Name________________________ Pd.____

AP Calculus

Chain Rule

1-3: Write the composite function in the form f(g(x)). [Identify the inner function u=g(x) and the outer function y=f(u).] Then find the derivative dy/dx.

1. $y=(2x^3+5)^4$

$y'=4(2x^3+5)^3(6x^2)$

$\boxed{y'=24x^2(2x^3+5)^3}$

2. $y=\sin(\cot x)$

$y'=\cos(\cot x)(-\csc^2 x)$

$=\boxed{-\cos(\tan x)\csc^2 x}$

3. $y=e^{\sqrt{x}}=e^{x^{1/2}}$

$y'=e^{x^{1/2}}\cdot\frac{1}{2}x^{-1/2}$

$=\dfrac{e^{x^{1/2}}}{2\sqrt{x}}=\boxed{\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}}$

4-11 Find the derivative of each function

4. $F(x)=(4x-x^2)^{100}$

$F'(x)=100(4x-x^2)^{99}(4-2x)$

$F'(x)=(400-200x)(4x-x^2)^{99}$

$F'(x)=200(2-x)(4x-x^2)^{99}$

5. $f(t)=\sin(e^t)+e^{\sin t}$

$f'(t)=\cos(e^t)\cdot e^t+e^{\sin t}\cdot\cos t$

$=e^t\cos(e^t)+e^{\sin t}\cos t$

6. $f(x)=(2x-3)^4(x^2+x+1)^5$

Product rule

$f'(x)=(2x-3)^4\cdot 5(x^2+x+1)^4(2x+1)+(x^2+x+1)^5\cdot 4(2x-3)^3\cdot 2$

$=5(2x-3)^4(x^2+x+1)^4(2x+1)+8(2x-3)^3(x^2+x+1)^5$

* video for further simplification

$=(x^2+x+1)^4(2x-3)^3[5(2x-3)(2x+1)+8(x^2+x+1)]$

$=(x^2+x+1)^4(2x-3)^3[5(4x^2-4x-3)+8x^2+8x+8]$

$=(x^2+x+1)^4(2x-3)^3(20x^2-20x-15+8x^2+8x+8)$

$=(x^2+x+1)^4(2x-3)^3(28x^2-12x-7)$

7. $y=\left(\dfrac{x^2+1}{x^2-1}\right)^3$

$y'=3\left(\dfrac{x^2+1}{x^2-1}\right)^2\left[\dfrac{(x^2-1)(2x)-(x^2+1)(2x)}{(x^2-1)^2}\right]$

$=\dfrac{3(x^2+1)^2}{(x^2-1)^2}\left[\dfrac{2x^3-2x-2x^3-2x}{(x^2-1)^2}\right]$

$=\dfrac{3(x^2+1)^2}{(x^2-1)^2}\left[\dfrac{-4x}{(x^2-1)^2}\right]$

$=\dfrac{-12x(x^2+1)^2}{(x^2-1)^4}$

8. $f(s)=\sqrt{\frac{s^2+1}{s^2+4}} = \left(\frac{s^2+1}{s^2+4}\right)^{1/2}$

$f'(s)=\frac{1}{2}\left(\frac{s^2+1}{s^2+4}\right)^{-1/2}\left[\frac{(s^2+4)\cdot 2s-(s^2+1)(2s)}{(s^2+4)^2}\right]$

$=\frac{1}{2}\frac{(s^2+1)^{-1/2}}{(s^2+4)^{-1/2}}\left[\frac{2s^3+8s-2s^3-2s}{(s^2+4)^2}\right]$

$=\frac{1}{2}\frac{(s^2+1)^{-1/2}}{(s^2+4)^{-1/2}}\left[\frac{6s}{(s^2+4)^2}\right] = \frac{6s(s^2+1)^{-1/2}}{2(s^2+4)^{3/2}}$

$=\boxed{\frac{3s}{(s^2+4)^{3/2}(s^2+1)^{1/2}}}$

9. $F(t)=e^{t\sin t}$

$F'(t)=e^{t\sin t}(t\cos t+\sin t)$

10. $y=\sec^2(m\theta) = (\sec(m\theta))^2$

$y'=2\sec(m\theta)\cdot\sec(m\theta)\tan(m\theta)\cdot m$

$=2m\sec^2(m\theta)\tan(m\theta)$

11. $f(x)=(3x+7)^{10}$

$f'(x)=10(3x+7)^9\cdot 3$

$=30(3x+7)^9$

Find y' and y".

12. $y=e^{\alpha x}\sin(\beta x)$ α and β are contstants

$y'=e^{\alpha x}\cdot\cos(\beta x)\cdot\beta+\sin(\beta x)\cdot e^{\alpha x}\cdot\alpha$

$=\beta e^{\alpha x}\cos(\beta x)+\alpha e^{\alpha x}\sin(\beta x)$

$=e^{\alpha x}[\beta\cos(\beta x)+\alpha\sin(\beta x)]$

$y''=e^{\alpha x}[\beta(-\sin(\beta x)\beta)+\alpha\cdot\cos(\beta x)\cdot\beta]+[\beta\cos(\beta x)+\alpha\sin(\beta x)]e^{\alpha x}\cdot\alpha$

$=e^{\alpha x}[-\beta^2\sin(\beta x)+\alpha\beta\cos(\beta x)]+\alpha e^{\alpha x}[\beta\cos(\beta x)+\alpha\sin(\beta x)]$

$=e^{\alpha x}[-\beta^2\sin(\beta x)+\alpha\beta\cos(\beta x)+\alpha\beta\cos(\beta x)+\alpha^2\sin(\beta x)]$

$=e^{\alpha x}[-\beta^2\sin(\beta x)+2\alpha\beta\cos(\beta x)+\alpha^2\sin(\beta x)]$