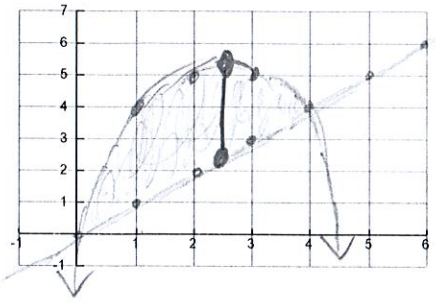
AP Calculus
Volume of Cross Sections

Name_____
Application of Integration Day 4

1-4: Find the volume of the solid whose base is the solid formed by $y = 5x - x^2$ and $y = x$.

Side = top - bottom
Side = $5x - x^2 - x$
side = $4x - x^2$



x	$y = 5x - x^2$
-1	
0	0
1	4
2	6
3	6
4	4

1. If the cross sections are perpendicular to the x-axis and the cross sections are squares.

$\int_0^4 (4x - x^2)^2 \, dx = \frac{512}{15}$

2. If the cross sections are perpendicular to the x-axis and the cross sections are equilateral triangles.

$\frac{\sqrt{3}}{4}\int_0^4 (4x - x^2)^2 \, dx$
$\frac{\sqrt{3}}{4}\left(\frac{512}{15}\right) = \frac{128\sqrt{3}}{15}$

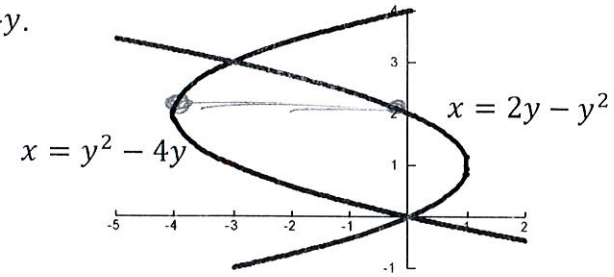
3. If the cross sections are perpendicular to the x-axis and the cross sections are right isosceles triangles.

$\frac{1}{2}\int_0^4 (4x - x^2)^2 \, dx$
$\frac{1}{2}\left(\frac{512}{15}\right) = \frac{256}{15}$

4. If the cross sections are perpendicular to the x-axis and the cross sections are semi-circles. $\frac{\pi}{8}\int_0^4 (4x - x^2)^2 \, dx$
$\frac{\pi}{8}\left(\frac{512}{15}\right) = \frac{64\pi}{15}$

5-8: Find the volume of the solid whose base is the solid formed by $x = 2y - y^2$ and $x = y^2 - 4y$.

Side = Right - left
side = $2y - y^2 - (y^2 - 4y)$
side = $2y - y^2 - y^2 + 4y$
Side = $6y - 2y^2$



5. If the cross sections are perpendicular to the y-axis and the cross sections are squares.

$\int_0^3 6y - 2y^2 \, dy = \frac{162}{5}$

6. If the cross sections are perpendicular to the y-axis and the cross sections are semi-circles. $\frac{\pi}{8}\int_0^3 6y - 2y^2 \, dy = \frac{\pi}{8}\left(\frac{162}{5}\right) = \frac{81\pi}{20}$

7. If the cross sections are perpendicular to the y-axis and the cross sections are right isosceles triangles.

$\frac{1}{2}\int_0^3 6y - 2y^2 \, dy$
$\frac{1}{2}\left(\frac{162}{5}\right) = \frac{81}{5}$

8. If the cross sections are perpendicular to the y-axis and the cross sections are equilateral triangles.

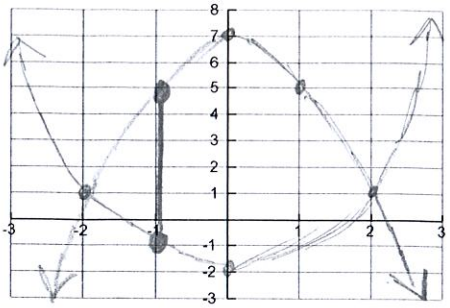
$\frac{\sqrt{3}}{2}\int_0^3 6y - 2y^2 \, dy$
$\frac{\sqrt{3}}{2}\left(\frac{162}{5}\right) = \frac{81\sqrt{3}}{5}$

9-12: Find the volume of the solid whose base is the solid formed by $y = \frac{1}{4}x^2 - 2$ and $y = -2x^2 + 7$.

Side = top − bottom

Side $= (-2x^2+7) - (\frac{1}{4}x^2 - 2)$

Side $= -2x^2 + 7 - \frac{1}{4}x^2 + 2 = -\frac{9}{4}x^2 + 9$



x	$y=-\frac{1}{4}x^2-2$	x	$y=-2x^2+7$
-3		-3	
-2	1	-2	1
-1	-1.75	-1	5
0	-2	0	7
1	-1.75	1	5
2	1	2	1
3		3	

9. If the cross sections are perpendicular to the x-axis and the cross sections are squares.

$\int_{-2}^{2} (-\frac{9}{4}x^2+9)^2\,dx = \frac{864}{5}$

10. If the cross sections are perpendicular to the x-axis and the cross sections are semi-circles. $\frac{\pi}{2}\int_{-2}^{2} (-\frac{9}{4}x^2+9)^2\,dx = \frac{\pi}{8}(\frac{864}{5}) = \frac{108\pi}{5}$

11. If the cross sections are perpendicular to the x-axis and the cross sections are equilateral triangles.

$\frac{\sqrt{3}}{4}\int_{-2}^{2} (-\frac{9}{4}x^2+9)^2\,dx = \frac{\sqrt{3}}{4}(\frac{864}{5}) = \frac{216\sqrt{3}}{5}$

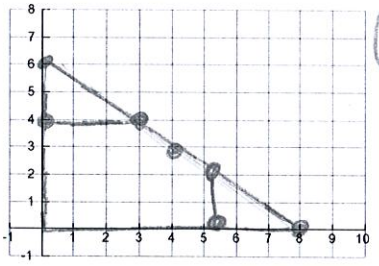
12. If the cross sections are perpendicular to the x-axis and the cross sections are right isosceles triangles.

$\frac{1}{2}\int_{-2}^{2} (-\frac{9}{4}x^2+9)^2\,dx = \frac{1}{2}(\frac{864}{5}) = \frac{432}{5}$

13-20: Find the volume of the solid whose base is the solid formed in the first quadrant by the x-axis, the y-axis and the line $y = 6 - \frac{3}{4}x$

y: Side = R − L = $8 - \frac{4}{3}y - 0 = 8 - \frac{4}{3}y$

x: Side = T − B = $6 - \frac{3}{4}x - 0 = 6 - \frac{3}{4}x$



$(y = 6 - \frac{3}{4}x)4$

$4y = 24 - 3x$

$3x = 24 - 4y$

$x = 8 - \frac{4}{3}y$

13. If the cross sections are perpendicular to the y-axis and the cross sections are squares.

$\int_0^6 (8-\frac{4}{3}y)^2\,dy = 128$

14. If the cross sections are perpendicular to the y-axis and the cross sections are semi-circles. $\frac{\pi}{8}\int_0^6 (8-\frac{4}{3}y)^2\,dy = \frac{\pi}{8}(128) = 16\pi$

15. If the cross sections are perpendicular to the y-axis and the cross sections are equilateral triangles.

$\frac{\sqrt{3}}{4}\int_0^6 (8-\frac{4}{3}y)^2\,dy = \frac{\sqrt{3}}{4}(128) = 32\sqrt{3}$

16. If the cross sections are perpendicular to the y-axis and the cross sections are right isosceles triangles.

$\frac{1}{2}\int_0^6 (8-\frac{4}{3}y)^2\,dy = \frac{1}{2}(128) = 64$

17. If the cross sections are perpendicular to the x-axis and the cross sections are squares.

$\int_0^8 (6-\frac{3}{4}x)^2\,dx = 96$

18. If the cross sections are perpendicular to the x-axis and the cross sections are semi-circles. $\frac{\pi}{8}\int_0^8 (6-\frac{3}{4}x)^2\,dx = \frac{\pi}{8}(96) = 12\pi$

19. If the cross sections are perpendicular to the x-axis and the cross sections are equilateral triangles.

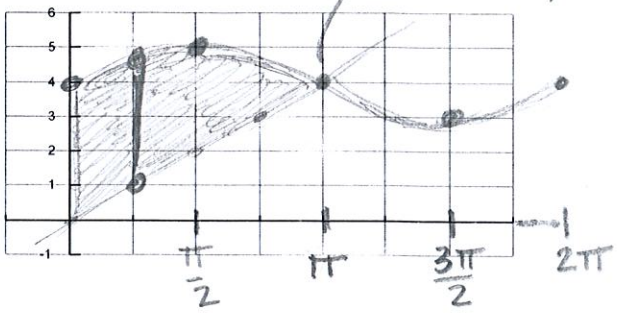
$\frac{\sqrt{3}}{4}\int_0^8 (6-\frac{3}{4}x)^2\,dx = \frac{\sqrt{3}}{4}(96) = 24\sqrt{3}$

20. If the cross sections are perpendicular to the x-axis and the cross sections are right isosceles triangles.

$\frac{1}{2}\int_0^8 (6-\frac{3}{4}x)^2\,dx = \frac{1}{2}(96) = 48$

21-24: Find the volume of the solid whose base is the solid formed by $y = sinx + 4$, the y-axis and $y = x$.

Side $= (\sin x + 4) - x = \sin x - x + 4$



21. If the cross sections are perpendicular to the x-axis and the cross sections are squares.

$\int_0^{3.57764} (\sin x - x + 4)^2 \, dx$

32.516

22. If the cross sections are perpendicular to the x-axis and the cross sections are equilateral triangles.

$\frac{\sqrt{3}}{4}\int_0^{3.57764} (\sin x - x + 4)^2 \, dx = \frac{\sqrt{3}}{4}(32.516)$

14.0798

23. If the cross sections are perpendicular to the x-axis and the cross sections are right isosceles triangles.

$\frac{1}{2}\int_0^{3.57764} (\sin x - x + 4)^2 \, dx = \frac{1}{2}(32.516)$

$= 16.258$

24. If the cross sections are perpendicular to the x-axis and the cross sections are semi-circles.

$\frac{\pi}{8}\int_0^{3.57764} (\sin x - x + 4)^2 \, dx$

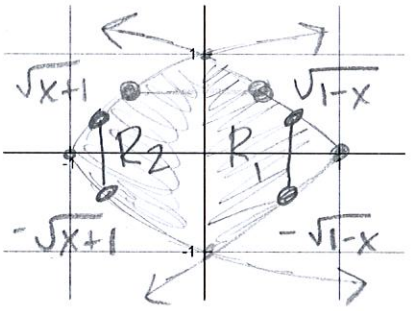
$\frac{\pi}{8}(32.516) = 12.769$

25-30: Find the volume of the solid whose base is the solid formed by $x = 1 - y^2$ and $x = y^2 - 1$.

y: Side $= R - L = (1 - y^2) - (y^2 - 1) = 2 - 2y^2$

x: Side $= T - B = \sqrt{1-x} - -\sqrt{1-x} = 2\sqrt{1-x}$

Side $= T - B = \sqrt{x+1} - -\sqrt{x+1} = 2\sqrt{x+1}$



$x = 1 - y^2$

$y^2 = 1 - x$

$y = \pm\sqrt{1-x}$

$x = y^2 - 1$

$y^2 = x + 1$

$y = \pm\sqrt{x+1}$

25. If the cross sections are perpendicular to the y-axis and the cross sections are squares.

$\int_{-1}^{1} (2 - 2y^2)^2 \, dy = \frac{64}{15}$

26. If the cross sections are perpendicular to the y-axis and the cross sections are semi-circles.

$\frac{\pi}{8}\int_{-1}^{1} (2 - 2y^2)^2 \, dy = \frac{\pi}{8}\left(\frac{64}{15}\right) = \frac{8\pi}{15}$

27. If the cross sections are perpendicular to the y-axis and the cross sections are right isosceles triangles.

$\frac{1}{2}\int_{-1}^{1} (2 - 2y^2)^2 \, dy = \frac{1}{2}\left(\frac{64}{15}\right) = \frac{32}{15}$

28. If the cross sections are perpendicular to the y-axis and the cross sections are equilateral triangles.

$\frac{\sqrt{3}}{4}\int_{-1}^{1} (2 - 2y^2)^2 \, dy = \frac{\sqrt{3}}{4}\left(\frac{64}{15}\right) = \frac{16\sqrt{3}}{15}$

29. If the cross sections are perpendicular to the x-axis and the cross sections are squares.

$\int_{-1}^{0} (2\sqrt{x+1})^2 \, dx + \int_{0}^{1} (2\sqrt{1-x})^2 \, dx$

$2 + 2$

$\boxed{4}$

30. If the cross sections are perpendicular to the y-axis and the cross sections are right isosceles triangles.

$\frac{1}{2}\int_{-1}^{0} (2\sqrt{x+1})^2 \, dx + \frac{1}{2}\int_{0}^{1} (2\sqrt{1-x})^2 \, dx$

$1 + 1$

$\boxed{2}$

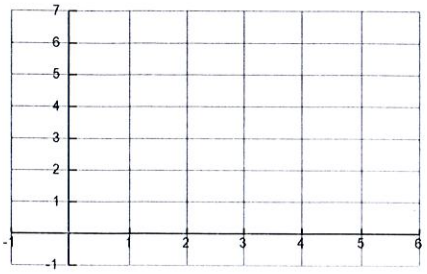
AP Calculus

Volume of Cross Sections

Name_____

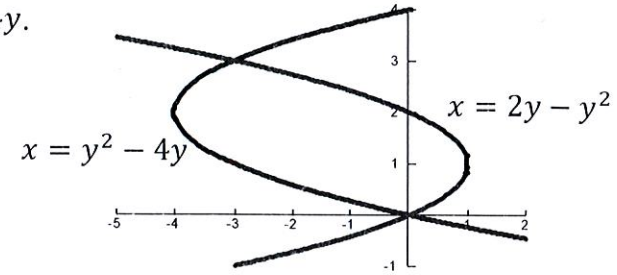
Application of Integration Day 4

1-4: Find the volume of the solid whose base is the solid formed by $y = 5x - x^2$ and $y = x$.



1. If the cross sections are perpendicular to the x-axis and the cross sections are squares.	2. If the cross sections are perpendicular to the x-axis and the cross sections are equilateral triangles.
3. If the cross sections are perpendicular to the x-axis and the cross sections are right isosceles triangles.	4. If the cross sections are perpendicular to the x-axis and the cross sections are semi-circles.

5-8: Find the volume of the solid whose base is the solid formed by $x = 2y - y^2$ and $x = y^2 - 4y$.



5. If the cross sections are perpendicular to the y-axis and the cross sections are squares.	6. If the cross sections are perpendicular to the y-axis and the cross sections are semi-circles.
7. If the cross sections are perpendicular to the y-axis and the cross sections are right isosceles triangles.	8. If the cross sections are perpendicular to the y-axis and the cross sections are equilateral triangles.