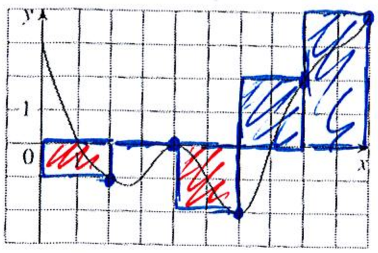
AP Calculus width $= \frac{10-0}{5} = 2$ Name________________

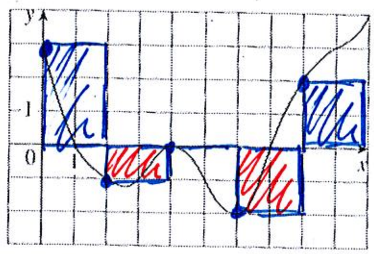
Definite Integrals by Approximation Integration Day 3

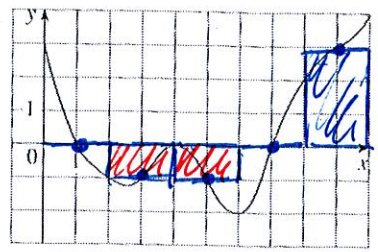
1. The graph of a function f is given. Estimate $\int_0^{10} f(x)dx$ using five subintervals with

a.)Right endpoints b.)left endpoints $= \boxed{4}$ c.)midpoints $= \boxed{2}$

$A = -2+0-4+4+8 = \boxed{6}$ $A = 6-2+0-4+4 =$ $A = 0-2-2+0+6$







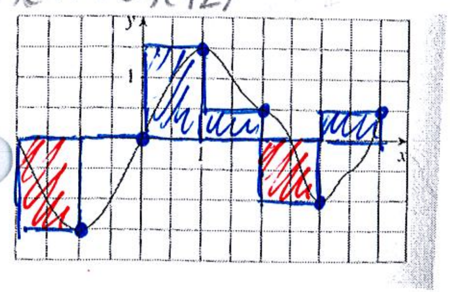
2. The graph of g is shown. Estimate $\int_{-2}^{4} g(x)dx$ with six subintervals using width $= \frac{4--2}{6} = 1$

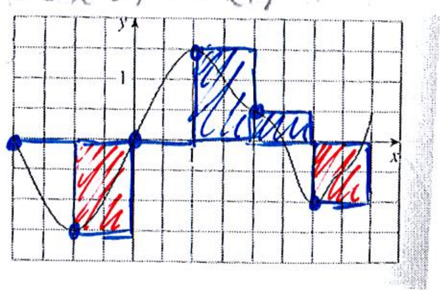
a.)Right endpoints $\boxed{0}$ b.)left endpoints $\boxed{-\frac{1}{2}}$ c.)midpoints $\boxed{\frac{1}{2}}$

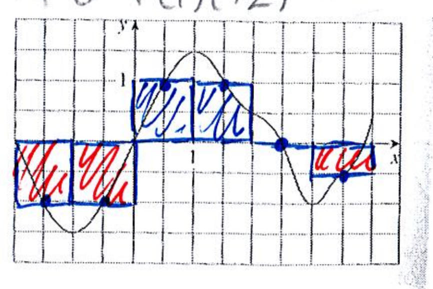
$A = (1)(-1.5)+0+(1)(1.5)+(1)(\frac{1}{2})+(1)(-1)+(1)(\frac{1}{2})$

$A = 0+(1)(-1.5)+0+(1)(1.5)+(1)(\frac{1}{2})+(1)(-1)$

$A = (1)(-1)+(1)(-1)+(1)(1)+(1)(1)+0+(1)(\frac{1}{2})$







3. The table gives the values of a function obtained from an experiment. Use them to estimate $\int_3^9 f(x)dx$ using three equal subintervals with $\frac{9-3}{3} = \frac{6}{3} = 2$

a.)Right endpoints b.)left endpoints c.)midpoints

x	3	4	5	6	7	8	9
f(x)	-3.4	-2.1	-.6	.3	.9	1.4	1.8

$2[-.6+.9+1.8] = \boxed{4.2}$

x	3	4	5	6	7	8	9
f(x)	-3.4	-2.1	-.6	.3	.9	1.4	1.8

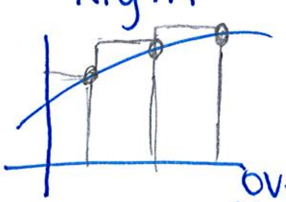
$2[-3.4-.6+.9] = \boxed{-6.2}$

x	3	4	5	6	7	8	9
f(x)	-3.4	-2.1	-.6	.3	.9	1.4	1.8

$2[-2.1+.3+1.4] = \boxed{-.8}$

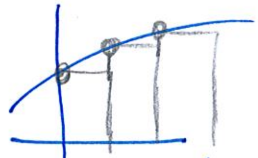
If the function is known to be an increasing function, can you say whether your estimates are less than or greater than the exact value of each integral?

Right



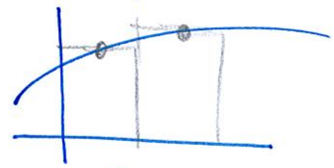
yes- over estimate

Left



yes- under estimate

Midpoints



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AP Calculus

Definite Integrals by Approximation

Name________________________________

Integration Day 3

4. The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.

a.) $\int_0^2 f(x)dx$

$\frac{2}{2}[1+3] = \boxed{4}$

b.) $\int_0^5 f(x)dx$

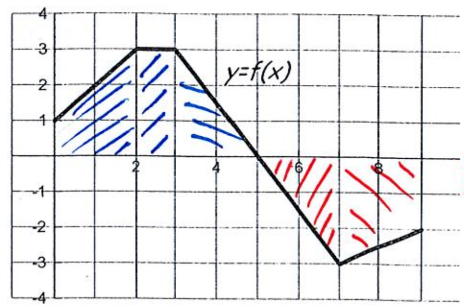
$4+3+\frac{1}{2}(2)(3) = \boxed{10}$

c.) $\int_5^7 f(x)dx$

$-\frac{1}{2}(2)(3) = \boxed{-3}$

d.) $\int_0^9 f(x)dx$

$10-3-5 = \boxed{2}$



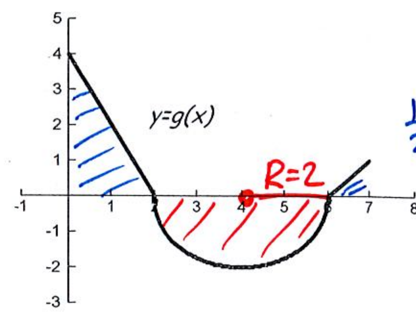
$\frac{2}{2}(3+2)=5$

5. The graph of g consists of two straight lines and a semi-circle. Use it to evaluate each integral.

a.) $\int_0^2 g(x)dx$ $\frac{1}{2}(2)(4) = \boxed{4}$

b.) $\int_2^6 g(x)dx$ $-\frac{1}{2}\pi(2)^2 = \boxed{-2\pi}$

c.) $\int_0^7 g(x)dx$ $4+\frac{1}{2}-2\pi = \boxed{\frac{9}{2}-2\pi}$



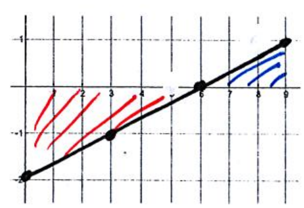
$\frac{1}{2}(1)(1)=\frac{1}{2}$

6-7: Evaluate the integral by interpreting it in terms of areas.

6. $\int_0^9 \left(\frac{1}{3}x-2\right)dx$

$-\frac{1}{2}(6)(2)+\frac{1}{2}(3)(1)$

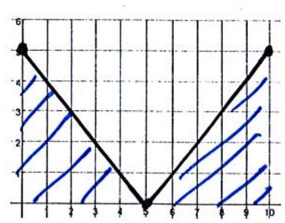
$-\frac{12}{2}+\frac{3}{2} = \boxed{\frac{-9}{2}}$



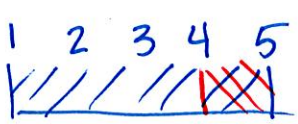
7. $\int_0^{10} |x-5|dx$

$\frac{1}{2}(5)(5)+\frac{1}{2}(5)(5)$

$\frac{25}{2}+\frac{25}{2} = \frac{50}{2} = \boxed{25}$



8. If $\int_1^5 f(x)dx = 12$ and $\int_4^5 f(x)dx = 3.6$, find $\int_1^4 f(x)dx =$



$\int_1^5 f(x)dx - \int_4^5 f(x)dx = \int_1^4 f(x)dx$

$12 - 3.6 = \int_1^4 f(x)dx$

$8.4 = \int_1^4 f(x)dx$

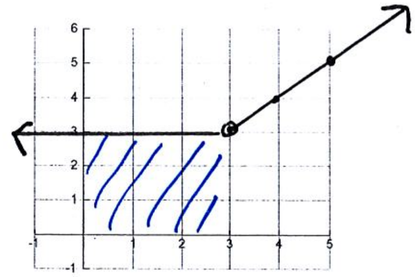
AP Calculus Name___________________________

Definite Integrals by Approximation Integration Day 3

9. If $\int_0^9 f(x)dx = 37$ and $\int_0^9 g(x)dx = 16$, find $\int_0^9 [2f(x) + 3g(x)]dx =$

$2(37) + 3(16) = \boxed{122}$

10. Find $\int_0^5 f(x)dx$ if $f(x) = \begin{cases} 3 & for\ \ x < 3 \\ x & for\ \ x \geq 3 \end{cases}$



$3(3) + \frac{2}{2}(3+5)$

$9 + 8 = \boxed{17}$

11. For the function f whose graph is shown, list the following quantities in increasing order, from smallest to largest, and explain your reasoning.

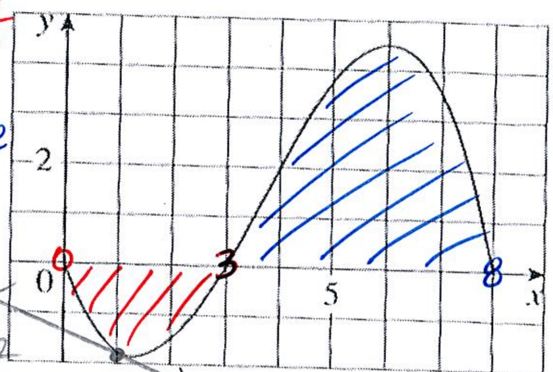
a.) $\int_0^8 f(x)dx$ = pos. and smaller than d

b.) $\int_0^3 f(x)dx$ = neg 3 or 4

c.) $\int_3^8 f(x)dx$ = all positive

d.) $\int_4^8 f(x)dx$ = all positive but less than c.

e.) $f'(1)$



$f'(1)$ $-1/3$ or $-1/2$

$\int_0^3 f(x)dx < f'(1) < \int_0^8 f(x)dx < \int_4^8 f(x)dx < \int_3^8 f(x)dx$

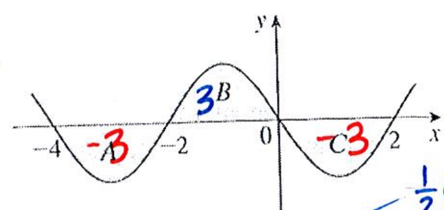
12. Each of the regions A, B, and C bounded by the graph of f and the x-axis has the area 3. Find the value of

$\int_{-4}^2 [f(x) + 2x + 5]dx = \int_{-4}^2 f(x) + 2\int_{-4}^2 x + \int_{-4}^2 5$

$-3 + 2(-6) + 30$

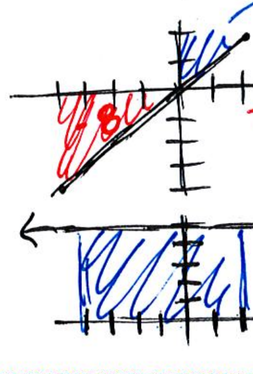
$-3 - 12 + 30$

$\boxed{15}$



$\frac{1}{2}(2)(2) = 2$

x



$-\frac{1}{2}(4)(4) = -8$

$f(x) = 5$

$6(5) = 30$