AP Calculus Trig. Derivatives Name________________

1-8: Differentiate

1. $y = 3x^2 - 2\cos x$ $y' = 6x + 2\sin x$	2. $y = 2\sec x - \csc x$ $y' = 2\sec x\tan x + \csc x\cot x$
3. $f(x) = \sin x + \frac{1}{2}\cot x$ $f'(x) = \cos x - \frac{1}{2}\csc^2 x$	4. $g(\theta) = e^\theta(\tan\theta - \theta)$ $g'(\theta) = e^\theta(\sec^2\theta - 1) + (\tan\theta - \theta)e^\theta$ $= e^\theta(\sec^2\theta - 1 + \tan\theta - \theta)$
5. $f(x) = \sqrt{x}\sin x$ $f'(x) = \sqrt{x}\cos x + \sin x \cdot \frac{1}{2}x^{-1/2}$ $= \sqrt{x}\cos x + \frac{\sin x}{2\sqrt{x}}$	6. $y = \sec\theta\tan\theta$ $y' = \sec\theta \cdot \sec^2\theta + \tan\theta \cdot \sec\theta\tan\theta$ $= \sec^3\theta + \sec\theta\tan^2\theta$ $= \sec\theta(\sec^2\theta + \tan^2\theta)$
7. $f(t) = \frac{\cot t}{e^t}$ $f'(t) = \frac{e^t(-\csc^2 t) - \cot t \cdot e^t}{(e^t)^2}$ $= \frac{-e^t\csc^2 t - e^t\cot t}{e^{2t}}$ $= \frac{e^t(-\csc^2 t - \cot t)}{e^{2t}} = \frac{-\csc^2 t - \cot t}{e^t}$	8. $y = \frac{\cos x}{1 - \sin x}$ $y' = \frac{(1-\sin x)(-\sin x) - \cos x(-\cos x)}{(1-\sin x)^2}$ $= \frac{-\sin x + \underbrace{[\sin^2 x + \cos^2 x]}_{=1}}{(1-\sin x)^2}$ $= \frac{-\sin x + 1}{(1-\sin x)^2} = \frac{1}{1-\sin x}$

Find an equation of the tangent line to the curve at the given point.

9. $y = \sec x \quad \left(\frac{\pi}{3}, 2\right)$ $y' = \sec x\tan x \Big\vert_{x=\frac{\pi}{3}}$ $y' = \sec\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{3}\right)$ $= 2 \cdot \sqrt{3}$ $y - 2 = 2\sqrt{3}\left(x - \frac{\pi}{3}\right)$	10. $y = e^x\cos x \quad (0,1)$ $y' = e^x(-\sin x) + \cos x \cdot e^x$ $= -e^x\sin x + e^x\cos x \Big\vert_{x=0}$ $= 1\cdot 0 + 1 \cdot 1 = 1$ $y - 1 = 1(x - 0)$

11. If $H(\theta) = \theta \sin\theta$, find $H'(\theta)$ and $H''(\theta)$. ← derivative of the derivative

$H'(\theta) = \theta\cos\theta + \sin\theta$

$H''(\theta) = \theta(-\sin\theta) + \cos\theta + \cos\theta$

$= -\theta\sin\theta + 2\cos\theta$

12. Suppose $f\left(\frac{\pi}{3}\right) = 4$ and $f'\left(\frac{\pi}{3}\right) = -2$ and let $g(x) = f(x)\sin x$ and $h(x) = \frac{\cos x}{f(x)}$. Find

A.) $g'\left(\frac{\pi}{3}\right) =$

$g(x) = f(x)\sin x$

$g'(x) = f(x)\cdot\cos x + \sin x \cdot f'(x)$

$g'(\frac{\pi}{3}) = f(\frac{\pi}{3})\cdot\cos(\frac{\pi}{3}) + \sin(\frac{\pi}{3})\cdot f'(\frac{\pi}{3})$

$= 4\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}(-2)$

$= 2 - \sqrt{3}$

B.) $h'\left(\frac{\pi}{3}\right) =$

$h(x) = \frac{\cos x}{f(x)}$

$h'(x) = \frac{f(x)(-\sin x) - \cos x \cdot f'(x)}{[f(x)]^2}$

$h'(\frac{\pi}{3}) = \frac{f(\frac{\pi}{3})(-\sin(\frac{\pi}{3})) - \cos(\frac{\pi}{3})\cdot f'(\frac{\pi}{3})}{[f(\frac{\pi}{3})]^2}$

$= \frac{4(-\frac{\sqrt{3}}{2}) - \frac{1}{2}(-2)}{4^2} = \frac{-2\sqrt{3}+1}{16}$

13-14 For what values of x does the graph of f have a horizontal tangent? Derivative = 0

13. $f(x) = x + 2\sin x$

$f'(x) = 1 + 2\cos x$

$0 = 1 + 2\cos x$

$-\frac{1}{2} = \cos x$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$

14. $f(x) = e^x \cos x$

$f'(x) = -e^x\sin x + e^x\cos x$

$0 = -e^x\sin x + e^x\cos x$

$0 = e^x(-\sin x + \cos x)$

$e^x \neq 0$ $\quad -\sin x + \cos x = 0$

$\sin x = \cos x$

$\boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$

15. $\frac{d^{99}}{dx^{99}}(\sin x)$ hmm...

$f'(x) = \cos x$ $\quad f^5(x)$

$f''(x) = -\sin x$ $\quad f^6(x)$ ...

$f'''(x) = -\cos x$ $\quad f^7(x)$ $\quad \leftarrow f^{99}(x) = -\cos x$

$f^4(x) = \sin x$ $\quad f^8(x)$