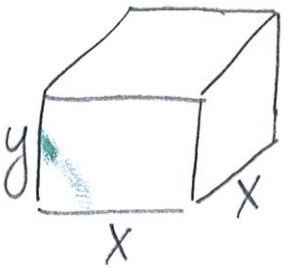
AP Calculus

Optimization $\boxed{40 \times 40 \times 20}$ ✓ $\underset{m'(39)\quad m'(41)}{\searrow - \overset{40}{|} + \nearrow}$

Name______________________ Pd.___

Day 7 Application of Derivatives

1. A box with a square base and open top must have a volume of 32,000 cm^3. Find the dimensions of the box that minimize the amount of material used.



$V = 3200$

$x^2y = 32000$

$y = \frac{32000}{x^2}$

Material = bottom + 4 sides

$= x^2 + 4xy$

$= x^2 + 4x\left(\frac{32000}{x^2}\right)$

Material $= x^2 + 128000x^{-1}$

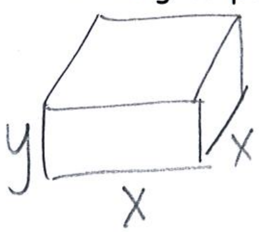
$Mat' = 2x - 128000x^{-2}$

$\frac{128000}{x^2} = 2x$

$2x^3 = 128000$

$x^3 = 64000 \quad x = 40$

2. If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.



$1200 = x^2 + 4xy$

$\frac{1200 - x^2}{4x} = \frac{\cancel{4x}y}{\cancel{4x}}$

$y = 300x^{-1} - \frac{1}{4}x$

$V = x^2y$

$V = x^2\left(300x^{-1} - \frac{1}{4}x\right)$

$V = 300x - \frac{1}{4}x^3$

$V' = 300 - \frac{3}{4}x^2$

$0 = 300 - \frac{3}{4}x^2$

$\cancel{\frac{3}{4}}x^2 = 300 \cdot \frac{4}{3}$

$x^2 = 400$

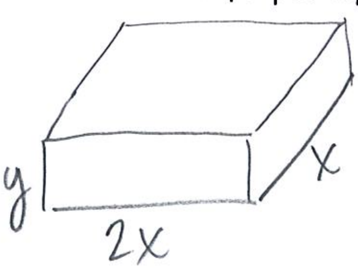
$\boxed{\begin{array}{l} x = 20 \\ y = 10 \end{array}}$

$\underset{v'(19)\quad v'(21)}{\nearrow + \overset{20}{|} - \searrow}$

$V = (20)^2(10)$ ✓

$\boxed{V = 4000\,cm^3}$

3. A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides cost \$6 per square meter. Find the cost of materials for the cheapest such container.



$V = 10$

$10 = 2x \cdot x \cdot y \quad \boxed{y = \frac{5}{x^2}}$

$C = 20x^2 + \frac{180}{x}$

$C(1.651) =$

$\boxed{\$163.54}$ ✓

$Cost = 10(2x \cdot x) + 6(2)(2x \cdot y) + 6(2)(x \cdot y)$

$Cost = 20x^2 + 24xy + 12xy$

$Cost = 20x^2 + 36xy$

$Cost = 20x^2 + 36x\left(\frac{5}{x^2}\right)$

$Cost = 20x^2 + 180x^{-1}$

$Cost' = 40x - 180x^{-2}$

$0 = 40x - 180x^{-2}$

$\frac{180}{x^2} = 40x$

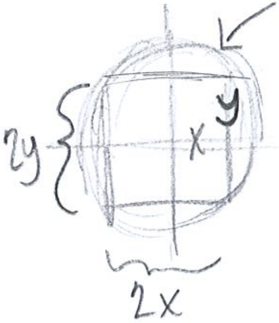
$40x^3 = 180$

$x^3 = 9/2$

$x = \sqrt[3]{9/2} \approx 1.651$

$\underset{C'(1)\quad C'(2)}{\searrow - \overset{1.651}{|} + \nearrow}$

4. Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius r.



$x^2 + y^2 = R^2$

$y^2 = R^2 - x^2$

$y = \sqrt{R^2 - x^2}$

$A =$ length $\cdot$ width

$A = 2x \cdot 2y$

$A = 2x \cdot 2\sqrt{R^2 - x^2}$

$A = 4x\sqrt{R^2 - x^2}$

$A' = 4x\frac{1}{2}(R^2 - x^2)^{-1/2}(-2x) + \sqrt{R^2 - x^2}(4)$

$A' = \frac{-4x^2}{\sqrt{R^2 - x^2}} + \frac{4\sqrt{R^2 - x^2}(\sqrt{R^2 - x^2})}{\sqrt{R^2 - x^2}}$

$0 = -4x^2 + 4(R^2 - x^2)$

$0 = -4x^2 + 4R^2 - 4x^2$

$0 = -8x^2 + 4R^2$

$\frac{8x^2}{8} = \frac{4R^2}{8} \quad x = \sqrt{\frac{1}{2}R^2} = R\sqrt{\frac{1}{2}}$

length $\cdot$ width ✓

$\boxed{2R\sqrt{\frac{1}{2}} \cdot 2R\sqrt{\frac{1}{2}}}$

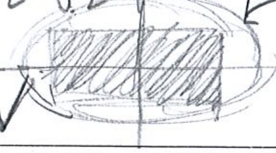
OR

$\boxed{\sqrt{2}R \cdot \sqrt{2}R}$

$A = 2x \cdot 2y = 2a\sqrt{\tfrac{1}{2}} \cdot 2b\sqrt{\tfrac{1}{2}}$

$= 4ab\left(\tfrac{1}{2}\right)$

$\boxed{2ab}$ ✓



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \rightarrow y^2 = b^2\left(1 - \frac{x^2}{a^2}\right)$ $\quad y = \sqrt{b^2\left(1 - \frac{(a\sqrt{\frac{1}{2}})^2}{a^2}\right)}$

$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$ $\quad y = \sqrt{b^2(1 - x^2/a^2)}$ $\quad y = \sqrt{b^2\left(1 - \frac{1}{2}\right)}$

AP Calculus Name $y = \sqrt{b^2(1-x^2/a^2)}$ Pd.__

Optimization Day 7 Application of Derivatives

5. Find the area of the largest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

$A_{max} = \text{length} \cdot \text{width}$

$A = 2\sqrt{b^2\left(1 - \frac{x^2}{a^2}\right)} \cdot 2x$

$A = 4x\sqrt{b^2 - \frac{b^2}{a^2}x^2}$

$A' = 4x \cdot \frac{1}{\cancel{2}}\left(b^2 - \frac{b^2}{a^2}x^2\right)^{-1/2}\left(\frac{-\cancel{2}b^2}{a^2}x\right) + \sqrt{b^2 - \frac{b^2}{a^2}x^2}\,(4)$ $\quad \boxed{y = b\sqrt{\tfrac{1}{2}}}$

$A' = \frac{-4x^2b^2}{a^2\sqrt{b^2 - \frac{b^2}{a^2}x^2}} + \frac{4\sqrt{b^2 - \frac{b^2}{a^2}x^2}\,\left(a^2\sqrt{b^2 - \frac{b^2}{a^2}x^2}\right)}{a^2\sqrt{b^2 - \frac{b^2}{a^2}x^2}}$

$0 = -4x^2b^2 + 4a^2\left(b^2 - \frac{b^2}{a^2}x^2\right)$

$0 = -4x^2b^2 + 4a^2b^2 - 4x^2b^2$

$0 = -8x^2b^2 + 4a^2b^2$

$\frac{8x^2b^2}{8b^2} = \frac{4a^2b^2}{8b^2}$

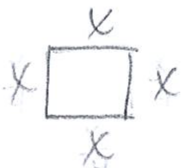
$x^2 = \frac{1}{2}a^2$

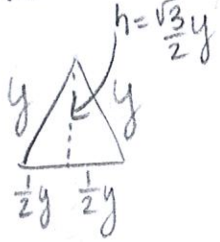
$x = \sqrt{\frac{1}{2}a^2} = \boxed{a\sqrt{1/2}}$

6. A piece of wire 10 m long is cute into two pieces. One piece is bent into a square and the other is bent into and equilateral triangle. How should the wire be cut so that the total area enclosed is a.) a maximum? b.) a minimum?

$\underline{\quad 4x \quad}\ \underline{\quad 3y \quad} = 10$

$4x + 3y = 10$





$4x = 10 - 3y$

$x = \frac{10}{4} - \frac{3}{4}y$

$A_{max} = \text{Area sq} + \text{Area } \Delta$

$A = x \cdot x + \frac{1}{2}(y)\left(\frac{\sqrt{3}}{2}y\right)$

$A = x^2 + \frac{\sqrt{3}}{4}y^2$

$A = \left(\frac{10}{4} - \frac{3}{4}y\right)^2 + \frac{\sqrt{3}}{4}y^2$

$A' = \frac{100}{16} - \frac{30}{8}y + \frac{9}{16}y^2 + \frac{\sqrt{3}}{4}y^2$

$16(0) = \left(\frac{100}{16} - \frac{30}{8}y + \frac{9}{16}y^2 + \frac{\sqrt{3}}{4}y^2\right)16$

$0 = 100 - 60y + 9y^2 + 4\sqrt{3}y^2$

omit... I am not getting an answer & can not find my mistake :(

7. A cylindrical can without a top is made to contain V cm^3 of liquid. Find the dimensions that will minimize the cost of the metal to make a can.

$V = \pi R^2 h$

$h = \frac{V}{\pi R^2}$

$C = \pi R^2 + 2\pi R h$

$C = \pi R^2 + 2\cancel{\pi} R\left(\frac{V}{\cancel{\pi} R^2}\right)$

$C = \pi R^2 + 2VR^{-1}$

$C' = 2\pi R - 2VR^{-2}$

$0 = 2\pi R - \frac{2V}{R^2}$

$\frac{2V}{R^2} = 2\pi R$

$\frac{2V}{2\pi} = \frac{2\pi R^3}{2\pi}$

$R^3 = \frac{V}{\pi}$

$R = \boxed{\sqrt[3]{\frac{V}{\pi}}}$ ✓

$h = \frac{V}{\pi R^2} = \boxed{\frac{V}{\pi\left(\sqrt[3]{\frac{V}{\pi}}\right)^2}}$ ✓