AP Calculus

Related Rates

Name______________________Pd.__

Day 1 Application of Derivatives

1. Each side of a square is increasing at a rate of 6 cm/s. At what rate is the area of the square increasing when the area of the square is 16 cm?

$A = S^2$

$\sqrt{16} = \sqrt{S^2}$

$S = 4$

Know: $\frac{ds}{dt} = 6\,\frac{cm}{s}$

Find: $\frac{dA}{dt} = \_\_\_$

When: $A = 16cm^2$ then $s = 4cm$

Equation: $\frac{d}{dt}\left[A = s^2\right]$

Derivative: $\frac{dA}{dt} = 2s\frac{ds}{dt}$

Substitution: $\frac{dA}{dt} = 2(4cm)(6\,\frac{cm}{s})$

$\boxed{\frac{dA}{dt} = 48\,\frac{cm^2}{s}}$ ✓

2. The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 3 cm/s. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing?

Know: $\frac{dl}{dt} = 8\,\frac{cm}{s}$ $\frac{dw}{dt} = 3\,\frac{cm}{s}$

Find: $\frac{dA}{dt} = \_\_\_$

When: $l = 20cm$, $w = 10cm$

Equation: $\left[A = l \cdot w\right]\frac{d}{dt}$

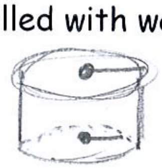
Derivative: $\frac{dA}{dt} = l\frac{dw}{dt} + w\frac{dl}{dt}$

Substitution: $\frac{dA}{dt} = (20cm)(3\,\frac{cm}{s}) + (10cm)(8\,\frac{cm}{s})$

$\frac{dA}{dt} = 60\,\frac{cm^2}{s} + 80\,\frac{cm^2}{s}$

$\boxed{\frac{dA}{dt} = 140\,\frac{cm^2}{s}}$ ✓

3. A cylindrical tank with radius 5 m is being filled with water at a rate of 3 m^3/min. How fast is the height of the water increasing?



Radius = 5m & never changes

Know: $\frac{dV}{dt} = 3\,\frac{m^3}{min}$

Find: $\frac{dh}{dt} = \_\_\_$

When: ?

$V = \pi R^2 h$

Equation: $V = \pi(5m)^2 h$

Derivative: $\left[V = 25\pi h\, m^2\right]\frac{d}{dt}$

$\frac{dV}{dt} = 25\pi\frac{dh}{dt}\,m^2$

Substitution: $\left(\frac{1}{25\pi m^2}\right)\frac{3m^{\cancel{3}}}{min} = \cancel{25\pi m^2}\frac{dh}{dt}\left(\frac{1}{\cancel{25\pi m^2}}\right)$

$\boxed{\frac{dh}{dt} = \frac{3}{25\pi}\,\frac{m}{min}}$ ✓

4. The radius of a sphere is increasing at a rate of 4 mm/s. How fast is the volume increasing when the diameter is 80 mm?

Know: $\frac{dR}{dt} = 4\,\frac{mm}{s}$

Find: $\frac{dV}{dt} = \_\_\_$

When: $d = 80mm$ then $R = 40mm$

Equation: $\left[V = \frac{4}{3}\pi R^3\right]\frac{d}{dt}$

Derivative: $\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi(\cancel{3}R^2)\frac{dR}{dt}$

$\frac{dV}{dt} = 4\pi R^2\frac{dR}{dt}$

Substitution: $\frac{dV}{dt} = 4\pi(40mm)^2(4\,\frac{mm}{s})$

$\frac{dV}{dt} = 4\pi(1600\,mm^2)(4\,\frac{mm}{s})$

$\boxed{\frac{dV}{dt} = 25{,}600\pi\,\frac{mm^3}{s}}$ ✓

5. If a snowball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 10 cm.

$$\boxed{\frac{dd}{dt} = \frac{-1}{20\pi}\frac{cm}{min}}$$ ✓

$\frac{dSA}{dt} = \frac{-1cm^2}{min}$

Know:

Equation: $SA = 4\pi R^2$

$SA = 4\pi\left(\frac{d}{2}\right)^2$

Substitution: $\frac{dSA}{dt} = 2\pi d\frac{dd}{dt}$

Find: $\frac{dd}{dt} =$ ___

Derivative: $SA = \cancel{4}\pi\frac{d^2}{\cancel{4}}$

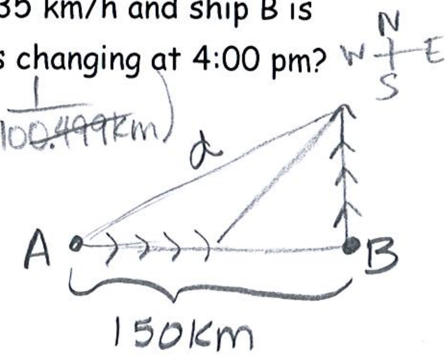
$-\frac{1\,cm^2}{min} = 2\pi(10cm)\frac{dd}{dt}$

When: $d = 10cm$

$\frac{d}{dt}[SA = \pi d^2]$

$\left(\frac{1}{20\pi cm}\right)\frac{-1cm^2}{min} = \cancel{20\pi cm}\frac{dd}{dt}\left(\frac{1}{\cancel{20\pi cm}}\right)$

6. At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 pm?



$\left(\frac{1}{100.499km}\right)\frac{2150\,km^2}{h} = \cancel{100.499km}\frac{dd}{dt}\left(\frac{1}{\cancel{100.499km}}\right)$

$$\boxed{\frac{dd}{dt} = 21.393\frac{km}{h}}$$ ✓

Know: $\frac{dA}{dt} = \frac{-35km}{h}$, $\frac{dB}{dt} = \frac{25km}{h}$

Equation: $\frac{d}{dt}[A^2 + B^2 = d^2]$

Substitution:

$2A\frac{dA}{dt} + 2B\frac{dB}{dt} = 2d\frac{dd}{dt}$

$150 - \frac{35km}{h}(4\cancel{h}) = 10km$

Find: $\frac{dd}{dt} =$ ___

Derivative:

$A\frac{dA}{dt} + B\frac{dB}{dt} = d\frac{dd}{dt}$

$0 + \frac{25km}{h}(4\cancel{h}) = 100km$

$t = 4$

When: $A = 10km$, $B = 100km$

$10km\left(-\frac{35km}{h}\right) + 100km\left(\frac{25km}{h}\right) = 100.499\,km\frac{dd}{dt}$

$10^2 + 100^2 = d$

$-350\frac{km^2}{h} + 2500\frac{km^2}{h} = 100.499km\frac{dd}{dt}$

$100.499 = d$

7. Two cars start moving from the same point. One travels south at 60 km/h and the other travels west at 25 km/h. At what rate is the distance between the cars increasing two hours later?

$x^2 + y^2 = d^2$

$50^2 + 120^2 = d^2$

$d = 130km$

$\frac{dy}{dt} = \frac{60\,km}{h}$ $\frac{dx}{dt} = \frac{25km}{h}$

Know:

Equation: $[x^2 + y^2 = d^2]\frac{d}{dt}$

Substitution:

$50km\left(\frac{25km}{h}\right) + 120km\left(\frac{60km}{h}\right) = 130km\frac{dd}{dt}$

Find: $\frac{dd}{dt}$

Derivative:

$1250\frac{km^2}{h} + 7200\frac{km^2}{h} = 130\,km\frac{dd}{dt}$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2d\frac{dd}{dt}$

$\frac{8450\frac{km^2}{h}}{130km} = \frac{130\,km\frac{dd}{dt}}{130km}$

When:

$x\frac{dx}{dt} + y\frac{dy}{dt} = d\frac{dd}{dt}$

$t = 2$ hours

$y = 120km$

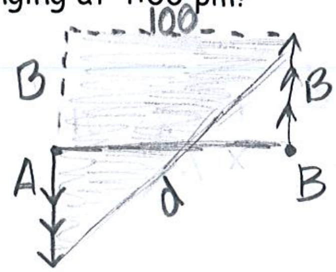
$x = 50\,km$

$$\boxed{65\frac{km}{h} = \frac{dd}{dt}}$$ ✓

8. At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 pm?

$(140+100)^2 + 100^2 = d^2$

$d = \sqrt{67600} = 260$



Know:

$\frac{dA}{dt} = 35\frac{km}{h}$ $\frac{dB}{dt} = 25\frac{km}{h}$

Equation:

$\frac{d}{dt}\left[(A+B)^2 + 100^2 = d^2\right]$

Substitution:

Find:

$\frac{dd}{dt} =$ ____

Derivative:

$2(A+B)'\left[\frac{dA}{dt} + \frac{dB}{dt}\right] + 0 = 2d\frac{dd}{dt}$

$(A+B)\left[\frac{dA}{dt} + \frac{dB}{dt}\right] = d\frac{dd}{dt}$

$(140+100)[35+25] = 260\frac{dd}{dt}$ ✓

$\frac{(240)(60)}{260} = \boxed{\frac{dd}{dt} = 55.385 \ km/h}$

When:

$t=4$ $A = 140km$ $B = 100km$

9. Water is leaking out of an inverted conical tank at a rate of 10,000 cm^3/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank.

$\frac{dV}{dt} = C - 10{,}000\frac{cm^3}{min}$ (C = Constant Rate)

Know:

$\frac{dh}{dt} = \frac{20cm}{min}$

Equation:

$V = \frac{1}{3}\pi R^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$

Substitution:

$\frac{dV}{dt} = \frac{1}{27}\pi(3h^2)\frac{dh}{dt}$

$\frac{dV}{dt} = \frac{1}{9}\pi h^2\frac{dh}{dt}$

$C - 10{,}000 = \frac{1}{9}\pi(200)^2(20)$

$C = \frac{800000\pi}{9} + 10{,}000 = \boxed{289252.603\frac{cm^3}{min}}$ ✓

$\frac{r}{h} = \frac{2}{6}$

$6r = 2h$

$r = \frac{1}{3}h$

Find:

$C =$ ____

$h = 2m = 200cm$

Derivative:

$V = \frac{1}{3}\pi\left(\frac{1}{9}h^2\right)h$

When:

$\frac{d}{dt}\left[V = \frac{1}{27}\pi h^3\right]$

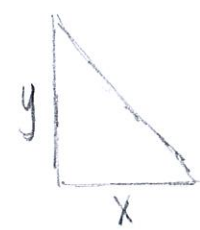
10. The top of a ladder slides down a vertical wall at a rate of 0.15 m/s. At the moment when the bottom of the ladder is 3 m from the wall, it slides away from the wall at a rate of 0.2 m/s. How long is the ladder?

$x^2 + y^2 = \ell^2$

$3^2 + 4^2 = \ell^2$

$25 = \ell^2$

$\boxed{\text{Ladder} = 5m}$ ✓



$\frac{dy}{dt} = -.15\frac{m}{s}$

Know:

$\frac{dx}{dt} = .2\frac{m}{s}$

Equation:

$\frac{d}{dt}\left[x^2 + y^2 = \ell^2\right]$

Substitution:

$(3)(.2) + y(-.15) = 0$

$\frac{.6}{.15} = \frac{.15y}{.15}$

$y = 4$

Find:

$\ell =$ ____

Derivative:

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

When:

$x = 3$