AP Calculus

Linear Approximation

Name_______________________Pd.___

Day 4 Application of Derivatives

1-4: Find the linearization L(x) of the function at a.

1. $f(x)=x^4+3x^2$, $a=-1$ $\boxed{1.}$ Point $(-1,4)$

$f(-1)=(-1)^4+3(-1)^2$

$f(-1)=1+3=4$

$f'(x)=4x^3+6x$

$f'(-1)=-4-6=-10$

$\boxed{2}$ $f'(-1)=\underline{-10}=m$

$y-4=-10(x+1)$

$\boxed{y=-10(x+1)+4}$ ✓

2. $f(x)=\sin x$, $a=\frac{\pi}{6}$ $\boxed{1.}$ Point $(\frac{\pi}{6},\frac{1}{2})$

$f(\frac{\pi}{6})=\sin\frac{\pi}{6}=\frac{1}{2}$

$f'(x)=\cos x$

$f'(\frac{\pi}{6})=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

$\boxed{2.}$ $m=f'(\frac{\pi}{6})=\underline{\frac{\sqrt{3}}{2}}$

$y-\frac{1}{2}=\frac{\sqrt{3}}{2}(x-\frac{\pi}{6})$

$\boxed{y=\frac{\sqrt{3}}{2}(x-\frac{\pi}{6})+\frac{1}{2}}$ ✓

3. $f(x)=\sqrt{x}$, $a=4$ $\boxed{1}$ Point $(4,2)$

$f(4)=\sqrt{4}=2$

$f'(x)=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$

$f'(4)=\frac{1}{2\sqrt{4}}=\frac{1}{4}$

$\boxed{2.}$ $m=f'(4)=\underline{\frac{1}{4}}$

$y-2=\frac{1}{4}(x-4)$

$\boxed{y=\frac{1}{4}(x-4)+2}$ ✓

4. $f(x)=x^{\frac{3}{4}}$, $a=16$ $\boxed{1}$ Point $(16,8)$

$f(16)=(\sqrt[4]{16})^3=8$

$f'(x)=\frac{3}{4}x^{-1/4}$

$f'(16)=\frac{3}{4\sqrt[4]{16}}=\frac{3}{8}$

$\boxed{2.}$ $m=f'(16)=\underline{3/8}$

$y-8=\frac{3}{8}(x-16)$

$\boxed{y=\frac{3}{8}(x-16)+8}$ ✓

5. Find the linear approximation of the function $f(x)=\sqrt{1-x}$ at a=0 and use it to approximate the numbers $\sqrt{0.9}$ and $\sqrt{0.99}$.

$\boxed{1.}$ Point $(0,1)$

$\boxed{2.}$ $m=f'(0)=-1/2$

$f(0)=\sqrt{1-0}=1$

$f(x)=(1-x)^{1/2}$

$f'(x)=\frac{1}{2}(1-x)^{-1/2}(-1)$

$f'(x)=\frac{-1}{2\sqrt{1-x}}$

$f'(0)=-1/2\sqrt{1}=-1/2$

$y-1=-\frac{1}{2}(x-0)$

$y=-\frac{1}{2}x+1$

$\sqrt{.9}=\sqrt{1-.1}$

$y(.1)=-\frac{1}{2}(.1)+1=\boxed{.95}$ ✓

$\sqrt{.99}=\sqrt{1-.01}$

$y(.01)=-\frac{1}{2}(.01)+1=\boxed{.995}$ ✓

6-10: Use a linear approximation (or differentials) to estimate the given number.

6. $(1.999)^4$ $f(x)=x^4$ $a=2$ $\boxed{1.}$ Point $(2,16)$

$f'(x)=4x^3$ $f'(2)=4(8)=32$

$\boxed{2.}$ $m=f'(2)=32$

$y-16=32(x-2)$

$y=32(x-2)+16$

$y(1.999)=32(1.999-2)+16$ ✓

$=32(-.001)+16=\boxed{15.968}$

7. $e^{-0.015}$ $f(x)=e^x$ $a=0$ $\boxed{1}$ Point $(0,1)$

$f'(x)=e^x$

$f'(0)=e^0=1$

$\boxed{2}$ $m=f'(0)=1$

$y=1(x-0)+1$

$y-1=1(x-0)$

$y(-.015)=1(-.015-0)+1$

$=\boxed{.985}$ ✓

8. $\sqrt[3]{1001}$ $f(x)=\sqrt[3]{x}$ $a=1000$ $\boxed{1}$ Point $(1000,10)$

$f(x)=x^{1/3}$ $f'(x)=\frac{1}{3}x^{-2/3}$

$\boxed{2}$ $m=f'(1000)=\underline{1/300}$

$f'(1000)=\frac{1}{3(\sqrt[3]{1000})^2}=\frac{1}{300}$

$y-10=1/300(x-1000)$

$y=1/300(x-1000)+10$

$y(1001)=1/300(1)+10$

$=\boxed{10.003}$ ✓

9. $\frac{1}{4.002}$ $f(x)=\frac{1}{x}$ $a=4$ $\boxed{1}$ $(4,1/4)$

$\boxed{2}$ $m=f'(4)=-1/16$

$f(x)=x^{-1}$

$f'(x)=-x^{-2}=-\frac{1}{x^2}$

$f'(4)=-1/16$

$y-1/4=-1/16(x-4)$

$y=-1/16(x-4)+1/4$

$y(4.002)=-1/16(.002)+1/4=\boxed{.249875}$ ✓

10. $\tan 44°$ $f(x)=\tan x$ $a=45°$ $\boxed{1}$ Point $(45°,1)$

$f'(x)=\sec^2 x$

$f'(45)=(\frac{2}{\sqrt{2}})^2=\frac{4}{2}=2$

$\boxed{2}$ $f'(x)=2$

$y-1=2(x-45°)$

$y=2(x-45°)+1$

$y(44°)=2(-1°)+1$

$=2\left[-1°\left(\frac{\pi}{180°}\right)\right]+1$ ✓

$-\frac{\pi}{90}+1=\boxed{.965}$

11. $\sqrt{99.8}$ $f(x)=\sqrt{x}$ $a=100$ $\boxed{1}$ $(100,10)$

$f'(x)=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}=\frac{1}{20}$

$\boxed{2}$ $m=f'(100)=\underline{1/20}$

$y-10=\frac{1}{20}(x-100)$

$y=\frac{1}{20}(x-100)+10$

$y(99.8)=\frac{1}{20}(-.2)+10$

$=\boxed{9.99}$ ✓