

Definite Integration

Integration Day 4

$$1. \int_{-1}^2 (x^3 - 2x) dx = \left. \frac{x^4}{4} - x^2 \right|_{-1}^2$$

$$= 4 - 4 - \left(\frac{1}{4} - 1 \right)$$

$$= \boxed{\frac{3}{4}}$$

$$2. \int_1^4 (5 - 2t + 3t^2) dt = \left. 5t - t^2 + t^3 \right|_1^4$$

$$= 20 - 16 + 64 - (5 - 1 + 1)$$

$$= \boxed{63}$$

$$3. \int_1^9 \sqrt{x} dx = \left. \frac{2}{3} x^{3/2} \right|_1^9 = \frac{2}{3} (9)^{3/2} - \frac{2}{3} (1)^{3/2}$$

$$= \frac{54}{3} - \frac{2}{3} = \boxed{\frac{52}{3}}$$

$$4. \int_{\pi/6}^{\pi} \sin \theta d\theta = \left. -\cos \theta \right|_{\pi/6}^{\pi}$$

$$= -\cos \pi - (-\cos \pi/6)$$

$$= \boxed{\frac{1 + \sqrt{3}}{2}}$$

$$5. \int_0^1 (u+2)(u-3) du = \int_0^1 u^2 - u - 6 du$$

$$= \left. \frac{u^3}{3} - \frac{u^2}{2} - 6u \right|_0^1$$

$$= \frac{1}{3} - \frac{1}{2} - 6 = \boxed{\frac{-37}{6}}$$

$$6. \int_0^4 (4-t)\sqrt{t} dt = \int_0^4 4t^{1/2} - t^{3/2} dt$$

$$= \left. \frac{8}{3} t^{3/2} - \frac{2}{5} t^{5/2} \right|_0^4$$

$$= \frac{8}{3} (4)^{3/2} - \frac{2}{5} (4)^{5/2} - [0 - 0] = \frac{64}{3} - \frac{64}{5} = \boxed{\frac{128}{15}}$$

$$7. \int_1^9 \frac{x-1}{\sqrt{x}} dx = \int_1^9 \sqrt{x} - \frac{1}{\sqrt{x}} dx = \int_1^9 x^{1/2} - x^{-1/2} dx$$

$$= \left. \frac{2}{3} x^{3/2} - 2x^{1/2} \right|_1^9$$

$$= \frac{2}{3} (9)^{3/2} - 2(9)^{1/2} - \left[\frac{2}{3} - 2 \right]$$

$$= \frac{54}{3} - 6 - \left(-\frac{4}{3} \right) = \boxed{\frac{40}{3}}$$

$$8. \int_0^{\pi/4} \sec^2 t dt = \left. \tan t \right|_0^{\pi/4}$$

$$= \tan \frac{\pi}{4} - \tan 0$$

$$= 1 - 0 = \boxed{1}$$

$$9. \int_0^{\pi/4} \sec \theta \tan \theta d\theta = \left. \sec \theta \right|_0^{\pi/4}$$

$$= \sec \left(\frac{\pi}{4} \right) - \sec 0$$

$$= \boxed{\frac{2}{\sqrt{2}} - 1}$$

$$10. \int_1^2 (1+2y)^2 dy = \int_1^2 (1+2y)(1+2y) dy$$

$$= \int_1^2 1 + 4y + 4y^2 dy$$

$$= \left. y + 2y^2 + \frac{4y^3}{3} \right|_1^2$$

$$= 2 + 8 + \frac{32}{3} - \left(1 + 2 + \frac{4}{3} \right) = \boxed{\frac{49}{3}}$$

Definite Integration

Integration Day 4

$$\begin{aligned}
 11. \int_0^3 (2 \sin x - e^x) dx &= -2 \cos x - e^x \Big|_0^3 \\
 &= -2 \cos 3 - e^3 - (-2 \cos 0 - e^0) \\
 &= -2 \cos 3 - e^3 - (-2 - 1) \\
 &= \boxed{-2 \cos 3 - e^3 + 3}
 \end{aligned}$$

$$\begin{aligned}
 12. \int_1^2 \frac{v^3 + 3v^6}{v^4} dv &= \int_1^2 \frac{1}{v} + 3v^2 dv \\
 &= \ln|v| + v^3 \Big|_1^2 \\
 &= \ln|2| + 8 - (\ln|1| + 1) \\
 &= \ln|2| + 8 - 1 \\
 &= \boxed{\ln 2 + 7}
 \end{aligned}$$

$$\begin{aligned}
 13. \int_0^1 (x^e + e^x) dx &= \frac{x^{e+1}}{e+1} + e^x \Big|_0^1 \\
 &= \frac{1^{e+1}}{e+1} + e - \left(\frac{0^{e+1}}{e+1} + e^0 \right) \\
 &= \boxed{\frac{1}{e+1} + e - 1}
 \end{aligned}$$

$$\begin{aligned}
 14. \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{8}{1+x^2} dx &= 8 \tan^{-1} x \Big|_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \\
 &= 8 \tan^{-1} \sqrt{3} - 8 \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) \\
 &= \frac{8\pi}{3} - \frac{8\pi}{6} = \boxed{\frac{4\pi}{3}}
 \end{aligned}$$

$$\begin{aligned}
 15. \int_{-1}^1 e^{u+1} du &= \int_{-1}^1 e \cdot e^u du = e \int_{-1}^1 e^u du \\
 &= e \cdot e^u \Big|_{-1}^1 = e \cdot e^1 - e \cdot e^{-1} \\
 &= \boxed{e^2 - 1}
 \end{aligned}$$

$$\begin{aligned}
 16. \int_0^{\pi} f(x) dx \quad \text{where } f(x) &= \begin{cases} \sin x & \text{if } 0 \leq x < \frac{\pi}{2} \\ \cos x & \text{if } \frac{\pi}{2} \leq x < \pi \end{cases} \\
 &= \int_0^{\pi/2} \sin x dx + \int_{\pi/2}^{\pi} \cos x dx \\
 &= -\cos x \Big|_0^{\pi/2} + \sin x \Big|_{\pi/2}^{\pi} \\
 &= -\cos \frac{\pi}{2} - (-\cos 0) + \sin \pi - \sin \frac{\pi}{2} \\
 &= 0 + 1 + 0 - 1 = \boxed{0}
 \end{aligned}$$