x$ $\boxed{\text{False}}$

D. $y=-\frac{1}{2}x\cos x$

$y'=-\frac{1}{2}x(-\sin x)+\cos x\left(-\frac{1}{2}\right)$

$y'=\frac{1}{2}x\sin x-\frac{1}{2}\cos x$

$y''=\frac{1}{2}x\cos x+\sin x\left(\frac{1}{2}\right)+\frac{1}{2}(+\sin x)$

$y''=\frac{1}{2}x\cos x+\sin x$

D. cont

$y''+y=\sin x$

$\cancel{\frac{1}{2}x\cos x}+\sin x+\cancel{-\frac{1}{2}x\cos x}=\sin x$

$\sin x=\sin x$

$\boxed{\text{True}}$

4-7: Match the differential equation with its direction field (labeled I-IV). Give reason for your answer.

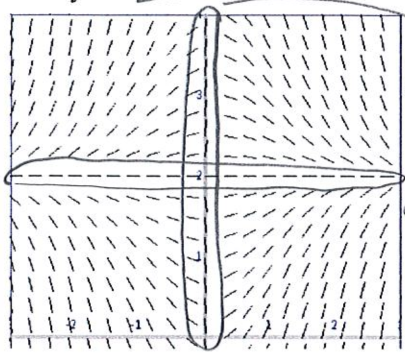
4. $y' = 2 - y$ III

5. $y' = x(2-y)$ I

6. $y' = x + y - 1$ IV

7. $y' = \sin x \sin y$ II

I.

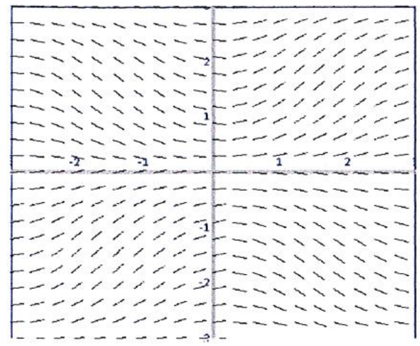


x=0 then $y'=0$

y=2 then $y'=0$

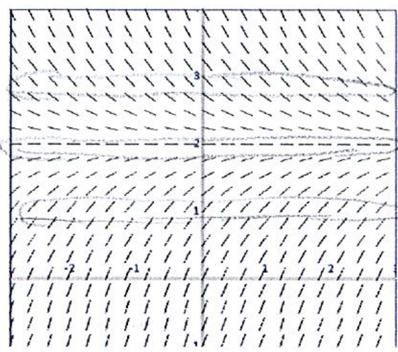
$y'=x(2-y)$

II.



left over :)

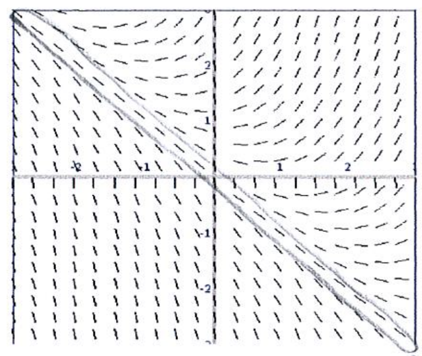
III.



$y'=2-y$ only depends on y.

So y=1,2,3,.. all the same

IV.



$y'=x+y-1$

$\cancel{-1+1}-1$

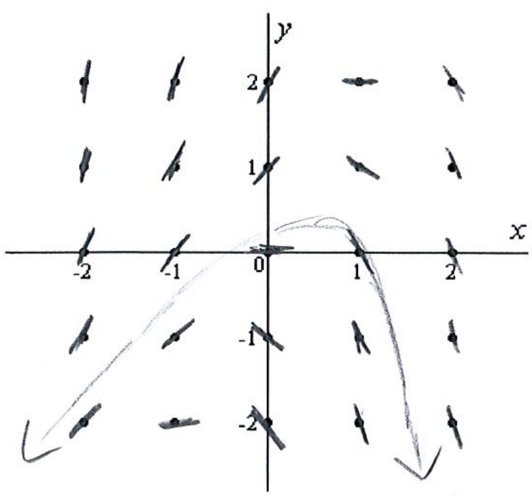
$\cancel{-2+2}-1$

$-3+3 \; -1$

All y's the same

8-9: Sketch the direction field of the differential equation. Then use it to sketch a solution curve that passes through the given point.

8. $y' = y - 2x$, $(1,0)$



$(-2,2) = 2+4$

$(-2,1) = 1+4$

9. $y' = x + y^2$, $(0,0)$