AP Calculus

L'Hospital's Rule (2)

Name______________________ Pd.____

Day 6 Curve Sketching

7. $\lim\limits_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx}$

$y=\left(1+\frac{a}{x}\right)^{bx}$

$\lim\limits_{x\to\infty}\ln y=\lim\limits_{x\to\infty}\ln\left(1+\frac{a}{x}\right)^{bx}$

$=\lim\limits_{x\to\infty} bx\ln\left(1+\frac{a}{x}\right)$

$=\lim\limits_{x\to\infty}\frac{\ln\left(1+\frac{a}{x}\right)}{\frac{1}{bx}}=\lim\limits_{x\to\infty}\frac{\ln(1+ax^{-1})}{\frac{1}{b}x^{-1}}$

$\overset{LH}{=}\lim\limits_{x\to\infty}\frac{\frac{1}{1+ax^{-1}}\cdot -a\cancel{x^{-2}}}{-\frac{1}{b}\cancel{x^{-2}}}$

$=\lim\limits_{x\to\infty}\frac{\frac{a}{1+ax^{-1}}}{\frac{1}{b}}=\lim\limits_{x\to\infty}\frac{ba}{1+\frac{a}{x}}$

$=\frac{ba}{1+0}$

$\lim\limits_{x\to\infty}\cancel{\ln} y=e^{ba}$

e^{ba}

8. $\lim\limits_{x\to1^+}x^{\frac{1}{1-x}}$

$y=x^{\frac{1}{1-x}}$

$\lim\limits_{x\to1^+}\ln y=\lim\limits_{x\to1^+}\ln x^{\frac{1}{1-x}}$

$=\lim\limits_{x\to1^+}\frac{1}{1-x}\cdot\ln x$

$=\lim\limits_{x\to1^+}\frac{\ln x}{1-x}$

$\overset{LH}{=}\lim\limits_{x\to1^+}\frac{\frac{1}{x}}{-1}$

$\lim\limits_{x\to1^+}\cancel{\ln} y=e^{-1}$

$\boxed{e^{-1}}$

9. $\lim\limits_{x\to0^+}(\cos x)^{\frac{1}{x^2}}$

$y=(\cos x)^{1/x^2}$

$\lim\limits_{x\to0^+}\ln y=\lim\limits_{x\to0^+}\ln(\cos x)^{1/x^2}$

$=\lim\limits_{x\to0^+}\frac{1}{x^2}\cdot\ln\cos x$

$=\lim\limits_{x\to0^+}\frac{\ln\cos x}{x^2}$

$\overset{LH}{=}\lim\limits_{x\to0^+}\frac{\frac{1}{\cos x}\cdot-\sin x}{2x}$

$=\lim\limits_{x\to0^+}\frac{\frac{-\sin x}{\cos x}}{2x}=\lim\limits_{x\to0^+}\frac{-\sin x}{2x\cos x}$

$\overset{LH}{=}\lim\limits_{x\to0^+}\frac{-\cos x}{2x(-\sin x)+2\cos x}=\frac{-1}{2}$

$\lim\limits_{x\to0^+}\cancel{\ln} y=e^{-\frac{1}{2}}$

$e^{-1/2}$

10. $\lim\limits_{x\to0^+}(\tan 2x)^{x}$

$y=(\tan 2x)^x$

$\lim\limits_{x\to0^+}\ln y=\lim\limits_{x\to0^+}\ln(\tan 2x)^x$

$=\lim\limits_{x\to0^+}x\ln(\tan 2x)$

$=\lim\limits_{x\to0^+}\frac{\ln(\tan 2x)}{\frac{1}{x}}$

$=\lim\limits_{x\to0^+}\frac{\frac{1}{\tan 2x}\cdot 2\sec^2 2x}{-x^{-2}}$

$=\lim\limits_{x\to0^+}\frac{\frac{2\sec^2 2x}{\tan 2x}}{\frac{-1}{x^2}}=\lim\limits_{x\to0^+}\frac{-2x^2\sec^2 2x}{\tan 2x}$

$=\lim\limits_{x\to0^+}\frac{-2x^2(2)\sec(2x)\sec(2x)\tan x+\sec^2(2x)(-4x)}{2\sec^2(2x)}$

$=\lim\limits_{x\to0^+}\frac{-4x^2\sec^2(2x)\tan(2x)-4x\sec^2(2x)}{2\sec^2(2x)}$

$=\lim\limits_{x\to0^+}\frac{2\sec^2(2x)[-2x^2\tan(2x)-2x]}{2\sec^2(2x)}=0$

$\lim\limits_{x\to0^+}\ln y=0$

$e^0=\boxed{1}$

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L'Hospital's Rule (2) | Day 6 Curve Sketching

1-10: Find the limit. Use L'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If L'Hospital's Rule doesn't apply, explain why.

1. $\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right)$

$\lim_{x\to 1} \frac{x\ln x-(x-1)}{\ln x(x-1)}$

$\lim_{x\to 1}\frac{x\ln x-x+1}{\ln x(x-1)} \overset{LH}{=} \lim_{x\to 1}\frac{x\cdot\frac{1}{x}+\ln x-1}{\ln x\cdot 1+(x-1)\cdot\frac{1}{x}}$

$=\lim_{x\to 1}\frac{\ln x}{\ln x+1-x^{-1}} = \lim_{x\to 1}\frac{\frac{1}{x}}{\frac{1}{x}+x^{-2}} = \frac{1}{1+1} = \boxed{\frac{1}{2}}$

2. $\lim_{x\to 0}(\csc x-\cot x)$

$\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right) = \lim_{x\to 0}\frac{1-\cos x}{\sin x}$

$\overset{LH}{=}\lim_{x\to 0}\frac{\sin x}{\cos x} = \boxed{0}$

3. $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{e^x-1}\right)$

$\lim_{x\to 0^+}\frac{e^x-1}{x(e^x-1)}-\frac{x}{x(e^x-1)}$

$\lim_{x\to 0^+}\frac{e^x-1-x}{x(e^x-1)} \overset{LH}{=} \lim_{x\to 0^+}\frac{e^x-1}{xe^x+(e^x-1)}$ $\frac{0}{0}$

$\overset{LH}{=}\lim_{x\to 0^+}\frac{e^x}{xe^x+e^x+e^x} = \frac{1}{0+1+1} = \boxed{\frac{1}{2}}$

4. $\lim_{x\to 0^+}\left(\cot x-\frac{1}{x}\right) = \lim_{x\to 0^+}\left(\frac{\cos x}{\sin x}-\frac{1}{x}\right)$

$=\lim_{x\to 0^+}\frac{x\cos x}{x\sin x}-\frac{\sin x}{x\sin x} = \lim_{x\to 0^+}\frac{x\cos x-\sin x}{x\sin x}$

$\overset{LH}{=}\lim_{x\to 0^+}\frac{-x\sin x+\cos x-\cos x}{x\cos x+\sin x}$ $\frac{0}{0}$

$\overset{LH}{=}\lim_{x\to 0^+}\frac{-x\cos x-\sin x-\sin x+\sin x}{x(-\sin x)+\cos x+\cos x} = \frac{0}{2}$

$=\boxed{0}$

5. $\lim_{x\to 0^+}(4x+1)^{\cot x}$

$y=(4x+1)^{\cot x}$

$\lim_{x\to 0^+}\ln y = \lim_{x\to 0^+}\ln(4x+1)^{\cot x}$

$=\lim_{x\to 0^+}\cot x\ln(4x+1)$

$=\lim_{x\to 0^+}\frac{\ln(4x+1)}{\tan x}$

$\overset{LH}{=}\lim_{x\to 0^+}\frac{\frac{1}{4x+1}\cdot 4}{\sec^2 x}$

$=\lim_{x\to 0^+}\frac{\frac{4}{4x+1}}{\sec^2 x}$

$\lim_{x\to 0^+}\cancel{\ln}y = \underset{e}{4}$

$\boxed{y=e^4}$

6. $\lim_{x\to 0}(1-2x)^{\frac{1}{x}}$

$y=(1-2x)^{1/x}$

$\lim_{x\to 0}\ln y = \lim_{x\to 0}\ln(1-2x)^{1/x}$

$=\lim_{x\to 0}\frac{1}{x}\ln(1-2x)$

$=\lim_{x\to 0}\frac{\ln(1-2x)}{x}$

$\overset{LH}{=}\lim_{x\to 0}\frac{\frac{1}{1-2x}\cdot -2}{1}$

$=\lim_{x\to 0}\frac{-2}{1-2x}$

$\lim_{x\to 0}\cancel{\ln y} = \underset{e}{-2}$

$\boxed{e^{-2}}$