AP Calculus

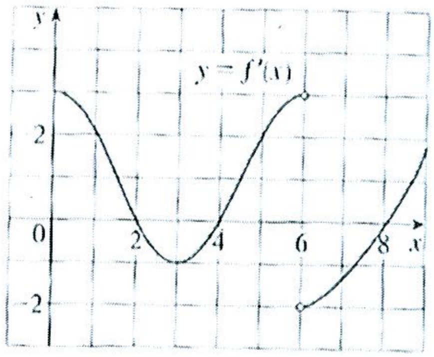
Curve Sketching

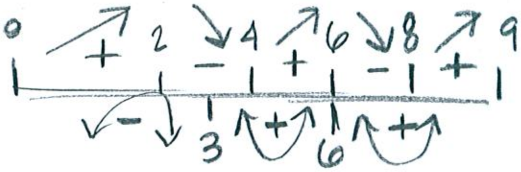
Name_____________________________Pd.____

Day 4 Curve Sketching

1-2: The graph of the derivative of f' of a continuous function f is shown

1.





a.) The open intervals of which f is increasing.? Decreasing?

Inc: (0,2)(4,6)(8,9) Dec: (2,4)(6,8)

b.) At what values of x does f have a local maximum? Local minimum?

Max: x=2 & x=6 min: x=4 & x=8

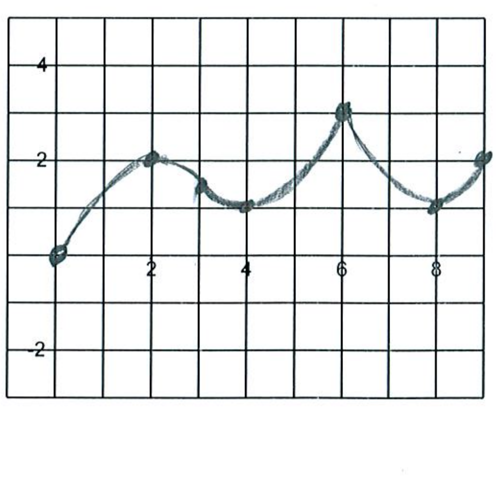
c.) The open intervals of which f is concave upward. Concave downward?

CC↑: (3,6)(6,9) CC↓: (0,3)

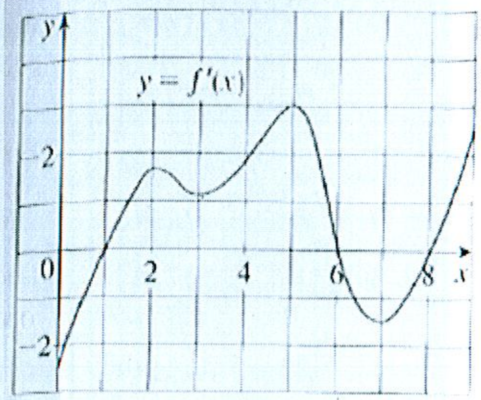
d.) State the x-coordinate(s) of the point(s) of inflection.

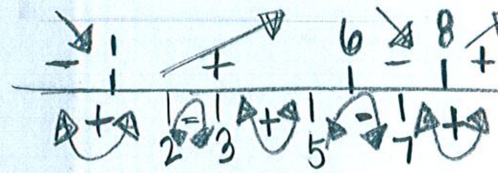
POI: x = 3

e.) Assuming that f(0)=0, sketch the graph of f.



2.





a.) The open intervals of which f is increasing.? Decreasing?

Inc: (1,6)(8,9) Dec: (0,1)(6,8)

b.) At what values of x does f have a local maximum? Local minimum?

Max: x=6 min: x=1 & x=8

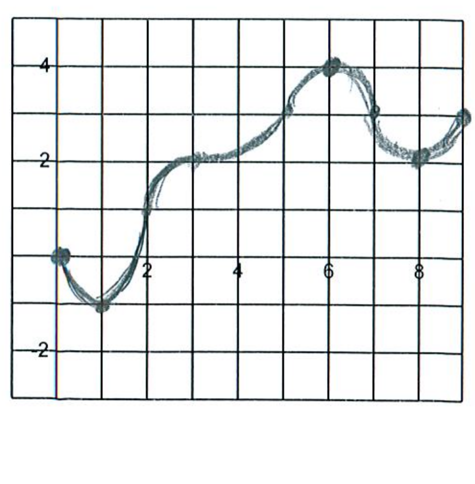
c.) The open intervals of which f is concave upward. Concave downward?

CC↑: (0,2)(3,5)(7,9) CC↓: (2,3)(5,7)

d.) State the x-coordinate(s) of the point(s) of inflection.

POI: x=2, 3, 5, & 7

e.) Assuming that f(0)=0, sketch the graph of f.



3-6: Find a)-d) for each:

3. $f(x) = x^3 - 12x + 2$

a.) Find intervals

increase: $(-\infty, -2) \cup (2, \infty)$

decrease: $(-2, 2)$

b.) Find

local max: $x = -2$

local min: $x = 2$

c.) Find intervals

concave up: $(0, \infty)$

concave down: $(-\infty, 0)$

P.O.I. $x = 0$

d.)Use a)-c) to sketch

$f'(x) = 3x^2 - 12$

$0 = 3(x^2 - 4)$

$x^2 = 4$

$x = \pm 2$

$-\infty$ + | −2 − | +2 + ∞ ; − | 0 +

$f(-2) = 18$

$f(0) = 2$

$f(2) = -14$

$f'(-3) = \text{pos}$

$f'(0) = \text{neg}$

$f'(3) = \text{pos}$

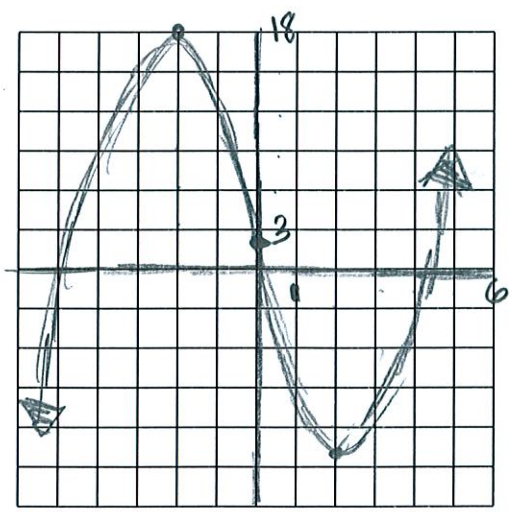
$f''(x) = 6x$

$6x = 0$

$x = 0$

$f''(-1) = \text{neg}$

$f''(1) = \text{pos}$



4. $G(x) = 5x^{\frac{2}{3}} - 2x^{\frac{5}{3}}$

$f(-\frac{1}{2}) = 3.78$

$f(0) = 0$

$f(1) = 3$

a.) Find intervals

increase: $(0, 1)$

decrease: $(-\infty, 0) \cup (1, \infty)$

b.) Find

local max: $x = 1$

local min: $x = 0$

c.) Find intervals

concave up: $(-\infty, -\frac{1}{2})$

concave down: $(-\frac{1}{2}, 0) \cup (0, \infty)$

P.O.I. $x = -\frac{1}{2}$

d.)Use a)-c) to sketch

− | 0 + | 1 − ; + | $-\frac{1}{2}$ − | 0 −

$f'(x) = \frac{10}{3}x^{-1/3} - \frac{10}{3}x^{2/3}$

$f'(x) = \frac{10}{3x^{1/3}} - \frac{10x^{2/3}(x^{1/3})}{3(x^{1/3})}$

$f'(x) = \frac{10(1-x)}{3x^{1/3}}$ $x = 1$ $x = 0$

$f'(-1) = \frac{+}{-} = -$

$f'(\frac{1}{2}) = \frac{+}{+} = +$

$f'(2) = \frac{-}{+} = -$

$f'(x) = \frac{10 - 10x}{3x^{1/3}}$

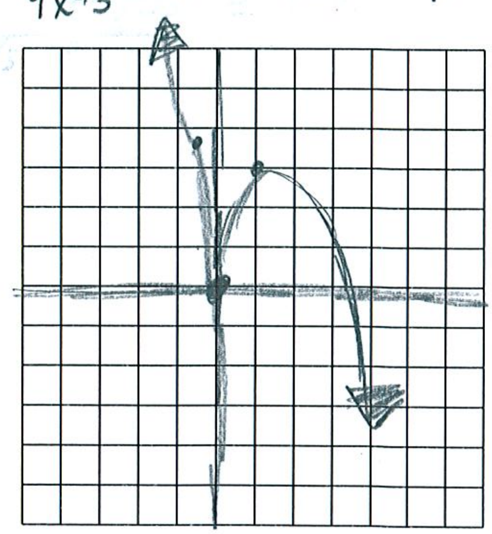
$f''(x) = \frac{3x^{1/3}[-10] - [10 - 10x]x^{-2/3}}{(3x^{1/3})^2}$

$f''(x) = \frac{\frac{-30x^{1/3}}{1} \cdot \frac{(x^{2/3})}{(x^{2/3})} - \frac{[10-10x]}{x^{2/3}}}{9x^{2/3}}$

$\frac{-30x - 10 + 10x}{x^{2/3}} \cdot \frac{1}{9x^{2/3}}$

$\frac{9x^{2/3}}{1} \cdot \frac{1}{9x^{2/3}}$

$\frac{-20x - 10}{9x^{4/3}}$ $x = -\frac{1}{2}$ $x = 0$



$6 - x \geq 0$
$6 \geq x$
$x \leq 6$

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5. $F(x) = x\sqrt{6-x} = x(6-x)^{1/2}$

a.) Find intervals
increase: $(-\infty, 4)$
decrease: $(4, 6)$

$f(4) = 4\sqrt{2} \approx 5.7$
$f(6) = 0$

b.) Find
local max: $x = 4$
local min: none

c.) Find intervals
concave up: $(-\infty, 6)$
concave down: none
P.O.I. none

d.)Use a)-c) to sketch

$f'(x) = x\frac{1}{2}(6-x)^{-1/2}(-1) + (6-x)^{1/2}(1)$

$f'(x) = \frac{-x}{2\sqrt{6-x}} + \frac{\sqrt{6-x}\,(2\sqrt{6-x})}{2\sqrt{6-x}}$

$f'(x) = \frac{-x + 2(6-x)}{2\sqrt{6-x}} = \frac{-x+12-2x}{2\sqrt{6-x}}$

$f'(x) = \frac{-3x+12}{2\sqrt{6-x}}$ $x = 4$ $x = 6$

$-\infty$ + 4 − 6

$f''(x) = \frac{2\sqrt{6-x}\,[-3] - (-3x+12)2\left(\frac{1}{2}\right)(6-x)^{-1/2}(-1)}{4(6-x)}$

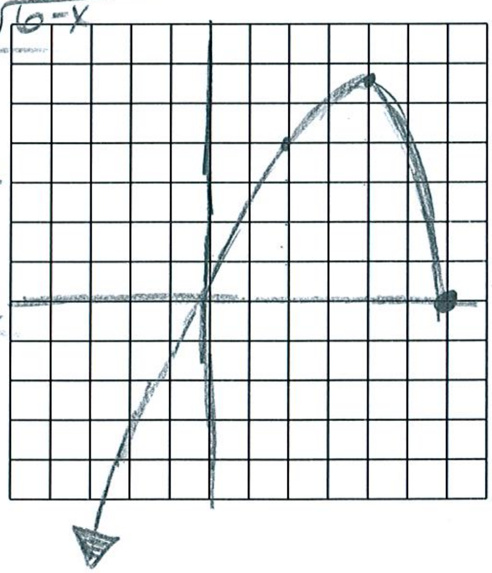
$= \frac{-6\sqrt{6-x} + \frac{(-3x+12)}{\sqrt{6-x}}}{\frac{4(6-x)}{1}}$

$\frac{-6(6-x) - 3x + 12}{4(6-x)^{3/2}}$

$\frac{-36 + 6x - 3x + 12}{4(6-x)^{3/2}}$

$\frac{3x - 24}{4(6-x)^{3/2}}$

$x = 8$
$x = 6$



$f(-1) = -3$
$f(0) = 0$
$f(2) = 7.6$

6. $C(x) = x^{\frac{1}{3}}(x+4) = x^{4/3} + 4x^{1/3}$

a.) Find intervals
increase: $(-1, \infty)$
decrease: $(-\infty, -1)$

b.) Find
local max: none
local min: $x = -1$

− −1 + 0 +

+ 0 − 2 +

c.) Find intervals
concave up: $(-\infty, 0)(2, \infty)$
concave down: $(0, 2)$
P.O.I. $x = 0$ & $x = 2$

d.)Use a)-c) to sketch

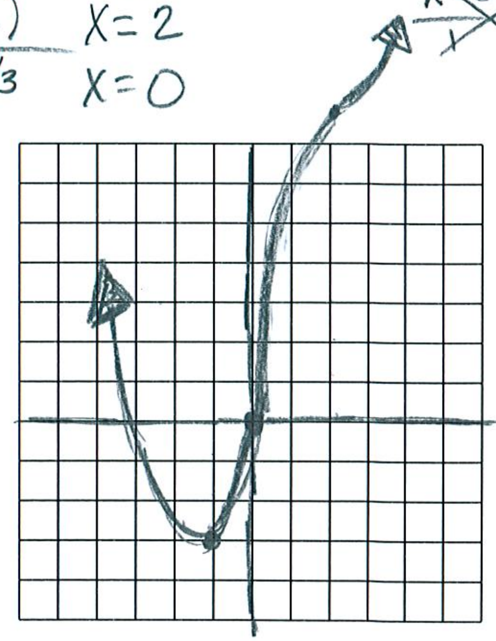
$C'(x) = \frac{4}{3}x^{1/3} + \frac{4}{3}x^{-2/3} = \frac{4}{3}\left(x^{1/3}\frac{(x^{2/3})}{(x^{2/3})} + \frac{1}{x^{2/3}}\right)$

$C'(x) = \frac{4}{3}\left(\frac{x+1}{x^{2/3}}\right)$ $x = -1$ $x = 0$

$C''(x) = \frac{4}{3}\left[\frac{x^{2/3}(1) - (x+1)\frac{2}{3}x^{-1/3}}{x^{4/3}}\right]$

$C''(x) = \frac{4}{3}\left[\frac{x^{2/3}\frac{(3x^{1/3})}{(3x^{1/3})} - \frac{2(x+1)}{3\,x^{1/3}}}{x^{4/3}}\right] = \frac{4}{3}\left[\frac{3x-2x-2}{3x^{1/3}}\right]\frac{\frac{1}{x^{4/3}}}{\frac{x^{4/3}}{1}}$

$= \frac{4}{3}\frac{(x-2)}{3x^{5/3}}$ $x = 2$ $x = 0$



7-8: Find a)-e) for each.

7. $f(x)=\frac{x^2-4}{x^2+4}$

a.) Find

VA: $x^2+4=0$ $x^2 \neq -4$ none

HA: $y=1$

$f(-1.15)=-\frac{1}{2}$

$f(0)=-1$

$f(1.15)=-\frac{1}{2}$

b.) Find intervals

Increase: $(0,\infty)$

Decrease: $(-\infty,0)$

c.) Find

Local Max(s): none

Local Min(s): $x=0$

d.) Find intervals

Concave Up: $(-\infty,\infty)$

Concave Down: none

Sign chart for f': $-$ | 0 | $+$

Sign chart for f'': $-$ | $-\sqrt{\frac{4}{3}}$ | $+$ | $\sqrt{\frac{4}{3}}$ | $-$

e.) Use a)-d) to sketch

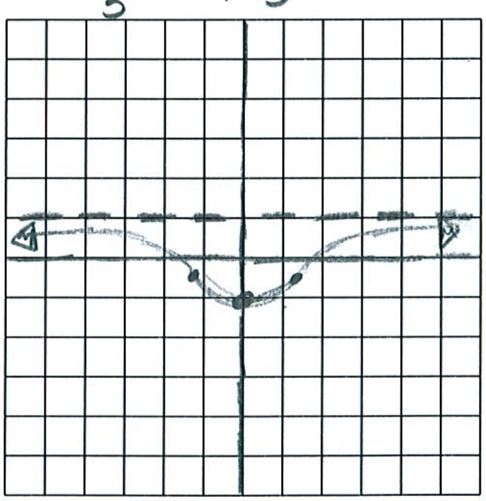
$f'(x)=\frac{(x^2+4)(2x)-(x^2-4)(2x)}{(x^2+4)^2}=\frac{2x^3+8x-2x^3+8x}{(x^2+4)^2}$

$f'(x)=\frac{16x}{(x^2+4)^2}$ $x=0$ $x=$ garbage

$f''(x)=\frac{(x^2+4)^2(16)-16x(2)(x^2+4)(2x)}{(x^2+4)^4}$

$\frac{(x^2+4)[16(x^2+4)-64x^2]}{(x^2+4)^3}=\frac{16x^2+64-64x^2}{(x^2+4)^3}$

$\frac{-48x^2+64}{(x^2+4)^3}$ $x^2=\frac{4}{3}=\pm\sqrt{4/3}$



8. $f(x)=\frac{e^x}{1-e^x}$

$f(1)=-1.58$

$f(-1)=.58$

a.) Find

VA: $1-e^x=0$ $x=0$

HA: $y=-1$

b.) Find intervals

Increase: $(-\infty,0)(0,\infty)$

Decrease: none

c.) Find

Local Max(s): none

Local Min(s): none

d.) Find intervals

Concave Up: $(-\infty,0)$

Concave Down: $(0,\infty)$

Sign chart for f': $+$ | 0 | $+$

Sign chart for f'': $+$ | 0 | $-$

e.) Use a)-d) to sketch

$f'(x)=\frac{(1-e^x)e^x-e^x(-e^x)}{(1-e^x)^2}=\frac{e^x(1-e^x+e^x)}{(1-e^x)^2}$

$f'(x)=\frac{e^x}{(1-e^x)^2}$ $e^x \neq 0$ $1-e^x=0$ $e^x=1$ $x=\ln(1)$ $x=0$

$f''(x)=\frac{(1-e^x)^2e^x-e^x(2)(1-e^x)(-e^x)}{(1-e^x)^4}$

$f''(x)=\frac{e^x(1-e^x)[1-e^x+2e^x]}{(1-e^x)^3}$

$f''(x)=\frac{e^x[1+e^x]}{(1-e^x)^3}$ $e^x \neq 0$ $1+e^x=0$ $e^x \neq -1$

$1-e^x=0$

$x=0$