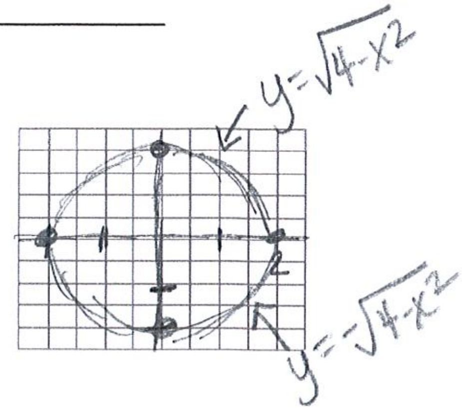
$Side = \sqrt{4-x^2} - -\sqrt{4-x^2} = 2\sqrt{4-x^2}\,dx$

AP Calculus
Cross Sections and Average Value

Name________________________________
Date__________________

1. Find the volume of the object with the base $x^2 + y^2 = 4$

a. With cross sections perpendicular to the $x-axis$ that are triangles with base and height equal to each other. Isoceles Δ



$\frac{1}{2}\int_{-2}^{2} (2\sqrt{4-x^2})^2 dx$

$\frac{1}{2}\int_{-2}^{2} 4(4-x^2)dx = \frac{1}{2}\left(\frac{128}{3}\right) = \boxed{\frac{64}{3}}$

b. With cross sections perpendicular to the $x-axis$ that are semi-circles.

$d = 2\sqrt{4-x^2} = side$ radius $= \sqrt{4-x^2} - 0 = \sqrt{4-x^2}$

$\frac{\pi}{8}\int_{-2}^{2} 4(4-x^2)dx$ $\frac{\pi}{2}\int_{-2}^{2} (\sqrt{4-x^2})^2$

$\frac{\pi}{8}\left(\frac{128}{3}\right) = \boxed{\frac{16\pi}{3}}$ $\frac{\pi}{2}\left(\frac{32}{3}\right) = \boxed{\frac{16\pi}{3}}$

c. With cross sections perpendicular to the $x-axis$ that are squares.

$\int_{-2}^{2} (2\sqrt{4-x^2})^2 dx$

$\int_{-2}^{2} 4(4-x^2)^2 dx = \boxed{\frac{128}{3}}$

d. With cross sections perpendicular to the $x-axis$ that are equilateral triangl

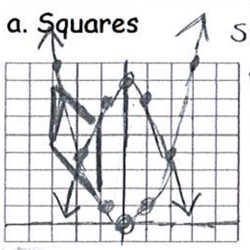
$\frac{\sqrt{3}}{4}\int_{-2}^{2} (2\sqrt{4-x^2})^2 dx = \frac{\sqrt{3}}{4}\left(\frac{128}{3}\right) = \boxed{\frac{32\sqrt{3}}{3}}$

For problems 2-3 set up and do not integrate.

2. The base of the volume is the region bounded by the curves $y = 8 - x^2$ and $y = x^2$.

The cross sections perpendicular to the x-axis are:

a. Squares



side = top - bottom
side = $8-x^2-x^2$
side = $8-2x^2$
$\int_{-2}^{2} (8-2x^2)^2 dx$
$\boxed{\frac{64}{3}}$

b. Equilateral triangles

$\frac{\sqrt{3}}{4}\int_{-2}^{2} (8-2x^2)^2 dx$

$\frac{\sqrt{3}}{4}\left(\frac{64}{3}\right) = \boxed{\frac{16\sqrt{3}}{3}}$

c. Isosceles right triangles with leg on the base

$\frac{1}{2}\int_{-2}^{2} (8-2x^2)^2 dx$

$\frac{1}{2}\left(\frac{64}{3}\right) = \boxed{\frac{32}{3}}$

d. Semi-circles

side = diameteR

$\frac{\pi}{8}\int_{-2}^{2} (8-2x^2)^2 dx$

$\frac{\pi}{8}\left(\frac{64}{3}\right) = \boxed{\frac{8\pi}{3}}$

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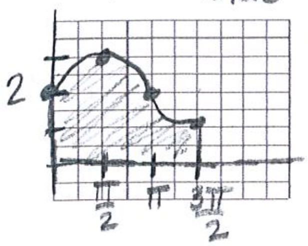
3. The base of the volume is the region bounded by the curve $y = 2 + \sin x$, the y-axis, $x=0$ and $x = \frac{3\pi}{2}$ The cross sections perpendicular to the x-axis are:

a. Square Side $= 2+\sin x - 0$

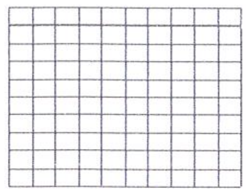


$= 2+\sin x$

$\int_0^{3\pi/2} (2+\sin x)^2 dx$

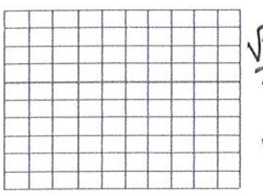
$\boxed{25.206}$

c. Isosceles right triangles with leg on the base



$\frac{1}{2}\int_0^{\frac{3\pi}{2}} (2+\sin x)^2 dx = \boxed{12.603}$

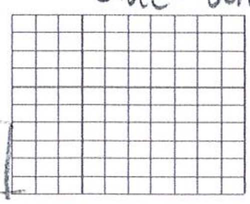
b. Equilateral triangles



$\frac{\sqrt{3}}{4}\int_0^{\frac{3\pi}{2}} (2+\sin x)^2$

$\frac{\sqrt{3}}{4}(25.206) = \boxed{10.914}$

d. Semi-circles Side = diameter

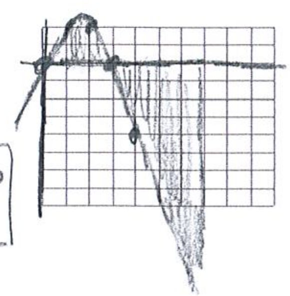


$\frac{\pi}{8}\int_0^{3\pi/2} (2+\sin x)^2 dx$

$\boxed{9.898}$

4. For the function $f(x) = -x^2 + 3x$

a. Find the area of the region bounded by $f(x)$, the x axis, and the vertical lines $x=1$, $x=7$. State any integrals you are using in proper notation. You can use a calculator to get the integrals.



$\int_1^3 (-x^2+3x) - 0\, dx + \int_3^7 0 - (-x^2+3x) dx = 3.\bar{3} + 45.\bar{3} = \boxed{\frac{146}{3}}$

b. Find the average value of the function $f(x)$ on the interval [1,7] using integration.

$\frac{1}{7-1}\left[\frac{146}{3}\right] = \frac{1}{6}\left(\frac{146}{3}\right) = \boxed{\frac{73}{9}}$

c. Sketch the function, the area you've found, and the average value, on the same graph. Use a horizontal line for the average value.

Find the average value of each:

5. $f(x) = 2x - 3x^2$, $[0,5]$

6. $f(x) = 2x - 3x^2$, $[5,10]$

7. $f(x) = \sin x$, $[0,\pi]$

8. $f(x) = \sin x$, $[0,2\pi]$

9. $f(x) = \frac{2}{x-3}$, $[1,5]$

10. $f(x) = \frac{2}{x-3}$, $[5,15]$

$\boxed{5}$ $\frac{1}{5}\int_0^5 2x - 3x^2 dx$

$\frac{1}{5}\left[x^2 - x^3\right|_0^5$

$\frac{1}{5}\left[25 - 125 - 0 + 0\right]$

$\frac{1}{5}[-100] = \boxed{-20}$

$\boxed{6.}$ $\frac{1}{10-5}\left[x^2 - x^3\right|_5^{10}$

$\frac{1}{5}\left[100 - 1000 - 25 + 125\right]$

$\frac{1}{5}[-800] = \boxed{-160}$

$\boxed{7.}$ $\frac{1}{\pi}\int_0^{\pi} \sin x dx$

$\frac{1}{\pi}\left[-\cos x\right|_0^{\pi}$

$\frac{1}{\pi}\left[-\cos\pi + \cos 0\right]$

$\frac{1}{\pi}[2] = \boxed{\frac{2}{\pi}}$

$\boxed{8.}$ $\frac{1}{2\pi - 0}\int_0^{2\pi} \sin x dx$

$\frac{1}{2\pi}\left[-\cos x\right|_0^{2\pi}$

$\frac{1}{2\pi}[0] = \boxed{0}$

$\boxed{9}$ $\frac{1}{4}\int_1^5 \frac{2}{x-3}$ $u = x-3$, $du = dx$

$\frac{1}{4} \cdot 2\int \frac{1}{x-3} dx$ $u(1) = 1-3 = -2$, $u(5) = 5-3 = 2$

$\frac{1}{2}\ln|u|\Big|_{-2}^{2}$

$\frac{1}{2}\left[\ln|2| - \ln|-2|\right] = \boxed{0}$

$\boxed{10}$ $\frac{1}{10}\int_5^{15} \frac{2}{x-3}$ $u = x-3$, $du = dx$

$\frac{1}{10}\int_2^{12} 2\left(\frac{1}{u}\right) du$ $u(5) = 2$, $u(15) = 12$

$\frac{2}{10}\ln u\Big|_2^{12}$

$\frac{1}{5}\left[\ln(12) - \ln(2)\right]$

$\boxed{\frac{1}{5}\ln(6)}$