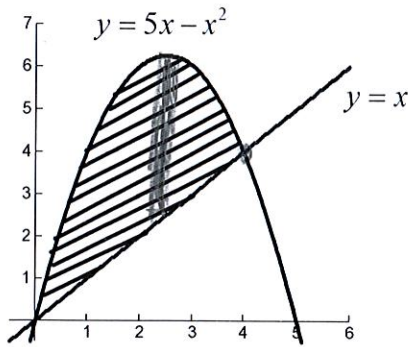


1-2: Find the area of the shaded region.

1.



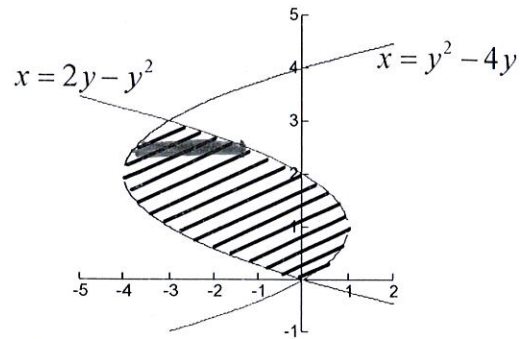
$$\int_0^4 (5x - x^2) - x \, dx$$

$$\int_0^4 -x^2 + 4x \, dx$$

$$-\frac{x^3}{3} + 2x^2 \Big|_0^4 = \frac{-64}{3} + 32 + 0 - 0$$

$$-\frac{64}{3} + \frac{96}{3} = \frac{32}{3} \checkmark$$

2.



$$\int_0^3 (2y - y^2) - (y^2 - 4y) \, dy$$

$$\int_0^3 -2y^2 + 6y \, dy$$

$$-\frac{2}{3}y^3 + 3y^2 \Big|_0^3 = -\frac{2}{3}(9) + 3(9) + 0 - 0$$

$$-6 + 27 = \frac{21}{1} \checkmark$$

5-7: Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

3.  $y = e^x$ ,  $y = x^2 - 1$ ,  $x = -1$ ,  $x = 1$

$$\int_{-1}^1 e^x - (x^2 - 1) \, dx$$

$$\int_{-1}^1 e^x - x^2 + 1 \, dx$$

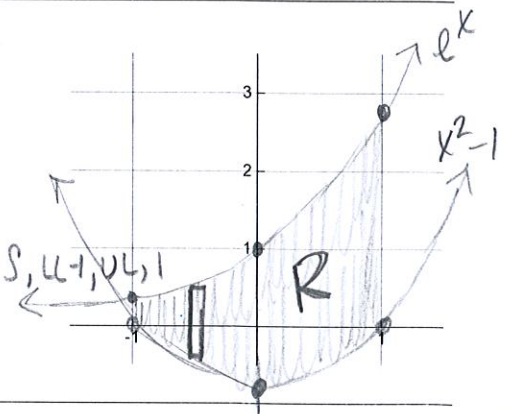
$$e^x - \frac{x^3}{3} + x \Big|_{-1}^1$$

$$e^1 - \frac{1}{3} + 1 - (e^{-1} - \frac{(-1)^3}{3} + (-1))$$

$$e - \frac{1}{3} + 1 - e^{-1} + \frac{1}{3} - (-1)$$

$$e - e^{-1} + \frac{4}{3} \approx 3.683$$

Calculator (83 or 84)  
 $Y_1 = e^x - x^2 + 1$ , graph, calc, 7: S, UL=1, UL=1  
 Calculator (89)  
 $\int(e^x - x^2 + 1, x, -1, 1)$



4.  $y = x^2 - 2x$ ,  $y = x + 4$

$$\int_{-1}^4 (x + 4) - (x^2 - 2x) \, dx$$

$$\int_{-1}^4 x + 4 - x^2 + 2x \, dx$$

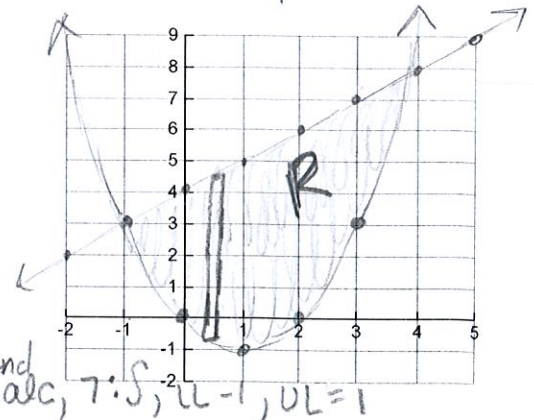
$$\int_{-1}^4 -x^2 + 3x + 4 \, dx$$

$$-\frac{x^3}{3} + \frac{3}{2}x^2 + 4x \Big|_{-1}^4$$

$$-\frac{64}{3} + \frac{48}{2} + 16 + (-\frac{(-1)^3}{3} - \frac{3}{2}(-1)^2 - 4(-1))$$

$$-\frac{64}{3} + 24 + 16 - \frac{1}{3} + \frac{3}{2} + 4 = -\frac{130}{6} + \frac{264}{6} - \frac{9}{6} = \frac{125}{6} \checkmark$$

Calc (83 or 84)  
 $Y_1 = -x^2 + 3x + 4$ , graph, calc, 7: S, UL=-1, UL=1  
 Calc (89)  $\int(-x^2 + 3x + 4, x, -1, 4)$



5.  $y = \frac{1}{x}, y = \frac{1}{x^2}, x = 2$

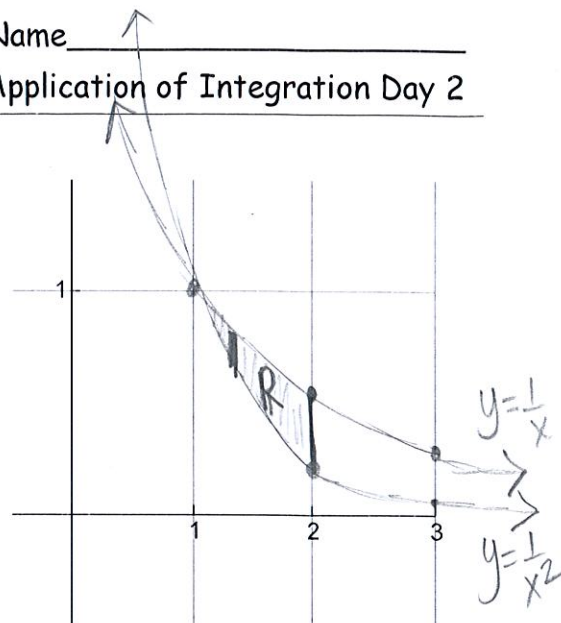
$$\int_1^2 \frac{1}{x} - x^{-2} dx$$

$$\ln|x| - \frac{x^{-1}}{-1} \Big|_1^2$$

$$\ln|x| + \frac{1}{x} \Big|_1^2$$

$$\ln(2) + \frac{1}{2} - \ln(1) - \frac{1}{1}$$

$$\boxed{\ln(2) - \frac{1}{2}} \checkmark$$



6.  $x = 1 - y^2, x = y^2 - 1$

$$\int_{-1}^1 (1 - y^2) - (y^2 - 1) dy$$

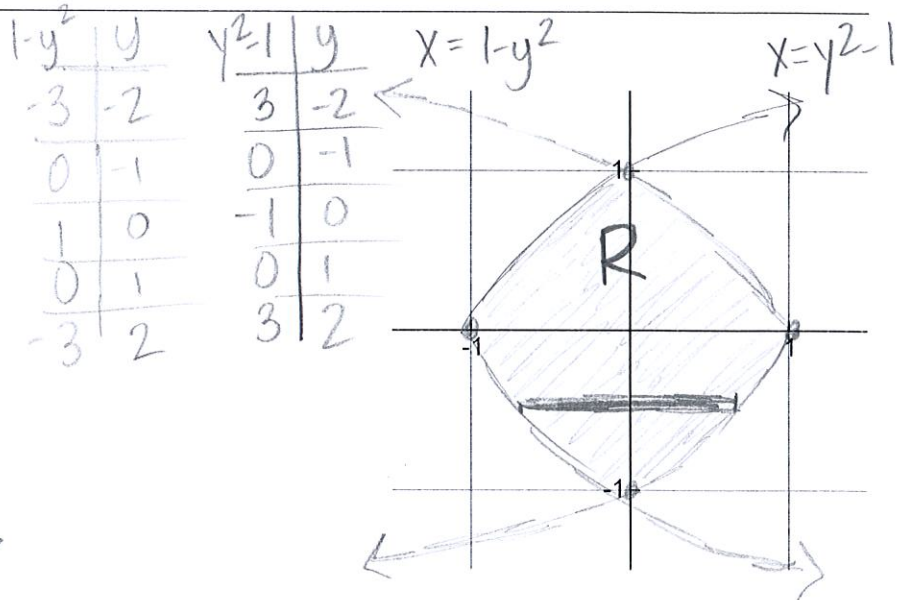
$$\int_{-1}^1 2 - 2y^2 dy$$

$$2y - \frac{2}{3}y^3 \Big|_{-1}^1$$

$$2 - \frac{2}{3} - 2(-1) + \frac{2}{3}(-1)$$

$$2 - \frac{2}{3} + 2 - \frac{2}{3}$$

$$4 - \frac{4}{3} = \frac{12}{3} - \frac{4}{3} = \boxed{\frac{8}{3}} \checkmark$$



7.  $4x + y^2 = 12, x = y$

$$\int_{-6}^2 (3 - \frac{1}{4}y^2) - y dy$$

$$3y - \frac{1}{4} \frac{y^3}{3} - \frac{y^2}{2} \Big|_{-6}^2$$

$$3y - \frac{1}{12}y^3 - \frac{1}{2}y^2 \Big|_{-6}^2$$

$$6 - \frac{1}{12}(8) - \frac{1}{2}(4) - 3(-6) + \frac{1}{12}(-6)^3 + \frac{1}{2}(36)$$

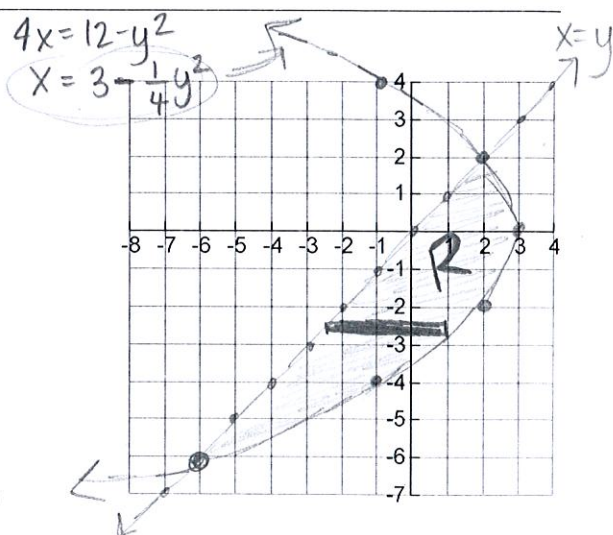
$$6 - \frac{2}{3} - 2 + 18 - 18 + 18$$

$$-\frac{2}{3} + 22 = -\frac{2}{3} + \frac{66}{3} = \boxed{\frac{64}{3}} \checkmark$$

$$y^2 = 12 - 4x$$

$$y = \pm \sqrt{12 - 4x}$$

x	y
3	0
2	±2
1	±√8
0	±√12
-1	±4
-2	±√20
-3	±√24
-4	
-5	
-6	±6



8-12: Sketch the region enclosed by the given curves and find its area.

8.  $y = 12 - x^2$ ,  $y = x^2 - 6$

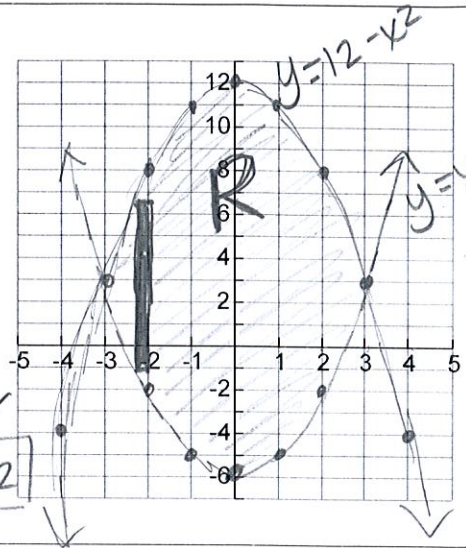
$$\int_{-3}^3 (12 - x^2) - (x^2 - 6) dx$$

$$\int_{-3}^3 18 - 2x^2 dx$$

$$18x - \frac{2}{3}x^3 \Big|_{-3}^3$$

$$54 - \frac{54}{3} - 18(-3) + \frac{2}{3}(-27)$$

$$54 - \frac{54}{3} + 54 - \frac{54}{3} = 108 - \frac{108}{3} = \frac{324}{3} - \frac{108}{3} = \frac{216}{3} = 72$$



9.  $x = 2y^2$ ,  $x = 4 + y^2$

$$\int_{-2}^2 (4 + y^2) - (2y^2) dy$$

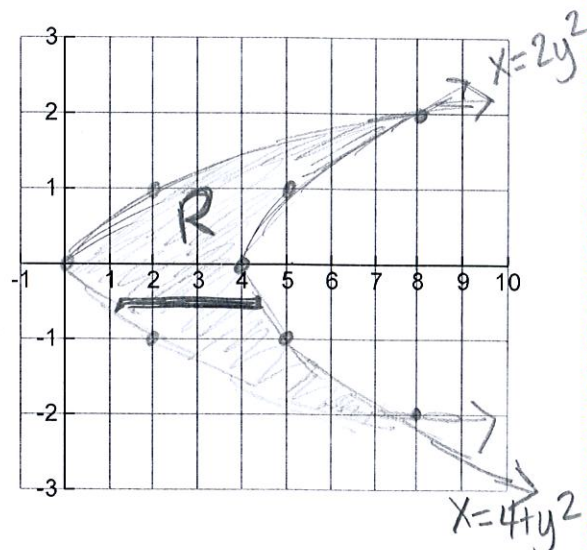
$$\int_{-2}^2 4 - y^2 dy$$

$$4y - \frac{y^3}{3} \Big|_{-2}^2$$

$$8 - \frac{8}{3} - 4(-2) + \frac{(-8)}{3}$$

$$16 - \frac{16}{3} + \frac{48}{3} - \frac{8}{3} = \frac{32}{3}$$

$x = 2y^2$	$y$
18	$\pm 3$
8	$\pm 2$
2	$\pm 1$
0	0
$x = 4 + y^2$	
13	$\pm 3$
8	$\pm 2$
5	$\pm 1$
4	0



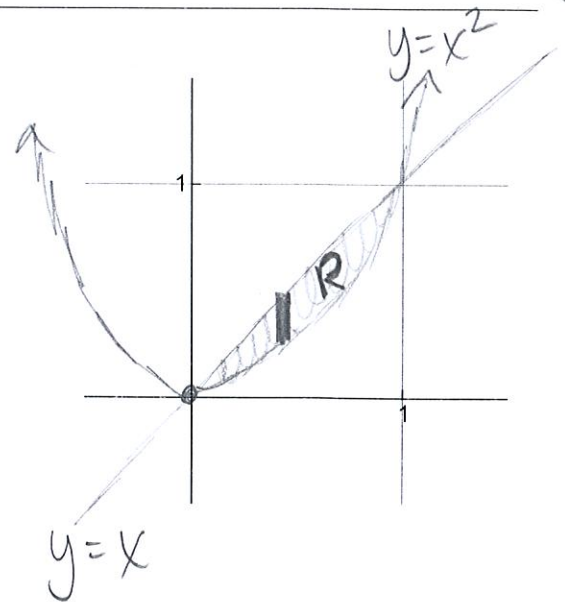
10.  $y = x^2$ ,  $y = x$

$$\int_0^1 x - x^2 dx$$

$$\frac{x^2}{2} - \frac{x^3}{3} \Big|_0^1$$

$$\frac{1}{2} - \frac{1}{3} - 0 + 0$$

$$\frac{3}{6} - \frac{2}{6} = \frac{1}{6}$$

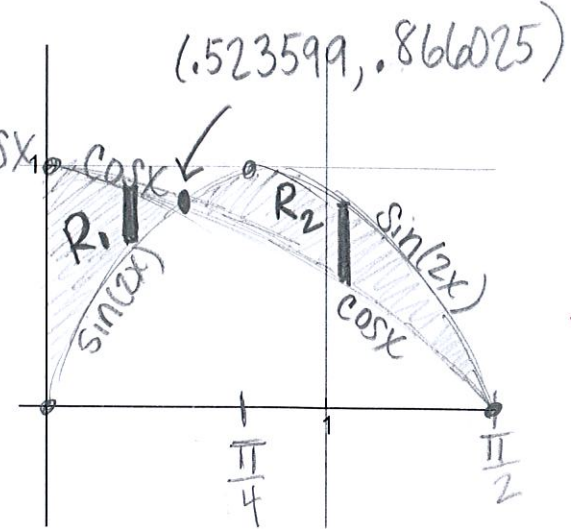


11.  $y = \cos x, y = \sin 2x, x = 0, x = \frac{\pi}{2}$

$$\int_0^{.523599} (\cos x) - \sin(2x) dx + \int_{.523599}^{\pi/2} \sin(2x) - \cos x dx$$

.25 + .25

$\boxed{\frac{1}{2}}$  ✓



12.  $y = \frac{1}{x}, y = x, y = \frac{1}{4}x, x > 0$

$$\int_0^1 x - \frac{1}{4}x dx + \int_1^2 \frac{1}{x} - \frac{1}{4}x dx$$

$\frac{3}{8} + \ln(2) - \frac{3}{8}$

$\boxed{\ln(2)}$  ✓

