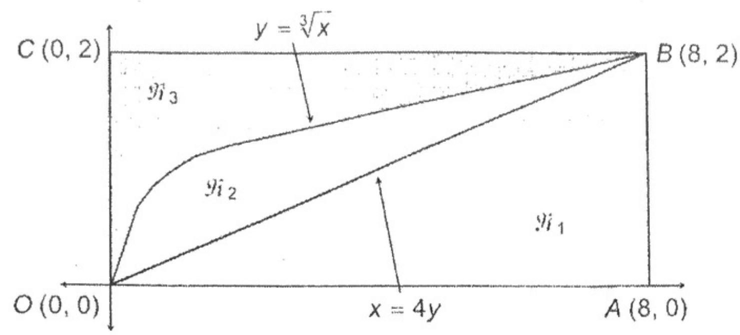
AP Calculus

Area & Volume

Name________________

Day 6 App of Int Pd.______ Day_____



Find the area of the given region.

1. $\mathcal{R}_1$

$$\int_0^8 \frac{x}{4}\,dx = 8$$

2. $\mathcal{R}_2$

$$\int_0^8 \sqrt[3]{x} - \frac{x}{4}\,dx = 4$$

3. $\mathcal{R}_3$

$$\int_0^8 2 - \sqrt[3]{x}\,dx = 4$$

Find the volume generated by rotating the given region about the given line

4. $\mathcal{R}_1$ about OA

$$\pi\int_0^8 \left(\frac{x}{4}\right)^2 dx = \frac{32\pi}{3}$$

5. $\mathcal{R}_1$ about AB

$$\pi\int_0^2 (8-4y)^2\,dy = \frac{128\pi}{3}$$

6. $\mathcal{R}_1$ about OC

$$\pi\int_0^2 8^2 - (4y)^2\,dy$$

$$= \frac{256\pi}{3}$$

7. $\mathcal{R}_2$ about OA

$$\pi\int_0^8 \left(\sqrt[3]{x}\right)^2 - \left(\frac{x}{4}\right)^2 dx$$

$$= \frac{128\pi}{15}$$

8. $\mathcal{R}_2$ about OC

$$\pi\int_0^2 (4y)^2 - (y^3)^2\,dy$$

$$= \frac{512\pi}{21}$$

9. $\mathcal{R}_2$ about line $y = -5$

$$\pi\int_0^8 \left(5 + \sqrt[3]{x}\right)^2 - \left(5 + \frac{x}{4}\right)^2 dx$$

$$= \frac{728\pi}{15}$$

10. $\mathcal{R}_3$ about AB

$$\pi\int_0^2 8^2 - (8 - y^3)^2\,dy$$

$$= \frac{320\pi}{7}$$

11. $\mathcal{R}_3$ about BC

$$\pi\int_0^8 (2 - \sqrt[3]{x})^2\,dx$$

$$= \frac{16\pi}{5}$$

12. $\mathcal{R}_3$ about OC

$$\pi\int_0^2 (y^3)^2\,dy = \frac{128\pi}{7}$$

Area & Volume Continued

Let $\mathcal{R}$ be the region bounded by the curves $y = \frac{1}{\sqrt{x}}$, $y = 1$, and $x = 4$.

a.) Find the area of $\mathcal{R}$.

$$\int_1^4 \left(1 - \frac{1}{\sqrt{x}}\right) dx = 1$$

b.) Suppose the line $x = k$ divides $\mathcal{R}$ into two regions of equal area. Find the value of k.

$$\int_1^k \left(1 - \frac{1}{\sqrt{x}}\right) dx = \frac{1}{2}(1)$$

$$k = 2.914$$

c.) Find the volume of the solid generated by revolving $\mathcal{R}$ about the $y - axis$.

$$\pi \int_{1/2}^{1} 4^2 - \left(\frac{1}{y^2}\right)^2 dy = \frac{17\pi}{3}$$

d.) Find the volume of the solid generated by revolving $\mathcal{R}$ about the line $y = 2$.

$$\pi \int_1^4 \left(2 - \frac{1}{\sqrt{x}}\right)^2 - (2-1)^2 dx = 7.497$$

e.) Find the volume of the solid whose base is the region $\mathcal{R}$ and whose cross sections cut by perpendicular planes to the $x - axis$ are equilateral triangles.

$$\frac{\sqrt{3}}{4} \int_1^4 \left(1 - \frac{1}{\sqrt{x}}\right)^2 dx = .167$$