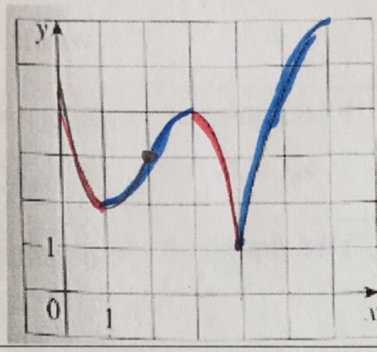
AP Calculus | Name Key Pd.___

Intervals of Inc/Dec & Concavity (1) | Day 2 Curve Sketching

1-2: Use the given graph of f to find the following

1.



a.) The open intervals of which f is increasing. $(1,3)\cup(4,6)$

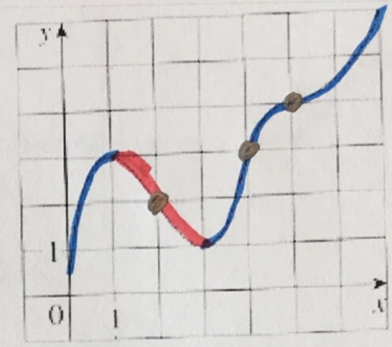
b.) The open intervals of which f is decreasing. $(0,1)\cup(3,4)$

c.) The open intervals of which f is concave upward. $(0,2)$

d.) The open intervals of which f is concave downward. $(2,4)\cup(4,6)$

e.) The coordinates of the points of inflection. $(2,3)$

2.



a.) The open intervals of which f is increasing. $(0,1)\cup(3,7)$

b.) The open intervals of which f is decreasing. $(1,3)$

c.) The open intervals of which f is concave upward. $(2,4)\cup(5,7)$

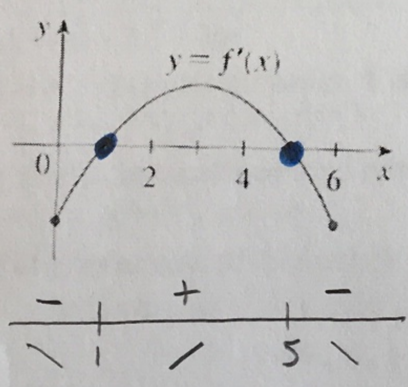
d.) The open intervals of which f is concave downward. $(0,2)\cup(4,5)$

e.) The coordinates of the points of inflection. $(2,2)$, $(4,3)$ + $(5,4)$

3-4: The graph of the derivative f' of a function f is shown.

↑ graph of derivative

3.



critical values $f'(x)=0$

a.) On what intervals is f increasing or decreasing?

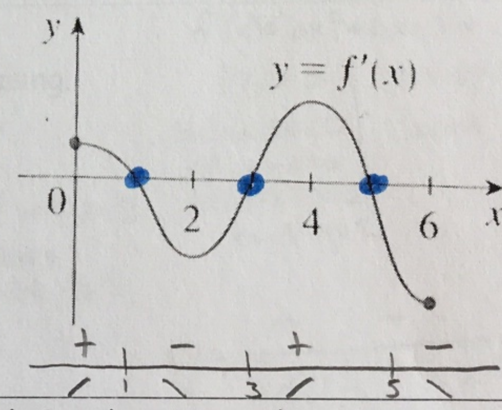
inc: $(1,5)$ dec: $(0,1)\cup(5,6)$

b.) At what values of x does f have a local maximum or minimum?

minimum at $x=1$

maximum at $x=5$

4.



critical values $f'(x)=0$

a.) On what intervals is f increasing or decreasing?

inc $(0,1)\cup(3,5)$ dec $(1,3)\cup(5,6)$

b.) At what values of x does f have a local maximum or minimum?

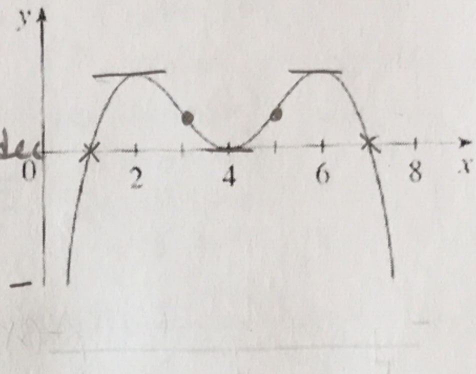
minimum at $x=3$

maximum at $x=1$ and $x=5$

5. In each part state the x-coordinate of the inflection points of f. Give a reason for your answers.

a.) The curve is the graph of f.

$x = 3$ and $x = 5$



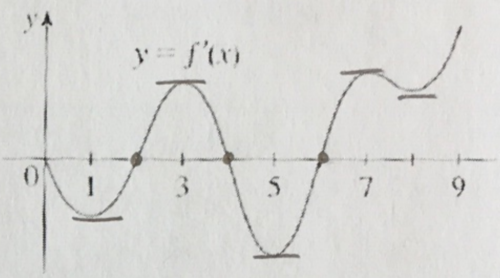
b.) The curve is the graph of f'. look at where f' changes from inc to dec or dec to inc

$x = 2,\ x = 4,\ x = 6$

c.) The curve is the graph of f''. look where f'' is changing from + to − or − to +

$x = 1,\ x = 7$

6. The graph of the first derivative f' of a function f is shown.



a.) On what intervals is f increasing or decreasing? Explain.

inc $(2,4) \cup (6,9)$ because $f'(x) > 0$

dec $(0,2) \cup (4,6)$ because $f'(x) < 0$

b.) At what values of x does f have a local maximum or minimum? Explain.

maximum: $x = 4$ (f'(x) changes from + to −)

minimum: $x = 2,\ x = 6$ (f'(x) changes from − to +)

c.) On what intervals is f concave upward or concave downward? Explain.

concave down: $(0,1) \cup (3,5) \cup (7,8)$ slope of $f'(x) < 0$

concave up: $(1,3) \cup (5,7) \cup (8,9)$ slope of $f'(x) > 0$

$f'(x)$: − | 2 | + | 4 | − | 6 | +

d.) What are the x-coordinates of the inflection points of f? Why?

$x = 1, 3, 5, 7, 8$ $f'(x)$ changes direction ∴ $f''(x)$ changes sign

$f''(x)$: − | 1 | + | 3 | − | 5 | + | 7 | − | 8 | +

7-10: Find a)-c) for each of the following:

7. $f(x) = 2x^3 + 3x^2 - 36x$

$f'(x) = 6x^2 + 6x - 36$

$f'(x) = 0$	$f'(x)$ DNE
$6x^2 + 6x - 36 = 0$	none
$x^2 + x - 6 = 0$	
$(x+3)(x-2) = 0$	
$x = -3 \quad x = 2$	

a.)Find the intervals on which f is increasing or decreasing.

inc: $(-\infty, -3) \cup (2, \infty)$ dec: $(-3, 2)$

b.)Find the local maximum and minimum values of f.

maximum of 81 at $x = -3$ minimum of −44 at $x = 2$

c.)Find the intervals of concavity and the inflection points.

concave up: $(-\frac{1}{2}, \infty)$ concave down $(-\infty, -\frac{1}{2})$

PoI: $(-\frac{1}{2}, \frac{37}{2})$

$f'(x)$: + | −3 | − | 2 | +

max $f(-3) = 81$

min $f(2) = -44$

$f''(x) = 12x + 6$

$f''(x) = 0$	$f''(x)$ DNE
$12x + 6 = 0$	none
$12x = -6$	
$x = -\frac{1}{2}$	

$f''(x)$: − | $-\frac{1}{2}$ | +

$f(-\frac{1}{2}) = \frac{37}{2}$

AP Calculus

Intervals of Inc/Dec & Concavity (1)

Name________________________ Pd.____

Day 2 Curve Sketching

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8. $f(x) = 4x^3 + 3x^2 - 6x + 1$

a.)Find the intervals on which f is increasing or decreasing.

increasing $(-\infty, -1) \cup (\frac{1}{2}, \infty)$ Decreasing $(-1, \frac{1}{2})$

b.)Find the local maximum and minimum values of f.

maximum of 6 at $x = -1$, minimum of $-\frac{3}{4}$ at $x = \frac{1}{2}$

c.)Find the intervals of concavity and the inflection points.

concave up $(-\frac{1}{4}, \infty)$ concave down $(-\infty, -\frac{1}{4})$

POI $(-\frac{1}{4}, \frac{21}{8})$

$f''(x) = 24x + 6$

$f''(x) = 0$ | $f''(x)$ DNE

$24x + 6 = 0$ | none

$x = \frac{-6}{24} = -\frac{1}{4}$

$f''(x)$: $-$ | $+$; $\cap$ $-\frac{1}{4}$ $\cup$

$f(-\frac{1}{4}) = \frac{21}{8}$

$f'(x) = 12x^2 + 6x - 6$

$f'(x) = 0$ | $f'(x)$ DNE

$12x^2 + 6x - 6 = 0$ | none

$2x^2 + x - 1 = 0$

$(2x - 1)(x + 1) = 0$

$x = \frac{1}{2}$ $x = -1$

$f'(x)$: $+$ | $-$ | $+$; -1, $\frac{1}{2}$

max $f(-1) = 6$

min $f(\frac{1}{2}) = -\frac{3}{4}$

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9. $f(x) = x^4 - 2x^2 + 3$

a.)Find the intervals on which f is increasing or decreasing.

increasing $(-1, 0) \cup (1, \infty)$ decreasing $(-\infty, -1) \cup (0, 1)$

b.)Find the local maximum and minimum values of f.

maximum of 3 at $x = 0$, minimum of 2 at $x = 1, -1$

c.)Find the intervals of concavity and the inflection points.

concave up $(-\infty, -\sqrt{\frac{1}{3}}) \cup (\sqrt{\frac{1}{3}}, \infty)$ concave down $(-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}})$

POI: $(-\sqrt{\frac{1}{3}}, \frac{22}{9})$ $(\sqrt{\frac{1}{3}}, \frac{22}{9})$

$f''(x) = 12x^2 - 4$

$f''(x) = 0$ | $f''(x)$ DNE

$12x^2 - 4 = 0$ | none

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}}$

$f''(x)$: $+$ | $-$ | $+$; $\cup$ $-\sqrt{\frac{1}{3}}$ $\cap$ $\sqrt{\frac{1}{3}}$ $\cup$

$f(-\sqrt{\frac{1}{3}}) = \frac{22}{9}$

$f(\sqrt{\frac{1}{3}}) = \frac{22}{9}$

$f'(x) = 4x^3 - 4x$

$f'(x) = 0$ | $f'(x)$ DNE

$4x^3 - 4x = 0$ | None

$4x(x^2 - 1) = 0$

$x = 0$ $x = \pm 1$

$f'(x)$: $-$ | $+$ | $-$ | $+$; -1, 0, 1

min $f(-1) = 2$

$f(1) = 2$

max $f(0) = 3$

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10. $f(x) = x^3 - 12x^2 + 36x$

a.)Find the intervals on which f is increasing or decreasing.

increasing $(-\infty, 2) \cup (6, \infty)$ decreasing $(2, 6)$

b.)Find the local maximum and minimum values of f.

maximum of 32 at $x = 2$, minimum of 0 at $x = 6$

c.)Find the intervals of concavity and the inflection points.

concave up $(4, \infty)$ concave down $(-\infty, 4)$

PoI: $(4, 16)$

$f''(x) = 6x - 24$

$f''(x) = 0$ | $f''(x)$ DNE

$6x - 24 = 0$ | none

$x = 4$

$f''(x)$: $-$ | $+$; $\cap$ 4 $\cup$

$f(4) = 16$

$f'(x) = 3x^2 - 24x + 36$

$f'(x) = 0$ | $f'(x)$ DNE

$3x^2 - 24x + 36 = 0$ | None

$x^2 - 8x + 12 = 0$

$(x - 6)(x - 2) = 0$

$x = 6, x = 2$

$f'(x)$: $+$ | $-$ | $+$; 2, 6

max: $f(2) = 32$

min: $f(6) = 0$