AP Calculus

Name________________ Pd.____

Formal Definition of Derivative

1-4: Use the formal definition of a derivative to find f'(x).

1. $f(x) = 2x^2 - 3$

$$f'(x) = \lim_{h\to 0} \frac{2(x+h)^2 - 3 - (2x^2-3)}{h}$$
$$= \lim_{h\to 0} \frac{2(x^2+2xh+h^2) - 3 - (2x^2-3)}{h}$$
$$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{-3} \cancel{-2x^2} \cancel{+3}}{h}$$
$$= \lim_{h\to 0} \frac{4xh+2h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(4x+2h)}{\cancel{h}} = \boxed{4x}$$

2. $f(x) = \frac{1}{x}$

$$f'(x) = \lim_{h\to 0} \frac{\frac{x}{x}\cdot\frac{1}{x+h} - \frac{1}{x}\cdot\frac{x+h}{x+h}}{h} = \lim_{h\to 0} \frac{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{\cancel{x} \cancel{-x} - h}{x(x+h)}}{h} = \lim_{h\to 0} \frac{\frac{-h}{x(x+h)}}{\frac{\cancel{h}}{1}} = \lim_{h\to 0} \frac{-\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}}$$
$$= \lim_{h\to 0} \frac{-1}{x(x+h)} = \boxed{\frac{-1}{x^2}}$$

3. $f(x) = \sqrt{5+x}$

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{5+x+h} - \sqrt{5+x}}{h} \left(\frac{\sqrt{5+x+h} + \sqrt{5+x}}{\sqrt{5+x+h} + \sqrt{5+x}}\right)$$
$$= \lim_{h\to 0} \frac{5+x+h-(5+x)}{h(\sqrt{5+x+h} + \sqrt{5+x})}$$
$$= \lim_{h\to 0} \frac{\cancel{5} + \cancel{x} + h \cancel{-5} \cancel{-x}}{h(\sqrt{5+x+h} + \sqrt{5+x})} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{5+x+h} + \sqrt{5+x})}$$
$$= \lim_{h\to 0} \frac{1}{\sqrt{5+x+h} + \sqrt{5+x}} = \boxed{\frac{1}{2\sqrt{5+x}}}$$

4. $f(x) = x^3 + 6x + 8$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^3 + 6(x+h) + 8 - (x^3+6x+8)}{h}$$
$$= \lim_{h\to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 + \cancel{6x} + 6h \cancel{+8} \cancel{-x^3} \cancel{-6x} \cancel{-8}}{h}$$
$$= \lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3 + 6h}{h}$$
$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2+3xh+h^2+6)}{\cancel{h}}$$
$$= \lim_{h\to 0} 3x^2 + 3xh + h^2 + 6 = \boxed{3x^2+6}$$

5. Find the equation of the tangent line to the graph of y=g(x) at x=5 if g(5)=-3 and g'(5)=4.

$$y + 3 = 4(x-5)$$

6. Find the equation of the tangent line to the graph of y=g(x) at x=-2 if g(-2)=7 and g'(-2)=-1.

$$y - 7 = -1(x+2)$$

7-8 Find an equation of the tangent line to the curve at the given point.

7. $y = 4x - 3x^2$, $(2, -4)$

$y' = \lim_{h\to 0} \frac{4(x+h) - 3(x+h)^2 - (4x - 3x^2)}{h}$

$= \lim_{h\to 0} \frac{4x + 4h - 3(x^2 + 2xh + h^2) - 4x + 3x^2}{h}$

$= \lim_{h\to 0} \frac{\cancel{4x} + 4h - \cancel{3x^2} - 6xh - 3h^2 - \cancel{4x} + \cancel{3x^2}}{h}$

$= \lim_{h\to 0} \frac{4h - 6xh - 3h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(4 - 6x - 3h)}{\cancel{h}}$

$= \lim_{h\to 0} 4 - 6x - 3h = 4 - 6x \Big|_{x=2} = -8$

$\boxed{y + 4 = -8(x-2)}$

8. $y = \sqrt{x}$, $(4, 2)$

$y' = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right)$

$y' = \lim_{h\to 0} \frac{x + h - \cancel{x}}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$

$= \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}} \Big|_{x=4} = \frac{1}{4}$

$\boxed{y - 2 = \frac{1}{4}(x-4)}$

9. If an equation of the tangent line to the curve y=f(x) at the point where a=2 is y=4x-5, find f(2) and f'(2).

$f(2) = 4(2) - 5 = 3$

$f'(2) = 4$

10. If the tangent line to y=f(x) at (4,3) passes through the point (0,2), find f(4) and f'(4).

$f(4) = 3$

$f'(4) = \frac{2-3}{0-4} = \frac{-1}{-4} = \frac{1}{4}$

11-19 Each limit represents some derivative of some function f. State such an f and what derivative is asked for in each case.

11. $\lim_{h\to 0} \frac{(1+h)^{10} - 1}{h}$ $f(x) = x^{10}$ want $f'(1)$	12. $\lim_{h\to 0} \frac{\ln(x+h+9) - \ln(x+9)}{h}$ $f(x) = \ln(x+9)$ want $f'(x)$	13. $\lim_{x\to \frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}}$ $f(x) = \tan x$ want $f'(\frac{\pi}{4})$
14. $\lim_{h\to 0} \frac{((x+h)^3 - 5(x+h) - 10) - (x^3 - 5x + 10)}{h}$ $f(x) = x^3 - 5x - 10$ want $f'(x)$	15. $\lim_{x\to 5} \frac{2^x - 32}{x - 5}$ $f(x) = 2^x$ want $f'(5)$	16. $\lim_{h\to 0} \frac{\sqrt[4]{16+h} - 2}{h}$ $f(x) = \sqrt[4]{x}$ want $f'(16)$
17. $\lim_{t\to 1} \frac{t^4 + t - 2}{t - 1}$ $f(x) = t^4 + t$ want $f'(1)$	18. $\lim_{h\to 0} \frac{\cos(\pi + h) + 1}{h}$ $f(x) = \cos x$ want $f'(\pi)$	19. $\lim_{h\to 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h}$ $f(x) = \frac{1}{x+2}$ want $f'(x)$