AP Calculus
Volume

Name________________________

Application of Integration Day 6

1-5: Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region and the solid.

1. $y = x^2$, $x = y^2$; about $y = 1$

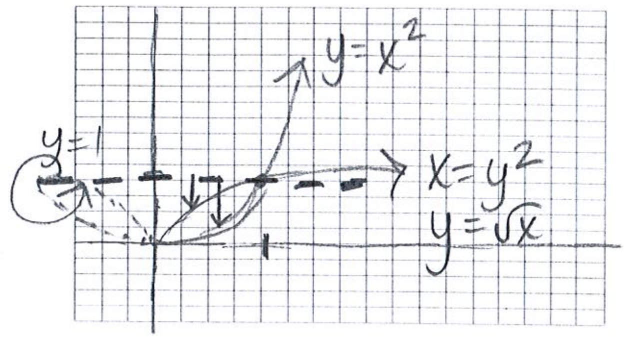
$\pi \int_0^1 (x^2-1)^2 - (\sqrt{x}-1)^2 \, dx$

$\pi \int_0^1 x^4 - 2x^2 + \cancel{1} - x + 2\sqrt{x} - \cancel{1} \, dx$

$\pi \int_0^1 x^4 - 2x^2 - x + 2x^{1/2} \, dx$

$\pi \left[\frac{1}{5}x^5 - \frac{2}{3}x^3 - \frac{1}{2}x^2 + \frac{2 \cdot 2x^{3/2}}{3}\right]_0^1$

$\pi \left[\frac{1}{5} - \frac{2}{3} - \frac{1}{2} + \frac{4}{3} - \cancel{0}\right]$ $\pi \left[\frac{6}{30} - \frac{20}{30} - \frac{15}{30} + \frac{40}{30}\right] = \boxed{\frac{11\pi}{30}}$ ✓



2. $y = e^{-x}$, $y = 1$, $x = 2$; about $y = 2$

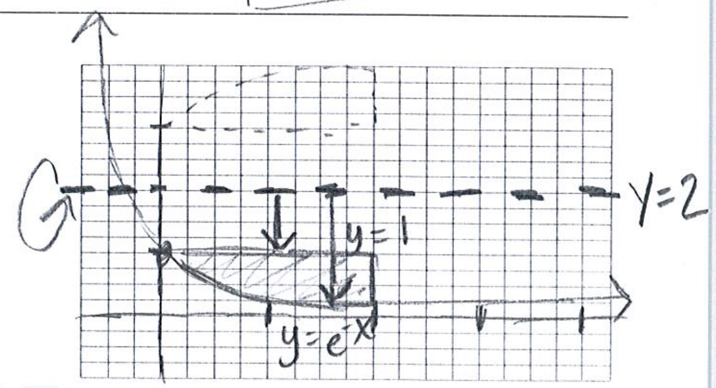
$\pi \int_0^2 (e^{-x}-2)^2 - (1-2)^2 \, dx$

$\pi \int_0^2 e^{-2x} - 4e^{-x} + 4 - 1 \, dx$

$\pi \int_0^2 e^{-2x} - 4e^{-x} + 3$

$\pi \left[-\frac{1}{2}e^{-2x} + 4e^{-x} + 3x\right]_0^2 = \pi\left[-\frac{1}{2}e^{-4} + 4e^{-2} + 6\right] - \left[-\frac{1}{2} + 4 + 0\right]$ ✓

$\pi\left[-\frac{1}{2}e^{-4} + 4e^{-2} + 6 + 1/2 - 4\right] = \boxed{\pi\left[-\frac{1}{2}e^{-4} + 4e^{-2} + \frac{5}{2}\right]}$

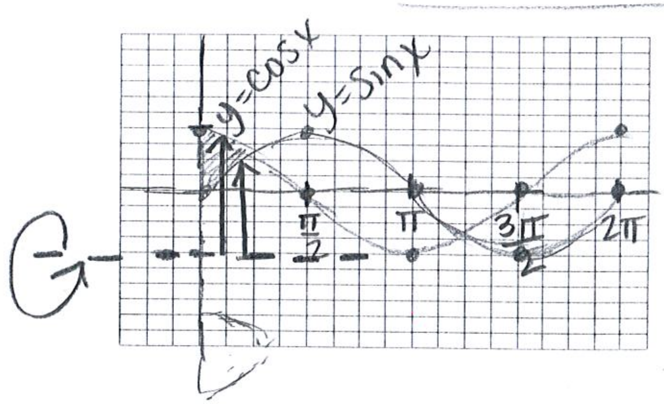


3. $y = \sin x$, $y = \cos x$, $0 \le x \le \frac{\pi}{4}$; about $y = -1$

$\pi \int_0^{\frac{\pi}{4}} (\cos x - -1)^2 - (\sin x - -1)^2 \, dx$

$\pi \int_0^{\frac{\pi}{4}} (\cos x + 1)^2 - (\sin x + 1)^2 \, dx$

$\pi \left(2\sqrt{2} - \frac{3}{2}\right)$



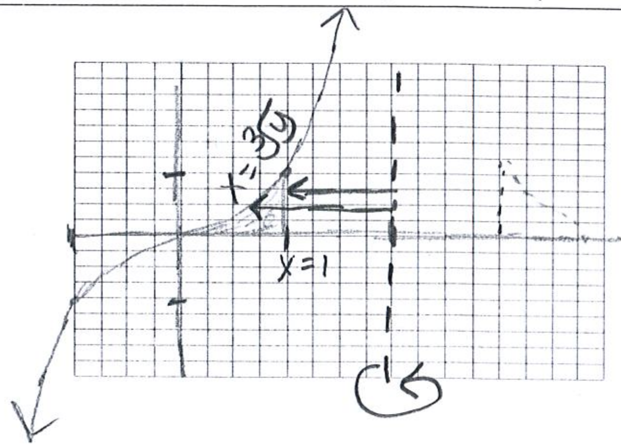
AP Calculus   Name______________________

Volume   Application of Integration Day 6

4. $y=x^3$, $y=0$, $x=1$; about $x=2$

$$\pi\int_0^1 (\sqrt[3]{y}-2)^2-(1-2)^2\,dy$$

$$\boxed{\frac{3\pi}{5}}$$

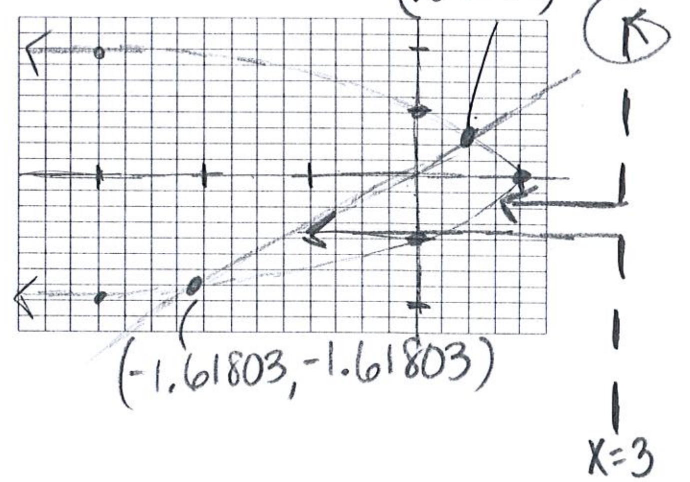


5. $y=x$, $x=1-y^2$; about $x=3$

$x=1-y^2$
$y^2=1-x$
$y=\pm\sqrt{1-x}$

$$\pi\int_{-1.61803}^{.618034} (y-3)^2-(1-y^2-3)^2\,dy$$

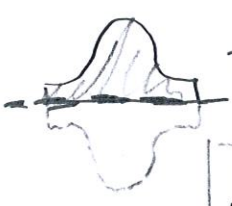
$$\boxed{35.124}$$



6: Set up an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Then use your calculator to evaluate the integral correct to five decimal places.

6. $y=e^{-x^2}$, $y=0$, $x=-1$, $x=1$;

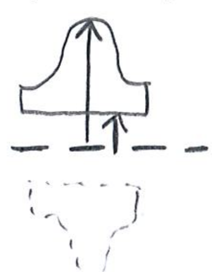
A.) About the x-axis

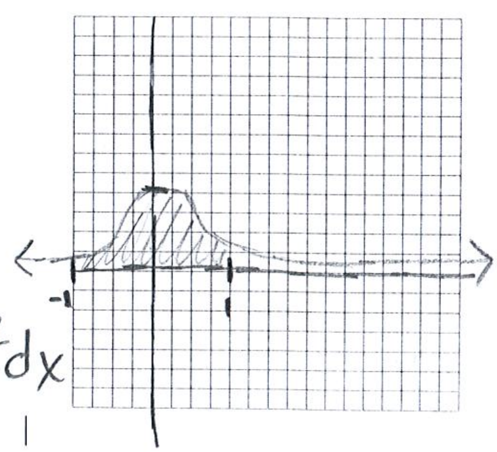


$$\pi\int_{-1}^{1} (e^{-x^2}-0)^2\,dx$$

$$\boxed{3.75825}$$

B.) About $y=-1$





$$\pi\int_{-1}^{1} (e^{-x^2}+1)^2-(0+1)^2\,dx$$

$$\boxed{13.14312}$$

AP Calculus

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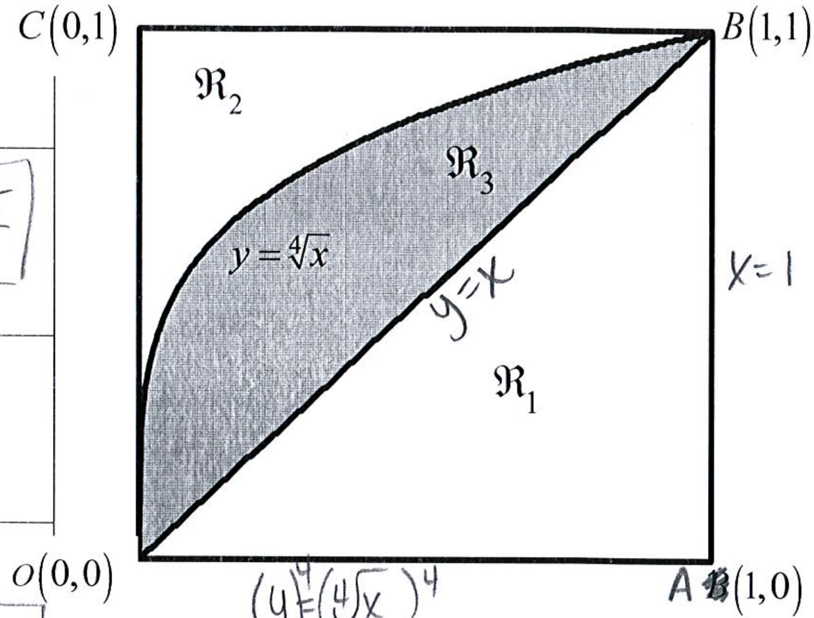
7-18: Refer to the figure and find the volume generated by rotating the given region about the specified line.



$(y)^4=(\sqrt[4]{x})^4$

$x=y^4$

dx 7. $\mathfrak{R}_1$ about OA

$\pi\int_0^1 (x-0)^2\,dx = \boxed{\frac{\pi}{3}}$ ✓

dy 8. $\mathfrak{R}_1$ about OC

$\pi\int_0^1 (1-0)^2-(y-0)^2 = \boxed{\frac{2\pi}{3}}$

dy 9. $\mathfrak{R}_1$ about AB

$\pi\int_0^1 (y-1)^2\,dy = \boxed{\frac{\pi}{3}}$

dx 10. $\mathfrak{R}_1$ about BC

$\pi\int (0-1)^2-(x-0)^2\,dx = \boxed{\frac{2\pi}{3}}$

dx 11. $\mathfrak{R}_2$ about OA

$\pi\int_0^1 (1-0)^2-(x^{1/4}-0)^2\,dx$

$\boxed{\frac{\pi}{3}}$

dy 12. $\mathfrak{R}_2$ about OC

$\pi\int_0^1 (y^4-0)^2\,dy$

$\boxed{\frac{\pi}{9}}$

dy 13. $\mathfrak{R}_2$ about AB

$\pi\int (0-1)^2-(y^4-1)^2\,dy$

$\frac{13\pi}{45}$

dx 14. $\mathfrak{R}_2$ about BC

$\pi\int_0^1 (\sqrt[4]{x}-1)^2\,dx$

$\boxed{\frac{\pi}{15}}$

15. $\mathfrak{R}_3$ about OA

dx

$\pi\int_0^1 (\sqrt[4]{x}-0)^2-(x-0)^2\,dx$

$\boxed{\frac{\pi}{3}}$

16. $\mathfrak{R}_3$ about OC

dy

$\pi\int_0^1 (y-0)^2-(y^4-0)^2\,dy$

$\boxed{\frac{2\pi}{9}}$

17. $\mathfrak{R}_3$ about AB

dy

$\pi\int_0^1 (y^4-1)^2-(y-1)^2\,dy$

$\boxed{\frac{17\pi}{45}}$

18. $\mathfrak{R}_3$ about BC

dx

$\pi\int_0^1 (x-1)^2-(\sqrt[4]{x}-1)^2\,dx$

$\boxed{\frac{4\pi}{15}}$