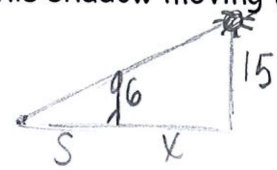
AP Calculus
Related Rates

Name________________________Pd._
Day 2 Application of Derivatives

1. A street light is mounted at the top of a 15-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole.



Know: $\frac{dx}{dt} = 5\frac{ft}{s}$

Find: $\frac{ds}{dt} =$ ___

When:
$x = 40$

Equation: $\frac{S}{6} = \frac{S+x}{15}$

Derivative: $15s = \cancel{6s} + 6x$
$-6s$

$\left[9s = 6x\right]\frac{d}{dt}$

$9\frac{ds}{dt} = 6\frac{dx}{dt}$

Substitution:
$9\frac{ds}{dt} = 6(5)$

$9\frac{ds}{dt} = 30$

$\frac{ds}{dt} = \frac{30}{9} = \frac{10}{3}$

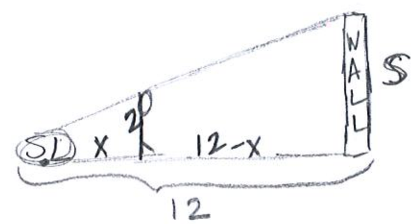
tip = $\frac{dx}{dt} + \frac{ds}{dt}$
shadow

$= 5 + \frac{10}{3}$ ✓

$= \frac{15}{3} + \frac{10}{3} = \boxed{\frac{25\,ft}{3\,s}}$

2. A spotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight toward the building at a speed of 1.6 m/s, how fast is the length of his shadow on the building decreasing when he is 4 m from the building?

$\frac{S}{12} = \frac{2}{8}$ $8s = 24$
$s = 3$



Know: $\frac{dx}{dt} = 1.6\frac{m}{s}$

Find: $\frac{ds}{dt} =$ ___

When:
$x = 8$
$s = 3$

Equation: $\frac{S}{12} = \frac{2}{x}$

Derivative: $\left[Sx = 24\right]\frac{d}{dt}$

$s\frac{dx}{dt} + x\frac{ds}{dt} = 0$

Substitution:
$3(1.6) + 8\frac{ds}{dt} = 0$

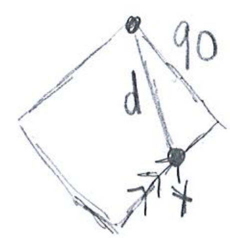
$8\frac{ds}{dt} = -4.8$

$\boxed{\frac{ds}{dt} = -.6\frac{m}{s}}$ ✓

3. A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 24 ft/s.

a.) At what rate is his distance from second base decreasing when he is halfway to first base?

b.) At what rate is his distance from third base increasing at the same moment?



Know: $\frac{dx}{dt} = 24\frac{ft}{s}$

Find: $\frac{dd}{dt}$

When:
$x = 45ft$

$d = \sqrt{45^2 + 90^2}$
$= 100.623$

Equation: $\left[x^2 + 90^2 = d^2\right]\frac{d}{dt}$

Substitution:

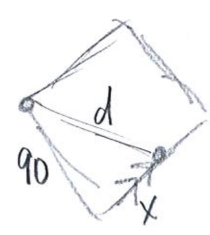
Derivative:

$\cancel{2}x\frac{dx}{dt} = \cancel{2}d\frac{dd}{dt}$

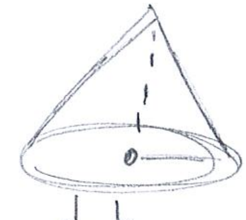
$45(24) = 100.623\frac{dd}{dt}$

a.) $\boxed{\frac{dd}{dt} = 10.733\frac{ft}{s}}$

b.) $\boxed{\text{same } \Delta \text{ so same answer}}$



4. Gravel is being dumped from a conveyor belt at a rate of 30 ft^3/min, and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?



$d = h$

$2r = h$

$r = \frac{h}{2}$

Know: $\frac{dV}{dt} = 30 \frac{ft^3}{min}$

Find: $\frac{dh}{dt} = $ ___

When: $h = 10 ft$

$V = \frac{1}{3}\pi R^2 h$

Equation: $V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$

Derivative: $V = \frac{1}{3}\pi \frac{h^2}{4} h$

$\frac{d}{dt}\left[V = \frac{1}{12}\pi h^3\right]$

$\frac{dV}{dt} = \frac{1}{12}\pi(3h^2)\frac{dh}{dt}$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

Substitution:

$30 = \frac{1}{4}\pi(10)^2 \frac{dh}{dt}$

$30 = 25\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{30}{25\pi} = \boxed{\frac{6}{5\pi} \frac{ft}{min}}$ ✓

5. A kite 100 ft above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string has been let out?

$x^2 + 100^2 = 200^2$

$x = 173.205$

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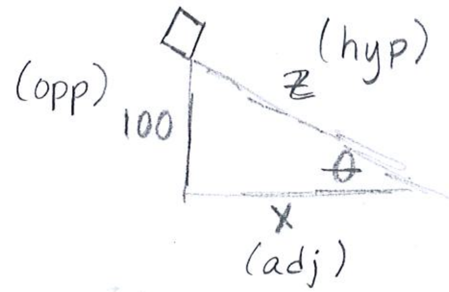


$\tan\theta = \frac{100}{173.205}$ $\theta = \tan^{-1}\left(\frac{100}{173.205}\right) = 30°$

Know: $\frac{dx}{dt} = 8 \frac{ft}{s}$

Find: $\frac{d\theta}{dt} = $ ___

When: $z = 200 ft$

$x = 173.205$

Equation: $\left[\tan\theta = \frac{100}{x}\right]\frac{d}{dt}$

Derivative: $\tan\theta = 100x^{-1}$

$\sec^2\theta \frac{d\theta}{dt} = -100x^{-2}\frac{dx}{dt}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{-100}{x^2}\frac{dx}{dt}$

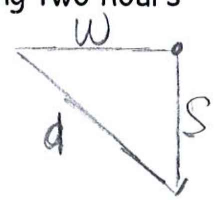
Substitution:

$\frac{d\theta}{dt} = \frac{\frac{-100}{x^2}\frac{dx}{dt}}{\sec^2\theta}$

$\frac{d\theta}{dt} = \frac{-100\cos^2\theta}{x^2}\frac{dx}{dt}$

$\frac{-100[\cos(30)]^2(8)}{(173.205)^2} = \boxed{-.02 \frac{Rad}{sec}}$ ✓

6. Two cars start moving from the same point. One travels south at 60 km/h and the other travels west at 25 km/h. At what rate is the distance between the cars increasing two hours later?



$\frac{dw}{dt} = 25 \frac{km}{h}$

$\frac{ds}{dt} = 60 \frac{km}{h}$

Know:

Find: $\frac{dd}{dt} = $ ___

When:

$t = 2$ $w = 50 km$

$s = 120 km$

$d = \sqrt{120^2 + 50^2}$

$d = 130$

Equation: $\left[w^2 + s^2 = d^2\right]\frac{d}{dt}$

Derivative:

$2w\frac{dw}{dt} + 2s\frac{ds}{dt} = 2d\frac{dd}{dt}$

$w\frac{dw}{dt} + s\frac{ds}{dt} = d\frac{dd}{dt}$

Substitution:

$50(25) + 120(60) = 130\frac{dd}{dt}$

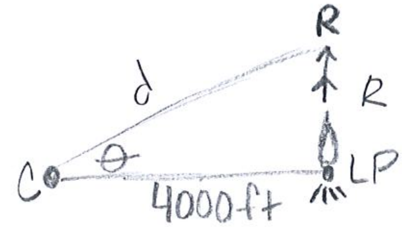
$8450 = 130\frac{dd}{dt}$

$\boxed{\frac{dd}{dt} = 65 km/h}$ ✓

7. A television camera is positioned 4000 ft from the base of a rocket launching pad. The angle of elevation of the camera has to change at the correct rate in order to keep the rocket in sight. Also, the mechanism for focusing the camera has to take into account the increasing distance from the camera to the rising rocket. Let's assume the rocket rises vertically and its speed is 600 ft/s when it has risen 3000 ft.

a.) How fast is the distance from the television camera to the rocket changing at that moment?

b.) If the television camera is always kept aimed at the rocket, how fast is the camera's angle of elevation changing at that same moment?



Know: $\frac{dR}{dt} = \frac{600ft}{s}$

Equation: $\frac{d}{dt}\left[R^2 + 4000^2 = d^2\right]$

Substitution:

Find: $\frac{dd}{dt}$

When: $R = 3000ft$

$d = \sqrt{4000^2 + 3000^2}$

$d = 5000$

Derivative:

$2R\frac{dR}{dt} = 2d\frac{dd}{dt}$

$R\frac{dR}{dt} = d\frac{dd}{dt}$

$3000(600) = 5000\frac{dd}{dt}$

a.) $\boxed{\frac{dd}{dt} = 360 ft/sec}$ ✓

$\tan\theta = \frac{3000}{4000}$

$\theta = \tan^{-1}(\frac{3}{4}) =$

$\theta = 36.87$

Find: $\frac{d\theta}{dt}$

Eqn: $\left[\tan\theta = \frac{R}{4000}\right]\frac{d}{dt}$

$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{4000}\frac{dR}{dt}$

$\frac{d\theta}{dt} = \frac{\cos^2\theta}{4000}\frac{dR}{dt} = \frac{[\cos 36.87]^2 600}{4000}$

b.) $\boxed{.096\frac{Rad}{s}}$ ✓

8. A plane flying with a constant speed of 300 km/h passes over a ground radar station at an altitude of 1 km and climbs at an angle of 30°. At what rate is the distance from the plane to the radar station increasing a minute later?

Know:

Equation:

Substitution:

Find:

Derivative:

When: