

$$\pi \int_{x_1}^{x_2} (\text{Radius})^2 dx$$

$$\pi \int_{x_1}^{x_2} (\text{function} - \text{LOR})^2 dx$$

1-10: Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region and the solid.

1. $y = 2 - \frac{1}{2}x$, $y = 0$, $x = 1$, $x = 2$; about the x-axis

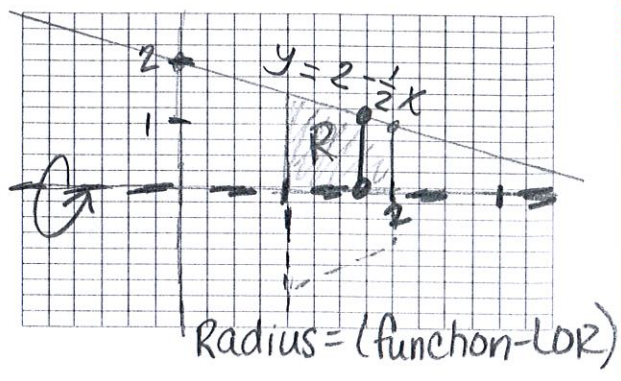
$$\pi \int_1^2 (2 - \frac{1}{2}x - 0)^2 dx$$

$$\pi \int_1^2 (2 - \frac{1}{2}x)^2 dx$$

$$\pi \int_1^2 4 - 2x + \frac{1}{4}x^2 dx$$

$$\pi [4x - \frac{2x^2}{2} + \frac{1}{4} \cdot \frac{x^3}{3} \Big|_1^2 = \pi [4x - x^2 + \frac{1}{12}x^3 \Big|_1^2]$$

$$\begin{aligned} &\rightarrow \pi [(8 - 4 + \frac{8}{12}) - (4 - 1 + \frac{1}{12})] \\ &\pi [4 + \frac{8}{12} - 3 - \frac{1}{12}] \\ &\pi [1 + \frac{7}{12}] \\ &\pi [\frac{12}{12} + \frac{7}{12}] \\ &\frac{19\pi}{12} \checkmark \end{aligned}$$



2. $y = 1 - x^2$, $y = 0$; about the x-axis

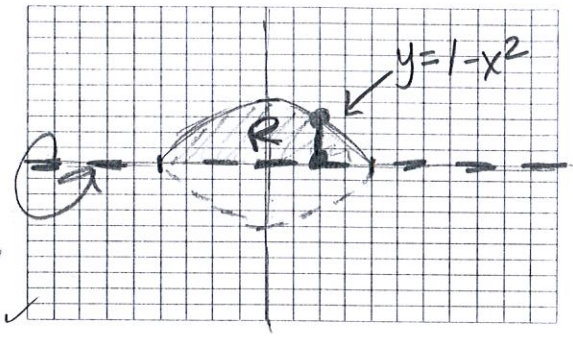
$$\pi \int_{-1}^1 (1 - x^2 - 0)^2 dx$$

$$\pi \int_{-1}^1 (1 - x^2)^2 dx$$

$$\pi \int_{-1}^1 1 - 2x^2 + x^4 dx$$

$$\pi [x - \frac{2x^3}{3} + \frac{x^5}{5} \Big|_{-1}^1]$$

$$\begin{aligned} &\rightarrow \pi [(1 - \frac{2}{3} + \frac{1}{5}) - (-1 + \frac{2}{3} - \frac{1}{5})] \\ &\pi [1 - \frac{2}{3} + \frac{1}{5} + 1 - \frac{2}{3} + \frac{1}{5}] \\ &\pi [2 - \frac{4}{3} + \frac{2}{5}] \\ &\pi [\frac{30}{15} - \frac{20}{15} + \frac{6}{15}] = \frac{16\pi}{15} \checkmark \end{aligned}$$



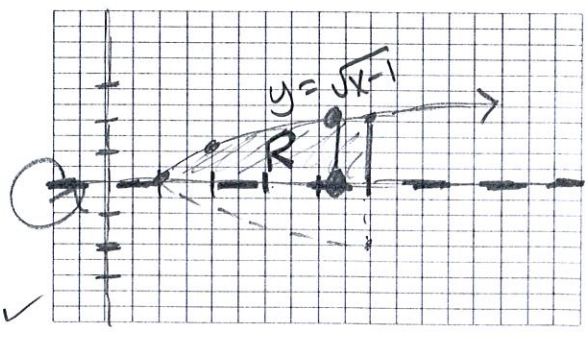
3. $y = \sqrt{x-1}$, $y = 0$, $x = 5$; about the x-axis

$$\pi \int_1^5 (\sqrt{x-1} - 0)^2 dx$$

$$\pi \int_1^5 x - 1 dx$$

$$\pi [\frac{1}{2}x^2 - x \Big|_1^5]$$

$$\pi [\frac{25}{2} - 5 - (\frac{1}{2} - 1)] = \pi [\frac{24}{2} - 4] = \pi [12 - 4] = 8\pi \checkmark$$



4. $y = \sqrt{25-x^2}$, $y = 0$, $x = 2$, $x = 4$; about the x-axis

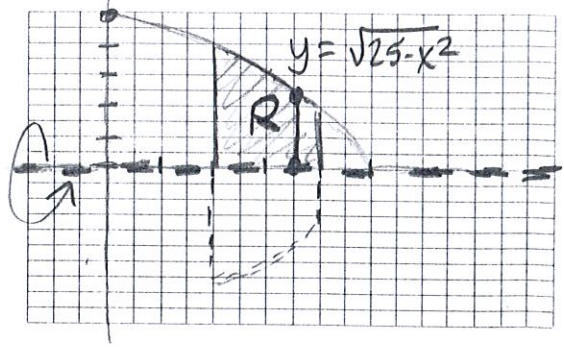
$$\pi \int_2^4 (\sqrt{25-x^2} - 0)^2 dx$$

$$\pi \int_2^4 25 - x^2 dx$$

$$\pi [25x - \frac{x^3}{3} \Big|_2^4]$$

$$\pi [(100 - \frac{64}{3}) - (50 - \frac{8}{3})]$$

$$\begin{aligned} &\rightarrow \pi [100 - \frac{64}{3} - 50 + \frac{8}{3}] \\ &\pi [50 - \frac{56}{3}] \\ &\pi [\frac{150}{3} - \frac{56}{3}] \\ &\frac{94\pi}{3} \checkmark \end{aligned}$$



$$x = 2\sqrt{y}$$

$$\frac{1}{2}x = \sqrt{y}$$

$$y = \frac{1}{4}x^2$$

5. $x = 2\sqrt{y}$, $x = 0$, $y = 9$; about the y-axis

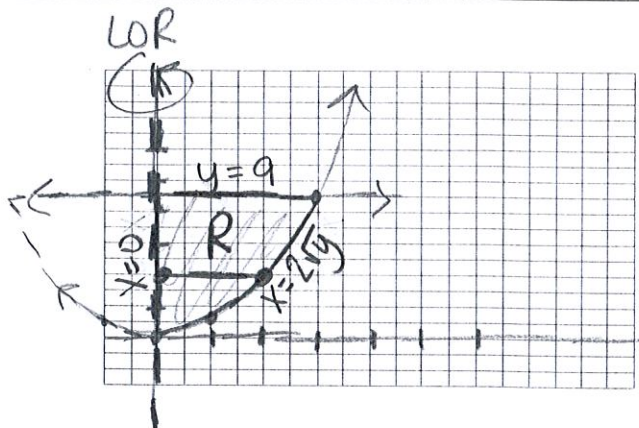
$$\pi \int_0^9 (2\sqrt{y} - 0)^2 dy$$

$$\pi \int_0^9 4y dy$$

$$\pi \left[\frac{4y^2}{2} \Big|_0^9 \right]$$

$$\pi \left[2y^2 \Big|_0^9 \right]$$

$$\pi [2(81) - 2(0)] = \boxed{162\pi} \checkmark$$



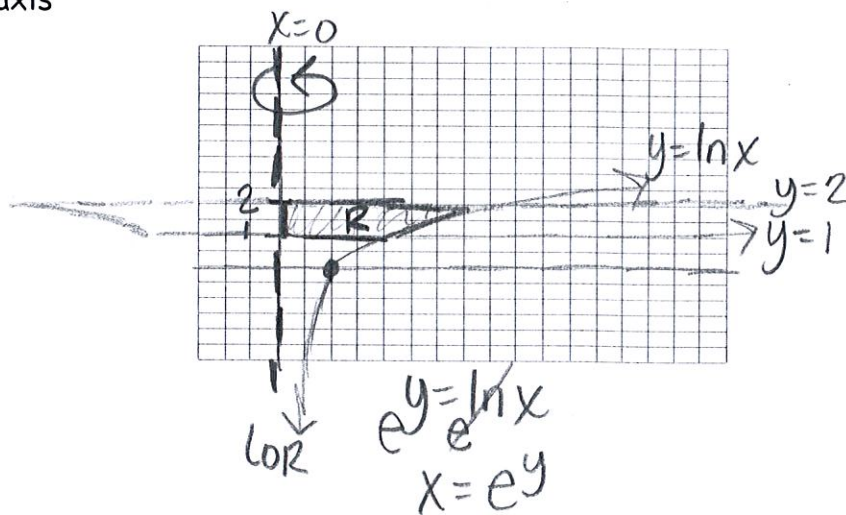
6. $y = \ln x$, $y = 1$, $y = 2$, $x = 0$; about the y-axis

$$\pi \int_1^2 (e^y - 0)^2 dy$$

$$\pi \int_1^2 e^{2y} dy$$

$$\pi \left[\frac{e^{2y}}{2} \Big|_1^2 \right]$$

$$\boxed{\frac{\pi}{2} [e^4 - e^2]} \checkmark$$



7. $y = x^3$, $y = x$, $x \geq 0$; about the x-axis

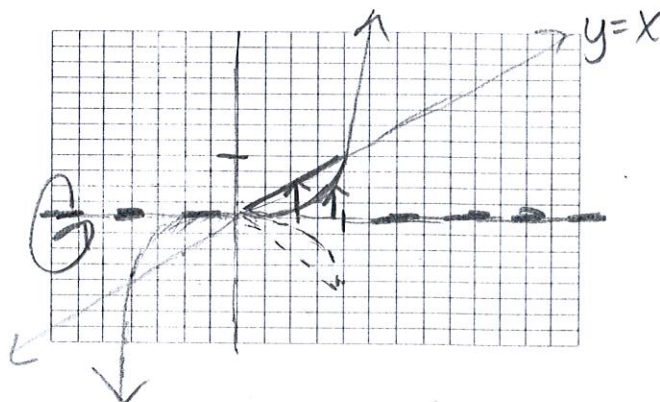
$$\pi \int_0^1 (x - 0)^2 - (x^3 - 0)^2 dx$$

$$\pi \int_0^1 x^2 - x^6 dx$$

$$\pi \left[\frac{x^3}{3} - \frac{x^7}{7} \Big|_0^1 \right]$$

$$\pi \left[\frac{1}{3} - \frac{1}{7} - 0 + 0 \right]$$

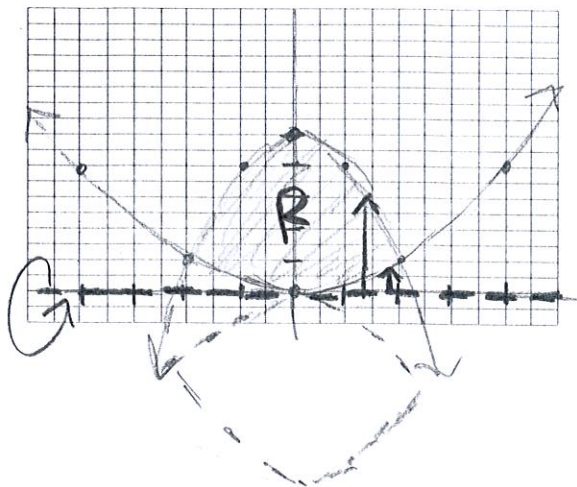
$$\pi \left[\frac{7}{21} - \frac{3}{21} \right] = \boxed{\frac{4\pi}{21}} \checkmark$$



8. $y = \frac{1}{4}x^2$, $y = 5 - x^2$; about the x-axis

$$\pi \int_{-2}^2 (5 - x^2 - 0)^2 - \left(\frac{1}{4}x^2 - 0\right)^2 dx$$

$$\boxed{\frac{176\pi}{3}} \checkmark$$



9. $y^2 = x$, $x = 2y$; about the y-axis

$$\pi \int_0^2 (2y - 0)^2 - (y^2 - 0)^2 dy$$

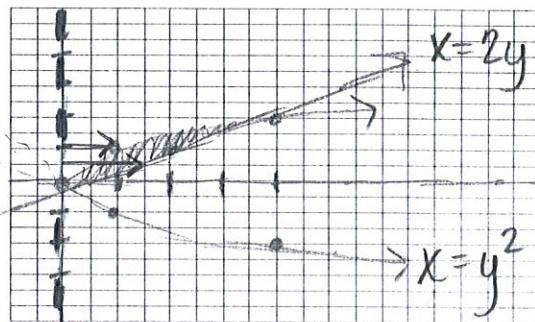
$$\boxed{\frac{64\pi}{15}} \checkmark$$

$$y^2 = x$$

$$y = \pm\sqrt{x}$$

$$x = 2y$$

$$y = \frac{1}{2}x$$



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LOR

10. $y = \frac{1}{4}x^2$, $x = 2$, $y = 0$; about the y-axis

$$\pi \int_0^1 (2 - 0)^2 - (\sqrt{4y} - 0)^2 dy$$

$$\boxed{2\pi} \checkmark$$

