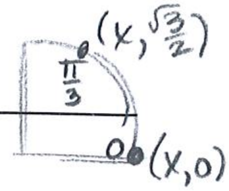
$$\frac{x^{1/3}}{x^{1/2}} = x^{\frac{1}{3}-\frac{1}{2}} = x^{\frac{2}{6}-\frac{3}{6}} = x^{-1/6}$$



AP Calculus

Definite Integrals and Rate of Change

Name________________________

Integration Day 7

1-4: Evaluate each integral.

1. $\int_1^4 \frac{\sqrt{y}-y}{y^2}\,dy = \int_1^4 \frac{y^{1/2}}{y^2} - \frac{y}{y^2}\,dy = \int_1^4 y^{-3/2} - \frac{1}{y}\,dy$

$$-\frac{2}{1}y^{-1/2} - \ln|y|\Big]_1^4 = -\frac{2}{\sqrt{y}} - \ln|y|\Big]_1^4$$

$$\left[\frac{-2}{\sqrt{4}} - \ln(4)\right] - \left[-\frac{2}{\sqrt{1}} - \ln(1)\right] = -1 - \ln 4 + 2 = \boxed{1 - \ln 4}$$ ✓

2. $\int_0^1 (5x - 5^x)\,dx$ $\quad \frac{5x^2}{2} - \frac{5^x}{\ln 5}\Big|_0^1$

$$\left[\frac{5}{2}(1)^2 - \frac{5^1}{\ln 5}\right] - \left[\frac{5}{2}(0)^2 - \frac{5^0}{\ln 5}\right]$$ ✓

$$\frac{5}{2} - \frac{5}{\ln 5} + \frac{1}{\ln 5} = \boxed{\frac{5}{2} - \frac{4}{\ln 5}}$$

3. $\int_1^{64} \frac{1+\sqrt[3]{x}}{\sqrt{x}}\,dx = \int_1^{64} \frac{1}{x^{1/2}} + \frac{x^{1/3}}{x^{1/2}}\,dx$

$$\int_1^{64} x^{-1/2} + x^{-1/6} = \frac{2}{1}x^{1/2} + \frac{6}{5}x^{5/6}\Big|_1^{64}$$

$$\left[2\sqrt{64} + \frac{6}{5}(\sqrt[6]{64})^5\right] - \left[2\sqrt{1} + \frac{6}{5}(\sqrt[6]{1})^5\right]$$

$$2(8) + \frac{6}{5}(2)^5 - 2 - \frac{6}{5} = 16 + \frac{192}{5} - 2 - \frac{6}{5} = 14 + \frac{186}{5} = \frac{70}{5} + \frac{186}{5} = \boxed{\frac{256}{5}}$$ ✓

4. $\int_0^{\frac{\sqrt{3}}{2}} \frac{dr}{\sqrt{1-r^2}}$

$$\sin^{-1} R\Big|_0^{\sqrt{3}/2} = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}(0)$$

$$\frac{\pi}{3} - 0$$

$$\boxed{\frac{\pi}{3}}$$ ✓

5. If w'(t) is the rate of growth of a child in pounds per year, what does $\int_5^{10} w'(t)\,dt$ represent?

$\int_5^{10} \frac{\text{pounds}}{\text{year}}$ = totals pounds grew from age 5 to age 10.

6. If oil leaks from a tank at a rate of r(t) gallons per minute at time t, what does $\int_0^{120} r(t)\,dt$ represent?

$\int_0^{120} \frac{\text{gallons}}{\text{minute}}$ = total gallons of oil that leak over 120 minutes

7. A honeybee population starts with 100 bees and increases at a rate of n'(t) bees per week. What does $100 + \int_0^{15} n'(t)\,dt$ represent?

$100 + \int_0^{15} \frac{\text{Bee's}}{\text{week}}$ = total bees after 15 weeks

What you start with + Bee's added over 15 weeks

8-9: The velocity function (in meters per second) is given for a particle moving along a line. Find a.) the displacement and b.) the distance traveled by the particle during the given time

8. $v(t) = 3t - 5, \quad 0 \le t \le 3$

displacement $= \int_0^3 3t - 5\,dt = \frac{3t^2}{2} - 5t\Big|_0^3$

$$\left[\frac{3}{2}(3)^2 - 15\right] - \left[\frac{3}{2}(0) - 0\right] = \frac{27}{2} - \frac{30}{2} = \boxed{-\frac{3}{2}}$$ ✓

distance $= \int_0^3 |3t-5|\,dt$ $\quad 3t - 5 = 0 \quad t = 5/3$

0 − 5/3 + 3

$$-\int_0^{5/3}(3t-5)\,dt + \int_{5/3}^{3}(3t-5)\,dt$$

$$-[-4.1\overline{6}] + 2.\overline{6} = \boxed{6.8\overline{3}}$$ ✓

9. $v(t) = t^2 - 2t - 8, \quad 1 \le t \le 6$

displacement $= \int_1^6 t^2 - 2t - 8\,dt = \boxed{-10/3}$ ✓

distance $= \int_1^6 |t^2 - 2t - 8|\,dt$

$t^2 - 2t - 8 = 0$

$(t+2)(t-4) = 0$

$t = 4, -2$

1 − 4 + 6

$$-\int_1^4 t^2 - 2t - 8 + \int_4^6 t^2 - 2t - 8 = -[-18] + 14.\overline{6} = \boxed{32.\overline{6}}$$ ✓

$\frac{1}{2}t^2+4t+5=0$ $t=-6.45$ & $t=-1.55$

10-11: The acceleration function (in m/s^2) and the initial velocity are given for a particle moving along a line. Find a.) the velocity at time t and b.) the distance traveled during the given time

10. $\int a(t)=t+4, \quad v(0)=5, \quad 0\le t\le 10$

$v(t)=\frac{t^2}{2}+4t+C$ $v(t)=\frac{1}{2}t^2+4t+5$

$5=\frac{0^2}{2}+4(0)+C$ $\int_0^{10}|v(t)|dt$

$C=5$ $\int_0^{10}\frac{1}{2}t^2+4t+5\,dt=\boxed{416.\bar{6}}$

11. $\int a(t)=2t+3, \quad v(0)=-4, \quad 0\le t\le 3$

$v(t)=t^2+3t+C$

$-4=0^2+3(0)+C$

$C=-4$

$v(t)=t^2+3t-4$

$(t-1)(t+4)$ → $t=1$, $t=-4$

0 − 1 + 3

$-\int_0^1 v(t)+\int_1^3 v(t)$

$-[-2.1\bar{6}]+12.6$

$14.8\bar{3}$

12. The linear density of a rod of length 4 m is given by $p(x)=9+2\sqrt{x}$ measured in kilograms per meter, where x is measured in meters from one end of the rod. Find the total mass of the rod. $\int_0^4 9+2\sqrt{x}\,dx=\boxed{46.\bar{6}\text{ kg}}$

13. Water flows from the bottom of a storage tank at a rate of $r(t)=200-4t$ liters per minute, where $0\le t\le 50$. Find the amount of water that flows from the tank during the first 10 minutes.

$\int_0^{10} 200-4t\,dt=\boxed{1000\text{ liters}}$

14. The velocity of a car was read from its speedometer at 10-second intervals and recorded in the table. Use the Midpoint Rule to estimate the distance traveled by the car.

t(s)	v(mi/h)	t(s)	v(mi/h)
0	0	60	56
10	38	70	53
20	52	80	50
30	58	90	47
40	55	100	45
50	51		

A = width · length

$=\frac{1}{180}[38+58+51+53+47]=\boxed{1.37\bar{2}\text{ mi}}$

$\frac{100-0}{5}=20\text{sec}\cdot\frac{1\text{min}}{60\text{sec}}\cdot\frac{1\text{hr}}{60\text{min}}=\frac{20}{3600}=\frac{1}{180}\text{hR}$

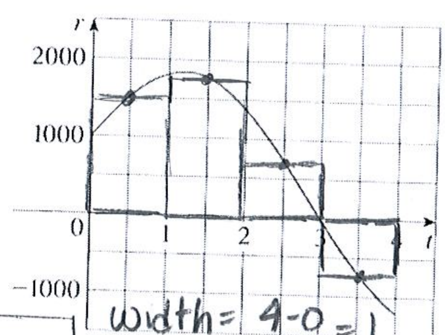
15. Water flows into and out of a storage tank. A graph of the rate of change r(t) of the volume of water in the tank, in liters per day, is shown. If the amount of water in the tank at time t=0 is 25,000 L, use the Midpoint Rule of estimate the amount of water in the tank four days later.

Amount of water = what you Start + how much Comes in oR out

$=25{,}000+\int_0^4 R(t)$

$=25{,}000+$ width · length

$=25{,}000+1[1500+1750+750-750]=\boxed{28{,}250\text{ LiteRS}}$



width $=\frac{4-0}{4}=1$