AP Calculus
L'Hospital's Rule (1)

Name________________________Pd.____
Day 5 Curve Sketching

1-3: Given that

$$\lim_{x\to a} f(x)=0 \qquad \lim_{x\to a} g(x)=0 \qquad \lim_{x\to a} h(x)=1$$

$$\lim_{x\to a} p(x)=\infty \qquad \lim_{x\to a} q(x)=\infty$$

Which of the following limits are indeterminate? For those that are not an indeterminate from, evaluate the limit where possible.

1.

a.) $\lim_{x\to a}\frac{f(x)}{g(x)}$ $\frac{0}{0}$ indeterminate

b.) $\lim_{x\to a}\frac{f(x)}{p(x)}$ $\frac{0}{\infty} = 0$

c.) $\lim_{x\to a}\frac{h(x)}{p(x)}$ $\frac{1}{\infty} = 0$

d.) $\lim_{x\to a}\frac{p(x)}{f(x)}$ $\frac{\infty}{0}$ DNE (∞)

e.) $\lim_{x\to a}\frac{p(x)}{q(x)}$ $\frac{\infty}{\infty}$ Indeterminate

2.

a.) $\lim_{x\to a}[f(x)p(x)]$ $0\cdot\infty$ indeterminate

b.) $\lim_{x\to a}[h(x)p(x)]$ $1\cdot\infty$ $=\infty$

c.) $\lim_{x\to a}[p(x)q(x)]$ $\infty\cdot\infty$ $=\infty$

3.

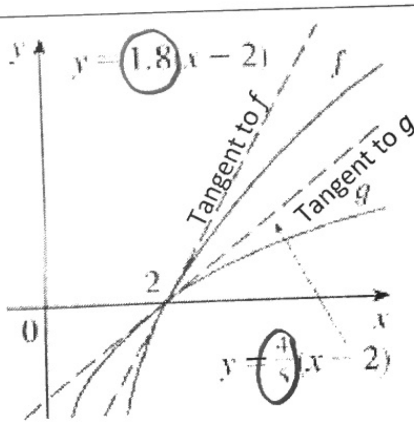
a.) $\lim_{x\to a}[f(x)-p(x)]$ $0-\infty$ $=-\infty$

b.) $\lim_{x\to a}[p(x)-q(x)]$ $\infty-\infty$ indeterminate

c.) $\lim_{x\to a}[p(x)+q(x)]$ $\infty+\infty$ $=\infty$

4-5: Use the graphs of f and g and their tangent lines at (2,0) to find $\lim_{x\to 2}\frac{f(x)}{g(x)}$.

4.

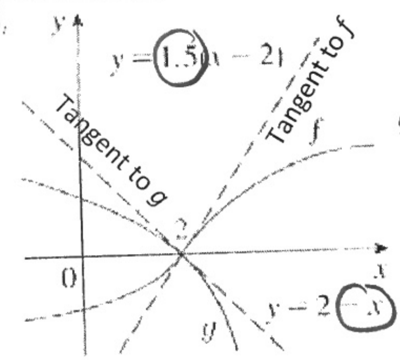


$\lim_{x\to 2}\frac{f(x)}{g(x)}$ $\frac{0}{0}$

$\overset{LH}{=}\lim_{x\to 2}\frac{f'(x)}{g'(x)}$ ← slope of tangent lines

$=\frac{1.8}{.8}=\boxed{2.25}$

5.



$\lim_{x\to 2}\frac{f(x)}{g(x)}=\frac{0}{0}$

$\overset{LH}{=}\lim_{x\to 2}\frac{f'(x)}{g'(x)}=\frac{1.5}{-1}$

$=\boxed{-1.5}$

6-20: Find the limit. Use L'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If L'Hospital's Rule doesn't apply, explain why.

6. $\lim_{x\to\frac{1}{2}}\frac{6x^2+5x-4}{4x^2+16x-9}$ $\frac{0}{0}$

$\overset{LH}{=}\lim_{x\to\frac{1}{2}}\frac{12x+5}{8x+16}=\frac{6+5}{4+16}$

$=\boxed{\frac{11}{20}}$

7. $\lim_{x\to\frac{\pi}{2}^+}\frac{\cos x}{1-\sin x}$ $\frac{0}{0}$

$\overset{LH}{=}\lim_{x\to\frac{\pi}{2}^+}\frac{-\sin x}{-\cos x}=\lim_{x\to\frac{\pi}{2}^+}\frac{\sin x}{\cos x}$

$=\lim_{x\to\frac{\pi}{2}^+}\tan x$

$=\boxed{-\infty}$

$\frac{\pi}{2}$

8. $\lim_{t\to 0}\frac{e^{2t}-1}{\sin t}$ $\frac{0}{0}$ $\overset{LH}{=}\lim_{t\to 0}\frac{2e^{2t}}{\cos t}$

$=\frac{2}{1}=\boxed{2}$

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9. $\lim_{\theta \to \frac{\pi}{2}} \frac{1-\sin\theta}{1+\cos 2\theta}$ $\frac{0}{0}$

$\overset{LH}{=} \lim_{\theta \to \frac{\pi}{2}} \frac{-\cos\theta}{-2\sin(2\theta)} = \lim_{\theta \to \frac{\pi}{2}} \frac{\cos\theta}{2\sin(2\theta)}$ $\frac{0}{0}$

$\overset{LH}{=} \lim_{\theta \to \frac{\pi}{2}} \frac{-\sin\theta}{4\cos(2\theta)} = \frac{-1}{-4}$

$= \boxed{\frac{1}{4}}$

10. $\lim_{x \to \infty} \frac{\ln x}{\sqrt{x}}$ $\frac{\infty}{\infty}$ $\overset{LH}{=} \lim_{x \to \infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} \cdot \frac{2\sqrt{x}}{1}$

$= \lim_{x \to \infty} \frac{2\sqrt{x}}{x} = \lim_{x \to \infty} \frac{2}{\sqrt{x}} = \boxed{0}$

11. $\lim_{x \to 0^+} \frac{\ln x}{x}$ $\frac{-\infty}{0}$ ← approaching 0

$\boxed{-\infty}$

12. $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$ $\frac{0}{0}$

$\overset{LH}{=} \lim_{x \to 0} \frac{e^x - 1}{2x}$ $\frac{0}{0}$ $\overset{LH}{=} \lim_{x \to 0} \frac{e^x}{2}$

$= \boxed{\frac{1}{2}}$

13. $\lim_{x \to 0} \frac{x - \sin x}{x - \tan x}$ $\frac{0}{0}$ $= \boxed{-\frac{1}{2}}$

$\overset{LH}{=} \lim_{x \to 0} \frac{1-\cos x}{1-\sec^2 x}$ $\frac{0}{0}$

$\overset{LH}{=} \lim_{x \to 0} \frac{\sin x}{-2\sec x \cdot \sec x \tan x} = \lim_{x \to 0} \frac{\sin x}{-2\sec^2 x \tan x}$ $\frac{0}{0}$

$\overset{LH}{=} \lim_{x \to 0} \frac{\cos x}{-2\sec^2 x \cdot \sec^2 x + \underbrace{\tan x(-4\sec x)\sec x \tan x}_{0}}$

14. $\lim_{x \to \infty} \frac{(\ln x)^2}{x}$ $\frac{\infty}{\infty}$ $\overset{LH}{=} \lim_{x \to \infty} \frac{2\ln x \cdot \frac{1}{x}}{1}$

$= \lim_{x \to \infty} \frac{2\ln x}{x}$ $\frac{\infty}{\infty}$

$\overset{LH}{=} \lim_{x \to \infty} \frac{2(\frac{1}{x})}{1} = \lim_{x \to \infty} \frac{2}{x}$

$= \boxed{0}$

15. $\lim_{x \to 0} \frac{\cos mx - \cos nx}{x^2}$ $\frac{0}{0}$

$\overset{LH}{=} \lim_{x \to 0} \frac{-\sin(mx)\cdot m + \sin(nx)\cdot n}{2x}$

$\overset{LH}{=} \lim_{x \to 0} \frac{-\cos(mx)\cdot m^2 + \cos(nx)\cdot n^2}{2}$

$= \boxed{\frac{-m^2+n^2}{2}}$

16. $\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$ $\frac{0}{0}$

$\overset{LH}{=} \lim_{x \to 0} \frac{e^x + e^{-x} - 2}{1-\cos x}$ $\frac{0}{0}$

$\overset{LH}{=} \lim_{x \to 0} \frac{e^x - e^{-x}}{\sin x}$ $\frac{0}{0}$

$\overset{LH}{=} \lim_{x \to 0} \frac{e^x + e^{-x}}{\cos x} = \frac{2}{1} = \boxed{2}$

17. $\lim_{x \to \infty} x \sin\left(\frac{\pi}{x}\right)$ $\infty \cdot 0$

$= \lim_{x \to \infty} \frac{\sin(\frac{\pi}{x})}{\frac{1}{x}}$ $\frac{0}{0}$ ($\frac{\pi}{x} = \pi x^{-1}$, $\frac{1}{x} = x^{-1}$)

$\overset{LH}{=} \lim_{x \to \infty} \frac{\cos(\frac{\pi}{x}) \cdot (-\pi \cancel{x^{-2}})}{-\cancel{x^{-2}}}$

$= \lim_{x \to \infty} \pi\cos(\frac{\pi}{x}) = \pi\cos(0)$

$= \boxed{\pi}$

18. $\lim_{x \to 0^+} \sin x \ln x$ $0 \cdot -\infty$

$= \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{\sin x}}$ $\frac{\infty}{\infty}$ $= \lim_{x \to 0^+} \frac{\ln x}{\csc x}$

$\overset{LH}{=} \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\csc x \cot x} = \lim_{x \to 0^+} \frac{-\sin x \tan x}{x}$ $\frac{0}{0}$

$\left(\frac{1}{\sin x \tan x}\right)$

$\overset{LH}{=} \lim_{x \to 0^+} \frac{-\sin x \cdot \sec^2 x + \tan x(-\cos x)}{1}$

$= \frac{-\sin(0)\sec^2(0) + \tan(0)(-\cos(0))}{1} = \boxed{0}$

18. $\lim_{x \to \infty} \sqrt{x} e^{-\frac{x}{2}} = \lim_{x \to \infty} \frac{\sqrt{x}}{e^{x/2}}$ $\frac{\infty}{\infty}$

$\overset{LH}{=} \lim_{x \to \infty} \frac{\frac{1}{2}x^{-1/2}}{\frac{1}{2}e^{x/2}} = \lim_{x \to \infty} \frac{\frac{1}{2\sqrt{x}}}{\frac{e^{x/2}}{2}}$

$= \lim_{x \to \infty} \frac{2}{2\sqrt{x}\, e^{x/2}} = \boxed{0}$

20. $\lim_{x \to \infty} x^3 e^{-x^2} = \lim_{x \to \infty} \frac{x^3}{e^{x^2}}$ $\frac{\infty}{\infty}$

$\overset{LH}{=} \lim_{x \to \infty} \frac{3x^2}{e^{x^2} \cdot 2x} = \lim_{x \to \infty} \frac{3x}{2e^{x^2}}$ $\frac{\infty}{\infty}$

$\overset{LH}{=} \lim_{x \to \infty} \frac{3}{2e^{x^2} \cdot 2x} = \boxed{0}$