Review: Curve Sketching $Y_1 = x^2 \cdot \cos(x^2)$ Name______________________

1. Let f be a function with a second derivative given by $f''(x) = x^2(x-3)(x-6)$. What are the x –coordinates of the points of inflection of the graph of f? $x^2=0$ $x-3=0$ $x-6=0$

$x=0$ $x=3$ $x=6$

(A) 0 *only*
(B) 3 *only*
(C) 0 *and* 6 *only*
(D) 3 *and* 6 *only*
(E) $0, 3,$ *and* 6

POI

$-\infty$ (+) 0 (+) 3 (−) 6 (+) ∞

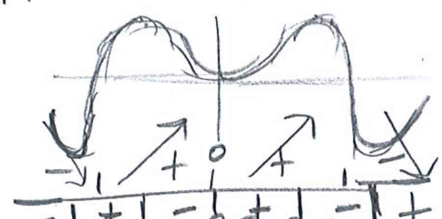
$f''(-1)=(+)(-)(-)=+$ $f''(1)=(+)(-)(-)=+$

$f''(4)=(+)(+)(-)=-$ $f''(7)=(+)(+)(+)=+$

Calculator 2. The derivative of the function f is given by $f'(x) = x^2 cos(x^2)$. How many points of inflection does the graph of f have on the open interval $(-2,2)$?

$Y_1 = x\cos(x^2)$ $(-2,2)$

(A) One
(B) Two
(C) Three
(D) Four
(E) Five



− + − + − +

POI POI POI POI POI

3. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

$f'(x) = 2xe^x + e^x(2)$

$f'(x) = 2e^x(x+1)$

$f''(x) = 2e^x(1) + (x+1)2e^x$

$f''(x) = 2e^x(1+x+1)$

$f''(x) = 2e^x(x+2)$

(A) $x < -2$
(B) $x > -2$
(C) $x < -1$
(D) $x > -1$
(E) $x < 0$

− −2 +

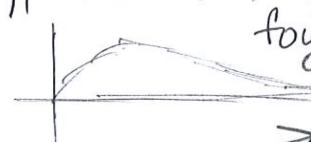
$\cancel{2e^x=0}$ garbage $x+2=0$ $x=-2$

$f''(0)=(+)(+)$

$f''(-3)(+)(-)$

Calculator 4. The function f has first derivative given by $f'(x) = \frac{\sqrt{x}}{1+x+x^3}$. What is the x –coordiante of the inflection point of the graph of f? $Y_1 = (\sqrt{x}) \div (1+x+x^3)(-1,2)$

(A) 1.008
(B) 0.473
(C) 0
(D) −0.278
(E) The graph of f has no inflection point.



found max on calc

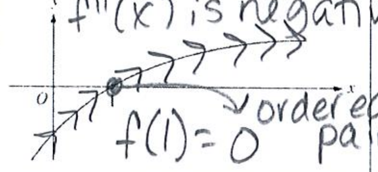
+ .473 −

5. The graph of a twice-differentiable function f is shown in the figure. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$
(B) $f(1) < f''(1) < f'(1)$
(C) $f'(1) < f(1) < f''(1)$
(D) $f''(1) < f(1) < f'(1)$
(E) $f''(1) < f'(1) < f(1)$

f is increasing $f'(x)$ = positive

f is concavedown $f''(x)$ is negative



ordered pair $f(1)=0$

$f''(1) < f(1) < f'(1)$

neg < 0 < pos

6. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$
(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
(C) $(0, \infty)$
(D) $(-\infty, 0)$
(E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

$f'(x) = 4x^3 + 2x$

$0 = 2x(x^2+1)$

$2x=0$ $x^2+1=0$

$\cancel{\sqrt{x^2}=\sqrt{-1}}$ garbage

− 0 +

$f'(-1)=(-)(+)$ $f'(1)=(+)(+)=+$

7. What are all values of x for which the function f defined by $f(x) = (x^2-3)e^{-x}$ is increasing?

− −1 + 3 −

(A) There are no such values of x.
(B) $x < -1$ and $x > 3$
(C) $-3 < x < 1$
(D) $-1 < x < 3$
(E) All values of x

$f'(-2)=-$ $f'(4)=-$

$f'(0)=+$

$f'(x) = (x^2-3)e^{-x}(-1) + e^{-x}(2x)$

$f'(x) = e^{-x}(-x^2+3+2x)$

$f'(x) = e^{-x}(-x^2+2x+3)$

$\cancel{e^{-x}=0}$ garbage

$-x^2+2x+3=0$

$x^2-2x-3=0$ $(x-3)(x+1)$ $x=3, x=-1$

Calculator 8. The graph of the function $y = x^3 + 6x^2 + 7x - 2cosx$ changes concavity at $x =$

(A) −1.58
(B) −1.63
(C) −1.67
(D) −1.89
(E) −2.33

$y' = 3x^2 + 12x + 7 - 2(-\sin x)$

$y' = 3x^2 + 12x + 7 + 2\sin x$

$y'' = 6x + 12 + 2\cos x$

$0 = 6x + 12 + 2\cos x$

$Y_1 = 6x + 12 + 2\cos x\ [-3,-1]$

$x = -1.8941$

$6x-12=0$ $x^5=0$
$6x=12$ $x=2$ $x=0$ $+ \; 0 \; - \; 2 \; +$

Calculator 9. If the derivative of f is given by $f'(x) = e^x - 3x^2$, at which of the following values of x does f have a relative maximum value? $Y_1 = e^x - 3x^2$ $[-1,4]$

(A) -0.46
(B) 0.20
(C) 0.91
(D) 0.95
(E) 3.73

10. At what value of x does the graph of $y = \frac{1}{x^2} - \frac{1}{x^3}$ have a point of inflection?

(A) 0
(B) 1
(C) 2
(D) 3
(E) At no value of x

$y = x^{-2} - x^{-3}$
$y' = -2x^{-3} + 3x^{-4}$
$y'' = 6x^{-4} - 12x^{-5}$
$= \frac{(x)6}{(x)x^4} - \frac{12}{x^5} = \frac{6x-12}{x^5}$

11. How many critical points does the function $f(x) = (x+2)^5(x-3)^4$ have?

(A) *One*
(B) *Two*
(C) *Three*
(D) *Five*
(E) *Nine*

$f'(x) = (x+2)^5(4)(x-3)^3(1) + (x-3)^4 5(x+2)^4(1)$
$f'(x) = (x+2)^4(x-3)^3[4(x+2)+5(x-3)]$
$f'(x) = (x+2)^4(x-3)^3[4x+8+5x-15]$
$0 = (x+2)^4(x-3)^3(9x-7)$

12. The graph of $y = \frac{-5}{x-2}$ is concave downward for all values of x such that

$+ \; 2 \; -$

(A) $x < 0$
(B) $x < 2$
(C) $x < 5$
(D) $x > 0$
(E) $x > 2$

$y' = \frac{(x-2)(0) - (-5)(1)}{(x-2)^2}$
$y' = \frac{5}{(x-2)^2}$
$y'' = \frac{(x-2)^2(0) - 5(2)(x-2)(1)}{(x-2)^4}$
$\frac{-10(x-2)}{(x-2)^4} = \frac{10}{-(x-2)^3}$

13. Let f be a polynomial function with degree greater than 2. If $a \neq b$ and $f(a) = f(b) = 1$, which of the following must be true for at least one value of x between a and b?

I. $f(x) = 0$
II. $f'(x) = 0$
III. $f''(x) = 0$

Rolle's Thm

(A) *None*
(B) *I only*
(C) *II only*
(D) *I and II only*
(E) *I, II, and III*

14. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2,4]$ occurs at $x =$

(A) 4
(B) 2
(C) 1
(D) 0
(E) -2

$f'(x) = 3x^2 - 6x$
$0 = 3x(x-2)$
$x=0$ $x=2$
$f(-2) = -8$
$f(0) = 12$
$f(2) = 8$
$f(4) = 28$ (Absolute max)

15. The function defined by $f(x) = x^3 - 3x^2$ for all real numbers x has a relative maximum at $x =$

(A) -2
(B) 0
(C) 1
(D) 2
(E) 4

$f'(x) = 3x^2 - 6x$
$0 = 3x(x-2)$
$x=0$ $x=2$
$+ \; 0 \; - \; 2 \; +$
$f'(1) = +(-)$
$f'(-1) = (-)(-) = +$
$f'(3) = (+)(+)$

16. If $f(x) = \frac{\ln x}{x}$, for all $x > 0$, which of the following is true?

$f'(x) = \frac{x(\frac{1}{x}) - \ln x(1)}{x^2}$

(A) f is increasing for all x greater than 0.
(B) f is increasing for all x greater than 1.
(C) f is decreasing for all x between 0 and 1.
(D) f is decreasing for all x between 1 and e.
(E) f is decreasing for all x greater than e.

$f'(x) = \frac{1 - \ln x}{x^2}$
$0 \; + \; e \; -$
$1 - \ln x = 0$ $\ln x = 1$ $x = e$ $x = 0$

Calculator 17. The graph of $y = 5x^4 - x^5$ has a point of inflection at

(A) $(0,0)$ *only*
(B) $(3,162)$ *only*
(C) $(4,256)$ *only*
(D) $(0,0)$ *and* $(3,162)$
(E) $(0,0)$ *and* $(4,256)$

$y' = 20x^3 - 5x^4$
$y'' = 60x^2 - 20x^3$
$0 = 20x^2(3-x)$
$x=0$ $x=3$

$+ \; 0 \; + \; 3 \; -$

$f''(-1) = (+)(+)$ $f''(4) = (+)(-)$
$f''(1) = (1)(+)$

18. At $x = 0$, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$?

(A) f is increasing.
(B) f is decreasing.
(C) f is discontinuous.
(D) f has a relative minimum.
(E) f has a relative maximum.

$f'(x) = 2x + e^{-2x}(-2)$
$f'(x) = 2x - 2e^{-2x}$
$f'(0) = 2(0) - 2e^0 = -2 = $ neg.