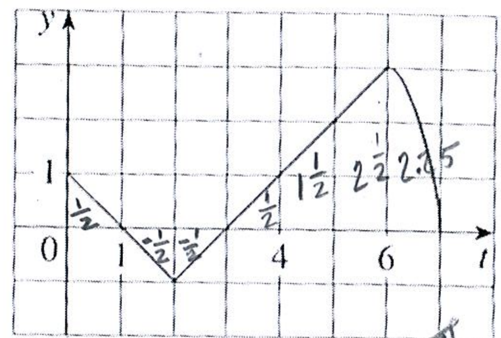
AP Calculus

Fundamental Theorem of Calculus

Name____________________________

Integration Day 5

1. Let $g(x)=\int_0^x f(t)dt$, where f is the function whose graph is shown.



A.) Evaluate g(x) for

x=0 $g(0)=\int_0^0 f(t)dt=0$

x=1 $g(1)=\int_0^1 f(t)dt=\frac{1}{2}$

x=2 $g(2)=\int_0^2 f(t)dt=0$

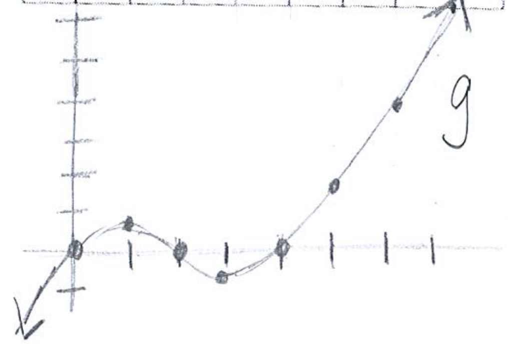
x=3 $g(3)=\int_0^3 f(t)dt=-\frac{1}{2}$

x=4 $g(4)=\int_0^4 f(t)dt=0$

x=5 $g(5)=\int_0^5 f(t)dt=1\frac{1}{2}$

x=6 $g(6)=\int_0^6 f(t)dt=4$

B.) Estimate g(7) $=6.25$

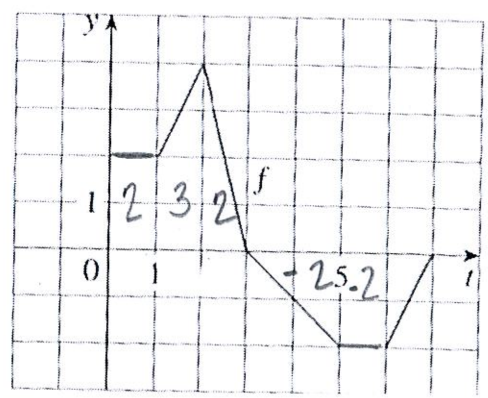


D.)Sketch a rough graph of g.

C.) Where does g have a maximum value? $x=1$

Where does it have a minimum value? $x=3$

2. Let $g(x)=\int_0^x f(t)dt$, where f is the function whose graph is shown.



A.) Evaluate

g(0) $g(0)=\int_0^0 f(t)dt=0$

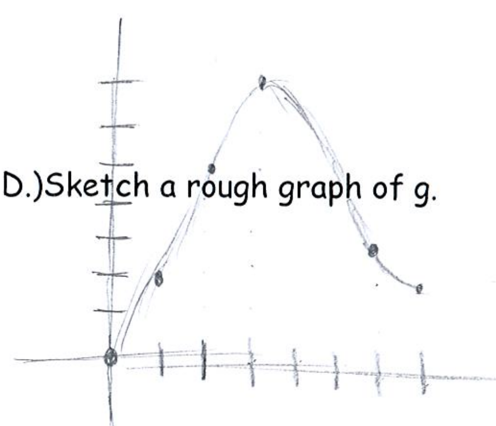
g(1) $g(1)=\int_0^1 f(t)dt=2$

g(2) $g(2)=\int_0^2 f(t)dt=5$

g(3) $g(3)=\int_0^3 f(t)dt=7$

g(6) $g(6)=\int_0^6 f(t)\,dt=3$

B.)On what intervals is g increasing? $(0,3)$



D.)Sketch a rough graph of g.

C.)Where does g have a maximum value?

$x=3$

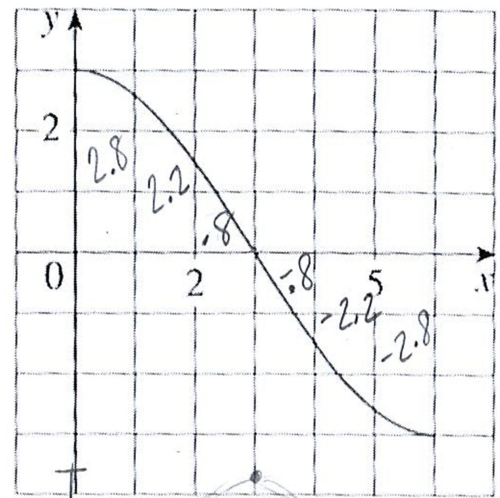
AP Calculus

Fundamental Theorem of Calculus

Name________________________________

Integration Day 5

3. Let $g(x)=\int_0^x f(t)dt$, where f is the function whose graph is shown.



A.) Evaluate

g(0) $g(0)=\int_0^0 f(t)dt=0$

g(6) $g(6)=\int_0^6 f(t)dt=0$

B.) Estimate g(x) for

x=1 $g(1)\approx 2.8$

x=2 $g(2)\approx 5$

x=3 $g(3)\approx 5.8$

x=4 $g(4)\approx 5$

x=5 $g(5)\approx 2.8$

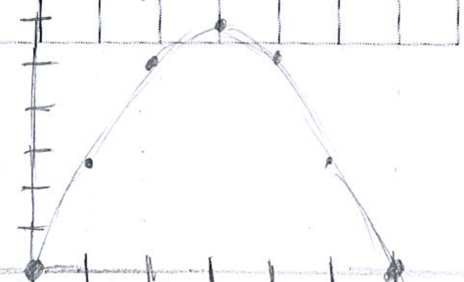
C.) On what interval is g increasing?

$(0,3)$

D.) Where does g have a maximum value?

$x=3$

E. Sketch a rough graph of g.



4-15: Use the 1st Fundamental Theorem of Calculus to find the derivative of the functions.

4. $\frac{d}{dx}g(x)=\frac{d}{dx}\int_1^x \frac{1}{t^3+1}dt$

$g'(x)=\boxed{\frac{1}{x^3+1}}$

5. $\frac{d}{dx}g(x)=\frac{d}{dx}\int_3^x e^{t^2-t}\,dt$

$g'(x)=\boxed{e^{x^2-x}}$

6. $\frac{d}{ds}g(s)=\frac{d}{ds}\int_5^s \left(t-t^2\right)^8 dt$

$g'(s)=\boxed{(s-s^2)^8}$

7. $\frac{d}{dR}g(r)=\frac{d}{dR}\int_0^r \sqrt{x^2+4}\,dx$

$g'(R)=\boxed{\sqrt{R^2+4}}$

8. $G(x)=\int_x^1 \cos\sqrt{t}\,dt = -\int_1^x \cos\sqrt{t}\,dt$

$G'(x)=\boxed{-\cos\sqrt{x}}$

9. $h(x)=\int_1^{e^x} \ln t\,dt$

$h'(x)=\ln e^x\cdot\frac{d}{dx}[e^x]=\boxed{xe^x}$

10. $h(x)=\int_1^{\sqrt{x}} \frac{z^2}{z^4+1}dz$

$h'(x)=\frac{(\sqrt{x})^2}{(\sqrt{x})^4+1}\cdot\frac{1}{2}x^{-1/2}=\boxed{\frac{x}{2\sqrt{x}[x^2+1]}}$

11. $y=\int_0^{\tan x} \sqrt{t+\sqrt{t}}\,dt$

$y'=\sqrt{\tan x+\sqrt{\tan x}}\,(\sec^2 x)$

$y'=\boxed{\sec^2 x\sqrt{\tan x+\sqrt{\tan x}}}$

AP Calculus

Fundamental Theorem of Calculus

Name________________________

Integration Day 5

12. $y=\int_0^{x^4}\cos^2\theta\,d\theta$

$y'=\cos^2(x^4)\cdot 4x^3=\boxed{4x^3\cos^2(x^4)}$

13. $y=\int_{1-3x}^{1}\frac{u^3}{1+u^2}\,du$ $-\int_1^{1-3x}\frac{u^3}{1+u^2}$

$y'=-\frac{(1-3x)^3}{1+(1-3x)^2}(-3)=\boxed{\frac{+3(1-3x)^3}{1+(1-3x)^2}}$

14. $y=\int_{\sin x}^{1}\sqrt{1+t^2}\,dt=-\int_1^{\sin x}\sqrt{1+t^2}\,dt$

$y'=-\sqrt{1+(\sin x)^2}\,[\cos x]=\boxed{-\cos x\sqrt{1+\sin^2 x}}$

15. $F(x)=\int_x^{\pi}\sqrt{1+\sec t}\,dt$

$F(x)=-\int_\pi^{x}\sqrt{1+\sec t}\,dt$

$F'(x)=\boxed{-\sqrt{1+\sec x}}$

16. If $f(x)=\int_0^x(1-t^2)e^{t^2}\,dt$, on what interval is f increasing?

$f'(x)=(1-x^2)e^{x^2}$

$0=1-x^2$ $\quad \cancel{0=e^{x^2}}$ garbage

$x^2=1$

$x=\pm 1$

$- \quad -1 \quad + \quad +1 \quad -$

$\boxed{\text{increasing } (-1,1)}$

17. On what interval is the curve $y=\int_0^x\frac{t^2}{t^2+t+2}\,dt$ concave down?

$y'=\frac{x^2}{x^2+x+2}$ $\quad y''=\frac{(x^2+x+2)(2x)-x^2(2x+1)}{(x^2+x+2)^2}=\frac{\cancel{2x^3}+2x^2+4x-\cancel{2x^3}-x^2}{(x^2+x+2)^2}$

$y''=\frac{x^2+4x}{(x^2+x+2)^2}=\frac{x(x+4)=0}{\cancel{x^2+x+2=0}}$ $\quad x=0\ x=-4$

$\boxed{\text{concave down } (-4,0)}$

$+ \quad -4 \quad - \quad 0 \quad +$

18. If $f(x)=\int_0^{\sin x}\sqrt{1+t^2}\,dt$ and $g(y)=\int_3^y f(x)\,dx$, find $g''\left(\frac{\pi}{6}\right)$.

$f'(x)=\sqrt{1+\sin^2 x}\,\cos x$

$f'(x)=\cos x\sqrt{1+\sin^2 x}$

$g'(y)=f(y)\,dy$

$g''(y)=f'(y)\,dy$

$g''(y)=\cos y\sqrt{1+\sin^2 y}$

$g''(\frac{\pi}{6})=\cos(\frac{\pi}{6})\sqrt{1+(\sin\frac{\pi}{6})^2}$

$=\frac{\sqrt{3}}{2}\sqrt{1+(\frac{1}{2})^2}=\frac{\sqrt{3}}{2}\sqrt{\frac{4}{4}+\frac{1}{4}}=\frac{\sqrt{3}}{2}\frac{\sqrt{5}}{2}=\boxed{\frac{\sqrt{15}}{4}}$

AP Calculus

Fundamental Theorem of Calculus

Name______________________________

Integration Day 5

19. If f(1)=12, f' is continuous, and $\int_1^4 f'(x)dx = 17$, what is the value of f(4)?

$f(x)\Big]_1^4 = 17$

$f(4) - f(1) = 17$

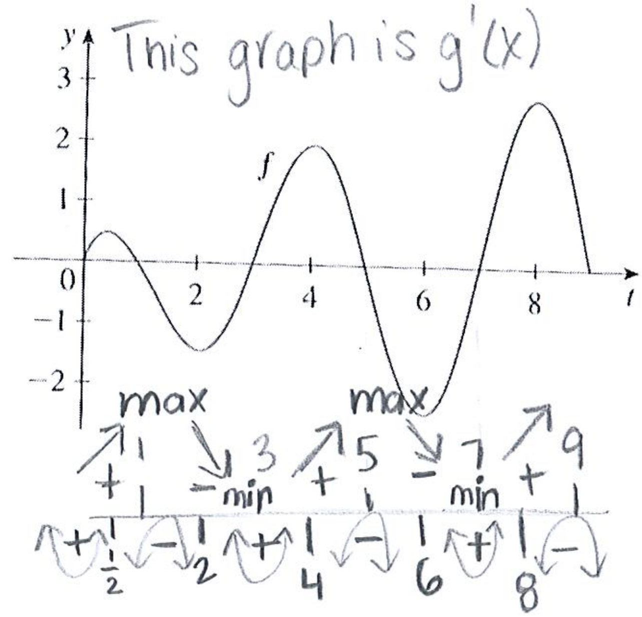
$f(4) - 12 = 17$

$f(4) = 17 + 12$

$\boxed{f(4) = 29}$

$g'(x) = f(x)$

20. Let $g(x) = \int_0^x f(t)dt$, where f is the function whose graph is shown.



A.) At what values of x doe the local maximum and minimum of g occur?

max: x=1 & x=5 min: x=3 & x=7

B.) Where does g attain its absolute maximum value?

x=9

C.) On what intervals is g concave downward?

concave down $(\frac{1}{2}, 2) \cup (4,6) \cup (8,9)$

D.) Sketch the graph of g.