AP Calculus

Intervals of Inc/Dec & Concavity (2)

Name______________________ Pd.____

Day 3 Curve Sketching

1-4: Find a)-c) for each of the following:

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1. $f(x)=\frac{x}{x^2+1}$

a.)Find the intervals on which f is increasing or decreasing.

inc. $(-1,1)$ dec $(-\infty,-1)\cup(1,\infty)$

b.)Find the local maximum and minimum values of f.

min of 0 at $x=-1$

max of 0 at $x=1$

c.)Find the intervals of concavity and the inflection points.

concave up $(-\sqrt{3},0)\cup(\sqrt{3},\infty)$

concave down $(-\infty,-\sqrt{3})\cup(0,\sqrt{3})$

POI: $(-\sqrt{3},-\frac{\sqrt{3}}{4})$ $(0,0)$ $(\sqrt{3},\frac{\sqrt{3}}{4})$

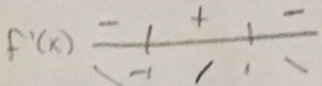
$f'(x)=\frac{(x^2+1)\cdot 1-x(2x)}{(x^2+1)^2}=\frac{x^2+1-2x^2}{(x^2+1)^2}=\boxed{\frac{-x^2+1}{(x^2+1)^2}}$

$f'(x)=0$ | $f'(x)$ DNE

$-x^2+1=0$ | $(x^2+1)^2=0$

$-x^2=-1$ | $x^2+1=0$

$x=\pm 1$ | $x^2=-1$ Nope



min: $f(-1)=0$

max: $f(1)=0$

$f''(x)=\frac{(x^2+1)^2(-2x)-(-x^2+1)(2(x^2+1)\cdot 2x}{(x^2+1)^4}=\frac{-2x(x^2+1)\cancel{^2}-(4x^3-4x)\cancel{(x^2+1)}}{(x^2+1)^{\cancel{4}\,3}}$

Factor out (x^2+1) and divide out

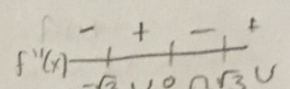
$=\frac{-2x^3-2x+4x^3-4x}{(x^2+1)^3}\boxed{\frac{2x^3-6x}{(x^2+1)^3}}$

$f''(x)=0$ | $f''(x)$ DNE

$2x^3-6x=0$ | $(x^2+1)^3=0$

$2x(x^2-3)=0$ | $x^2+1=0$

$x=0$ $x=\pm\sqrt{3}$ | $x^2=-1$ none



POI:

$f(-\sqrt{3})=-\frac{\sqrt{3}}{4}$ $f(\sqrt{3})=\frac{\sqrt{3}}{4}$

$f(0)=0$

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2. $f(x)=\cos^2 x-2\sin x,\quad 0\le x\le 2\pi$

a.)Find the intervals on which f is increasing or decreasing.

inc $(\frac{\pi}{2},\frac{3\pi}{2})$ dec $(0,\frac{\pi}{2})\cup(\frac{3\pi}{2},2\pi)$

b.)Find the local maximum and minimum values of f.

min of -2 at $x=\frac{\pi}{2}$ max of 2 at $x=\frac{3\pi}{2}$

c.)Find the intervals of concavity and the inflection points.

concave up $(\pi/6,5\pi/6)$

concave down $(0,\pi/6)\cup(5\pi/6,2\pi)$

POI: $(\pi/6,-\frac{1}{4})$ $(5\pi/6,-\frac{1}{4})$

$f'(x)=2\cos x(-\sin x)-2\cos x=-2\cos x\sin x-2\cos x=\boxed{-2\cos x(\sin x+1)}$

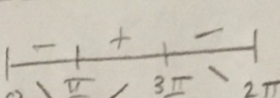
$f'(x)=0$ | $f'(x)$ DNE

$-2\cos x\sin x-2\cos x=0$ | none

$-2\cos x(\sin x+1)=0$

$\cos x=0$ $\sin x=-1$

$\frac{\pi}{2},\frac{3\pi}{2}$



min $f(\frac{\pi}{2})=-2$

max $f(\frac{3\pi}{2})=2$

$f''(x)=-2\cos x(\cos x)+(\sin x+1)(2\sin x)=-2\cos^2 x+2\sin^2 x+2\sin x$

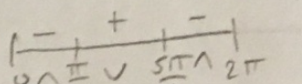
$f''(x)=0$ | $f''(x)$ DNE

$0=-2\cos^2 x+2\sin^2 x+2\sin x$ | none

(use calc)

$x=.5235$ $x=2.618$

$\frac{\pi}{6}$ $\frac{5\pi}{6}$



POI

$f(\frac{\pi}{6})=\frac{3}{4}-1=-\frac{1}{4}$

$f(\frac{5\pi}{6})=-\frac{1}{4}$

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3. $f(x)=e^{2x}+e^{-x}$

a.)Find the intervals on which f is increasing or decreasing.

Inc $(-.231,\infty)$ dec $(-\infty,-.231)$

b.)Find the local maximum and minimum values of f.

minimum at $x=-.231$

c.)Find the intervals of concavity and the inflection points.

concave up $(-\infty,\infty)$

POI: none

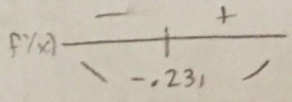
$f'(x)=2e^{2x}-e^{-x}$

$f'(x)=0$ | $f'(x)$ DNE

use calc | none

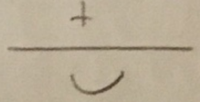
$2e^{2x}-e^{-x}=0$

$x=-.231$



min $f(-.231)$

$f''(x)=4e^{2x}+e^{-x}$



$f''(x)=0$ | $f''(x)$ DNE

use calc | none

$4e^{2x}+e^{-x}=0$

none

(always positive!)

∴ concave up

4. $f(x) = x^2 \ln x$ $(0,\infty)$

$f'(x) = x^2 \cdot \frac{1}{x} + \ln x \cdot 2x = x + 2x\ln x = x(1+2\ln x)$

a.)Find the intervals on which f is increasing or decreasing.

inc $(e^{-1/2}, \infty)$ dec $(0, e^{-1/2})$

$f'(x)=0$ | $f'(x)$ DNE

$x(1+2\ln x)=0$ | none

$x=0$ $1+2\ln x=0$

$\ln x = -\frac{1}{2}$

$x = e^{-1/2}$

$f'(x)$ − + ; 0 ∖ $e^{-1/2}$ /

b.)Find the local maximum and minimum values of f.

Minimum at $x=e^{-1/2}$

$f''(x) = x\left(\frac{2}{x}\right) + (1+2\ln x) = 2 + 1 + 2\ln x = 3 + 2\ln x$

c.)Find the intervals of concavity and the inflection points.

$f''(x)=0$ | $f''(x)$ DNE

$3+2\ln x=0$ | none

$\ln x = -\frac{3}{2}$

$x = e^{-3/2}$

$f''(x)$ − + ; ∩ $e^{-3/2}$ ∪

concave up $(e^{-3/2}, \infty)$

Concave down $(0, e^{-3/2})$

POI at $x = e^{-3/2}$

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5. $f(x) = \sin x + \cos x,$ $0 \le x \le 2\pi$

$f'(x) = \cos x - \sin x$

a.)Find the intervals on which f is increasing or decreasing.

$f'(x)=0$ | $f'(x)$ DNE

$\cos x - \sin x = 0$ | none

$\cos x = \sin x$

$\pi/4, \frac{5\pi}{4}$

$f'(x)$ + − + ; 0 / $\pi/4$ ∖ $5\pi/4$ / 2π

max at $x = \pi/4$

min at $x = 5\pi/4$

inc $(0,\frac{\pi}{4}) \cup (\frac{5\pi}{4}, 2\pi)$ dec $(\frac{\pi}{4}, \frac{5\pi}{4})$

b.)Find the local maximum and minimum values of f.

Min at $x=\frac{5\pi}{4}$ max at $x=\frac{\pi}{4}$

$f''(x) = -\sin x - \cos x$

$f''(x)=0$ | $f''(x)$ DNE

$-\sin x - \cos x = 0$ | none

$-\sin x = \cos x$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

$f''(x)$ − + − ; 0 ∩ $\frac{3\pi}{4}$ ∪ $\frac{7\pi}{4}$ ∩ 2π

c.)Find the intervals of concavity and the inflection points.

Concave down $(0,\frac{3\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi)$

Concave up $(\frac{3\pi}{4}, \frac{7\pi}{4})$

PoI at $x = \frac{3\pi}{4}, \frac{7\pi}{4}, 2\pi$

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6. $f(x) = x^2 - x - \ln x$ $(0,\infty)$

$f'(x) = 2x - 1 - \frac{1}{x}$

a.)Find the intervals on which f is increasing or decreasing.

$f'(x)=0$ | $f'(x)$ DNE

use calc | $x=0$ not on domain

$x=-\frac{1}{2}, x=1$

not in domain

$f'(x)$ − + ; 0 ∖ 1 /

inc $(1,\infty)$ dec $(0,1)$

b.)Find the local maximum and minimum values of f.

min at $x=1$

$f''(x) = 2 + \frac{1}{x^2}$

$f''(x)$ + ; 0 ∪

c.)Find the intervals of concavity and the inflection points.

$f''(x)=0$ | $f''(x)$ DNE

none always positive | $x=0$ not on domain

Concave up $(-\infty,\infty)$ PoI: none

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7. $f(x) = \dfrac{x^2}{x^2+3}$

$f'(x) = \dfrac{(x^2+3)(2x) - x^2(2x)}{(x^2+3)^2} = \dfrac{2x^3+6x-2x^3}{(x^2+3)^2} = \dfrac{6x}{(x^2+3)^2}$

a.)Find the intervals on which f is increasing or decreasing.

$f'(x)=0$ | $f'(x)$ DNE

$6x=0$ | $(x^2+3)^2=0$

$x=0$ | $x^2+3=0$

$x^2=-3$

none

$f'(x)$ − + ; ∖ 0 /

min: $f(0)=0$

inc $(0,\infty)$ dec $(-\infty,0)$

b.)Find the local maximum and minimum values of f.

minimum of 0 at $x=0$

$f''(x) = \dfrac{(x^2+3)^2 \cdot 6 - 6x(2(x^2+3)\cdot 2x)}{(x^2+3)^4} = \dfrac{\cancel{(x^2+3)}[6x^2+18-24x^2]}{(x^2+3)^{\cancel{4}3}}$

$= \dfrac{-18x^2+18}{(x^2+3)^3}$

c.)Find the intervals of concavity and the inflection points.

$f''(x)=0$ | $f''(x)$ DNE

$-18x^2+18=0$ | none

$x^2-1=0$

$x=\pm 1$

$f''(x)$ − + − ; ∖ −1 / 1 ∖

$f(-1)=\frac{1}{4}$ $f(1)=\frac{1}{4}$

concave up $(-1,1)$

Concave down $(-\infty,-1) \cup (1,\infty)$

PoI: $(-1,\frac{1}{4})$ $(1,\frac{1}{4})$

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Day 3 Curve Sketching

8-10: Find the local maximum and minimum values of f using both the First and Second Derivatives Tests. Which method do you prefer?

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8. $f(x)=1+3x^2-2x^3$

Critical values

$f'(x)=6x-6x^2$

$f'(x)=0$ | $f'(x)$ DNE

$6x-6x^2=0$ | none

$6x(1-x)=0$

$x=0 \quad x=1$

1st deriv test

− + −

\ 0 / 1 \

min at x=0

max at x=1

2nd deriv test

$f''(x)=6-12x$

$f''(0)=6>0$ ∴ minimum

$f''(1)=-6<0$ ∴ maximum

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9. $f(x)=\dfrac{x^2}{x-1}$ x=1 not in domain

Critical values

$f'(x)=\dfrac{(x-1)2x-x^2}{(x-1)^2}=\dfrac{2x^2-2x-x^2}{(x-1)^2}=\dfrac{x^2-2x}{(x-1)^2}$

$f'(x)=0$ | $f''(x)$ DNE

$x^2-2x=0$ | $(x-1)^2=0$

$x(x-2)=0$ | $x=1$ not in domain

$x=0 \quad x=2$

1st deriv test

+ − | − +

/ 0 \ 1 \ 2 /

VA

max at x=0

min at x=2

2nd deriv test

$f''(x)=\dfrac{(x-1)^2(2x-2)-(x^2-2x)\cdot 2(x-1)}{(x-1)^4}$

$=\dfrac{\cancel{(x-1)}[(x-1)(2x-2)-2(x^2-2x)]}{(x-1)^{\cancel{4}3}}$

$=\dfrac{\cancel{2x^2}\cancel{-2x}\cancel{-2x}+2\cancel{-2x^2}\cancel{+4x}}{(x-1)^3}$

$=\dfrac{2}{(x-1)^3}$

$f''(0)=-2<0$ ∴ max

$f''(2)=\frac{1}{2}>0$ ∴ min

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10. $f(x)=\sqrt{x}-\sqrt[4]{x}$ $(0,\infty)$

Critical values

$f'(x)=\frac{1}{2}x^{-1/2}-\frac{1}{4}x^{-3/4}=\dfrac{1}{2x^{1/2}}-\dfrac{1}{4x^{3/4}}$ Find LCD $=4x^{3/4}$

$=\left(\dfrac{2x^{1/4}}{2x^{1/4}}\right)\dfrac{1}{2x^{1/2}}-\dfrac{1}{4x^{3/4}}=\dfrac{2x^{1/4}-1}{4x^{3/4}}$

$f'(x)=0$ | $f'(x)$ DNE

$2x^{1/4}-1=0$ | $x=0$ not on domain

$x^{1/4}=\frac{1}{2} \quad x=\frac{1}{16}$

1st deriv

− +

0 \ $\frac{1}{16}$ /

min at $x=\frac{1}{16}$

2nd deriv

$f''(x)=-\frac{1}{4}x^{-3/2}+\frac{3}{16}x^{-7/4}$

$=\dfrac{-1}{4x^{3/2}}+\dfrac{3}{16x^{7/4}}$

$f''\left(\frac{1}{16}\right)>0$

∴ minimum

11.

a.) Find the critical numbers of $f(x)=x^4(x-1)^3$.

$f'(x)=x^4\cdot 3(x-1)^2+(x-1)^3\cdot 4x^3$ factor out GCF

$=x^3(x-1)^2[x\cdot 3+(x-1)\cdot 4]$

$=x^3(x-1)^2(3x+4x-4)=x^3(x-1)^2(7x-4)$

$f'(x)=0$

$x^3(x-1)^2(7x-4)=0$

$x^3=0 \quad (x-1)^2=0 \quad (7x-4)=0$

$\boxed{x=0, \quad x=1 \quad x=\frac{4}{7}}$

b.) What does the Second Derivatives test tell you about the behavior of f at these critical numbers?

$f'(x)=3x^4(x-1)^2+4x^3(x-1)^3$

$f''(x)=3x^4\cdot 2(x-1)+(x-1)^2\cdot 12x^3+4x^3\cdot 3(x-1)^2+(x-1)^3\cdot 12x^2$

$=6x^4(x-1)+12x^3(x-1)^2+12x^3(x-1)^2+12x^2(x-1)^3$

$f''(0)=0$ (no info)

$f''(1)=0$ (nada)

$f''\left(\frac{4}{7}\right)>0$ (minimum

c.) What does the First Derivatives Test tell you

+ − + +

/ 0 \ $\frac{4}{7}$ / 1 /

max at x=0

min at $x=\frac{4}{7}$