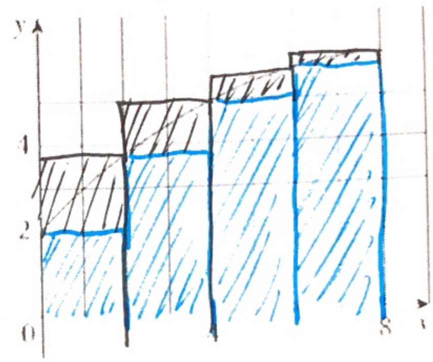
AP Calculus

Approximating Integrals by Estimating Area

Name________________________

Integration Day 1

1. A.) By reading values from the given graph of f, use four rectangles to find a under estimate (L_4) and an upper estimate (R_4) for the area under the graph of f from x=0 to x=8. In each case sketch the rectangles you use.



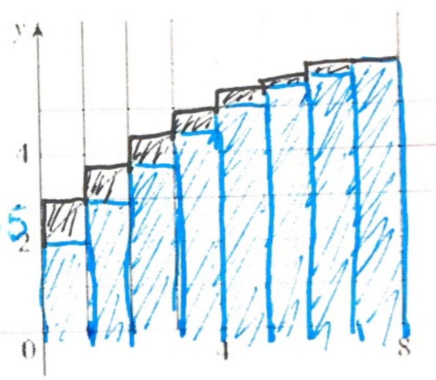
under estimate (L_4)=

$2(2)+2(3.8)+2(5)+2(5.8)=33.2$

upper estimate (R_4)=

$2(3.8)+2(5)+2(5.8)+2(6)=41.2$

B.) Find new estimates using eight rectangles



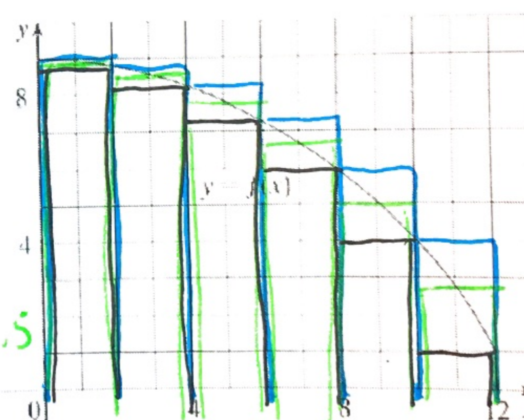
under estimate (L_8)=

$1(2+3+3.8+4.5+5+5.5+5.8+5.9)=35.5$

upper estimate (R_8)=

$1(3+3.8+4.5+5+5.5+5.8+5.9+6)=39.5$

2. A.) Use six rectangles to find estimates of each type for the area under the graph of f from x=0 to x=12.



(i) $L_6=2(9+8.75+8.25+7.25+6+4)=86.5$

(ii) $R_6=2(8.75+8.25+7.25+6+4+1)=70.5$

(iii) $M_6=2(8.9+8.5+7.8+6.75+5+2.8)=79.5$

B.) Is L_6 an underestimate or overestimate of the area? Overestimate

C.) Is R_6 an underestimate or overestimate of the area? underestimate

D.) Which of the numbers L_6, R_6, or M_6 gives the best estimate? Explain?

M_6 - contains area above and below the function

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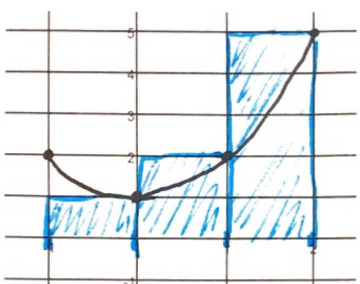
3. A.) Estimate the area under the graph of $f(x)=1+x^2$ from x= -1 to x=2 using three rectangles and right endpoints. Then improve your estimate by using six rectangles. Sketch the curve and the approximating rectangles.

B.) Repeat part (a) using left endpoints.

C.) Repeat part (a) using midpoints.

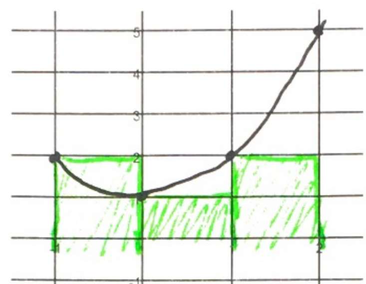
D.) From your sketches in parts (a)-(c), which appears to be the best estimate? M_6

A.)



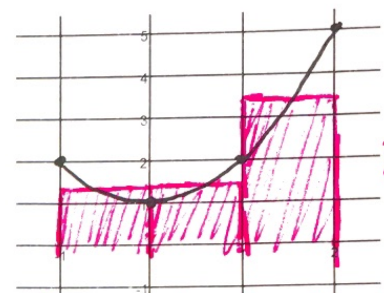
$1(1+2+5)=8$

B.)



$1(2+1+2)=5$

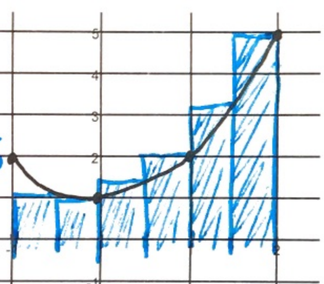
C.)



5.75

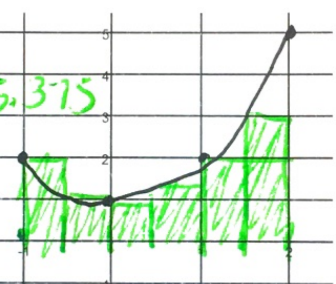
$1(f(-.5)+f(.5)+f(1.5))$

6.875



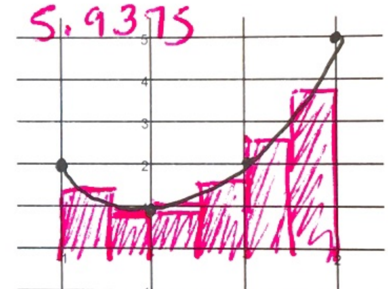
$\frac{1}{2}(f(-.5)+f(0)+f(.5)+f(1)+f(1.5)+f(2))$

= 5.375



$\frac{1}{2}(f(-1)+f(-.5)+f(0)+f(.5)+f(1)+f(1.5))$

5.9375



$\frac{1}{2}(f(-.75)+f(-.25)+f(.25)+f(.75)+f(1.25)+f(1.75))$

4. The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find the estimates for the distance that she traveled during these three seconds by finding L_6 and R_6.

t(s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v (ft/s)	0	6.2	10.8	14.9	18.1	19.4	20.2

$L_6 = .5(0+6.2+10.8+14.9+18.1+19.4)$
$= 34.7$ ft

$R_6 = .5(6.2+10.8+14.9+18.1+19.4+20.2)$
$= 44.8$ ft.

5. Oil leaked from a tank at a rate of r(t) liters per hour. The rate decreased as time passed and values of the rate at two-hour time intervals are shown in the table. Find lower and upper estimates for the total amount of oil that leaked out.

t(h)	0	2	4	6	8	10
r(t) (L/h)	8.7	7.6	6.8	6.2	5.7	5.3

$L_5 = 2(8.7+7.6+6.8+6.2+5.7) = 70$ L

$R_5 = 2(7.6+6.8+6.2+5.7+5.3) = 64.4$ L

6. The velocity graph of a braking car is shown. Use it to estimate the distance traveled by the car while the brakes are applied. $1(56+40+25+17+10+5)=153$ ft