AP Calculus

Rolle's Theorem & Mean Value Theorem

Name________________________________Pd.

Day 1 Curve Sketching Unit

1-4: Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.

1. $f(x) = 5 - 12x + 3x^2$, $[1,3]$

1. $f(x) = 5 - 12x + 3x^2$ continuous $[1,3]$
2. $f(x)$ differentiable $(1,3)$
3. $f(1) = 5-12+3 = -4$ $f(3) = 5-36+27 = -4$

$\therefore f'(x) = 0$ at least 1 time on $[1,3]$

$f'(x) = -12 + 6x = 0$ $6x = 12$ $\boxed{x=2}$ ✓

2. $f(x) = x^3 - x^2 - 6x + 2$, $[0,3]$

1. $f(x)$ continuous $[0,3]$ 2. $f(x)$ diff. $(0,3)$
3. $f(0) = 2$ $f(3) = 27 - 9 - 18 + 2 = 2$

$\therefore f'(x) = 0$ at least 1 time on $[0,3]$

$0 = 3x^2 - 2x - 6$

$x = -1.120$ & $\boxed{x = 1.786}$ ✓

3. $f(x) = \sqrt{x} - \frac{1}{3}x$, $[0,9]$

1. $f(x)$ continuous $[0,9]$ 2. $f(x)$ diff $(0,9)$
3. $f(0) = 0 - \frac{1}{3}(0) = 0$ $f(9) = 3 - \frac{1}{3}(9) = 0$

$\therefore f'(x) = 0$ at least 1 time on $[0,9]$

$f'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{3} = 0$ $\frac{1}{2\sqrt{x}} = \frac{1}{3}$ $3 = 2\sqrt{x}$ ✓

$\frac{3}{2} = \sqrt{x}$ $x = \frac{9}{4} = \boxed{2\frac{1}{4}}$

4. $f(x) = \cos(2x)$, $\left[\frac{\pi}{8}, \frac{7\pi}{8}\right]$

1. $f(x)$ continuous $\left[\frac{\pi}{8}, \frac{7\pi}{8}\right]$ & 2. diff $\left(\frac{\pi}{8}, \frac{7\pi}{2}\right)$
3. $f\left(\frac{\pi}{8}\right) = \cos\left(\frac{2\pi}{8}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $\therefore f'(x) = 0$ at least 1 time on $\left[\frac{\pi}{8}, \frac{7\pi}{8}\right]$

$f\left(\frac{7\pi}{8}\right) = \cos\left(\frac{14\pi}{8}\right) = \cos\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$f'(x) = -2\sin(2x)$ $2x = \frac{\pi}{2}$ & $\frac{3\pi}{2}$

$0 = \sin(2x)$ $\boxed{x = \frac{\pi}{4} \text{ \& } \frac{3\pi}{4}}$

✓ 5. Let $f(x) = 1 - x^{\frac{2}{3}}$. Show that f(-1)=f(1) but there is no number c in (-1,1) such that $f'(c) = 0$. Why does this **not** contradict Rolle's theorem?

$f(x) = 1 - (\sqrt[3]{x})^2$ $f(-1) = 1 - 1 = 0$ $f(1) = 1 - 1 = 0$

$f'(x) = -\frac{2}{3}x^{-1/3} = \frac{-2}{3\sqrt[3]{x}}$

$f'(0) = \frac{-2}{0}$ = does not exist

Because $f(x)$ is not differentiable at $x=0$ so $f(x)$ is not differentiable on $(-1,1)$.

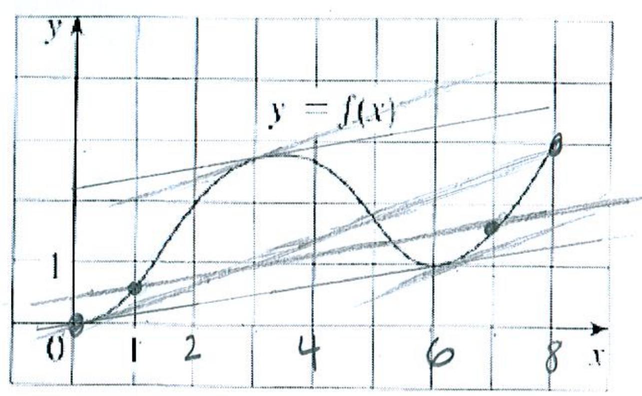
6-7: Use the graph to the right to answer the questions.

✓ 6. Use the graph of f to estimate the values of c that satisfy the conclusion of the Mean Value Theorem for the interval [0,8].

$x = .3, 3,$ & $6.3$

✓ 7. Use the graph of f to estimate the values of c that satisfy the conclusion of the Mean Value Theorem for the interval [1,7].

$x = 3.2$ & $x = 6.1$



8-11: Verify that the function satisfies the hypotheses of the Mean Value Theorem of the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

8. $f(x) = 2x^2 - 3x + 1$, $[0,2]$ Continuous & differentiable

$\frac{f(2)-f(0)}{2-0} = \frac{3-1}{2} = 1$

$f'(x) = 4x - 3$

$1 = 4x - 3$

$4 = 4x$

$\boxed{x = 1}$ ✓

9. $f(x) = x^3 - 3x + 2$, $[-2,2]$ Continuous & differentiable

$\frac{f(2)-f(-2)}{2--2} = \frac{4-0}{4} = 1$

$f'(x) = 3x^2 - 3$

$1 = 3x^2 - 3$

$4 = 3x^2$

$\frac{4}{3} = x^2$ ✓

$x = \pm\sqrt{4/3} \approx \boxed{\pm 1.155}$

10. $f(x) = \ln(x)$, $[1,4]$ Cont & diff

$\frac{f(4)-f(1)}{4-1} = \frac{\ln(4)-\ln(1)}{3}$

$f'(x) = \frac{1}{x} = \frac{\ln(4)}{3}$ $3 = \ln(4)x$ ✓

$x = 3/\ln(4) \approx \boxed{2.164}$

11. $f(x) = \frac{1}{x}, = x^{-1}$ $[1,3]$ Continuous & diff.

$\frac{f(3)-f(1)}{3-1} = \frac{\frac{1}{3}-1}{2} = \frac{\frac{1}{3}-\frac{3}{3}}{2} = \frac{-\frac{2}{3}}{2}\cdot\frac{1}{2} = -\frac{1}{3}$

$f'(x) = -1x^{-2} = \frac{-1}{x^2} = \frac{-1}{3}$ $-3 = -x^2$ $x^2 = 3$ $x = \pm\sqrt{3}$ ✓ $\boxed{x = \sqrt{3}} \approx 1.732$

12-13: Find the number c that satisfies the conclusion of the Mean Value Theorem on the given interval. Graph the function, the secant line through the endpoints, and the tangent line at $(c, f(c))$. Are the secant line and the tangent line parallel?

12. $f(x) = \sqrt{x}$, $[0,4]$

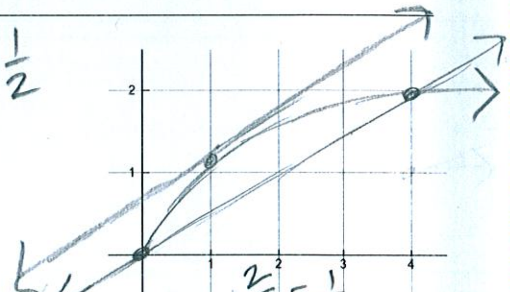
$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}} = \frac{1}{2}$

$2 = 2\sqrt{x}$

$1 = \sqrt{x}$

$\boxed{x = 1}$ ✓

$\frac{f(4)-f(0)}{4-0} = \frac{2}{4} = \frac{1}{2}$



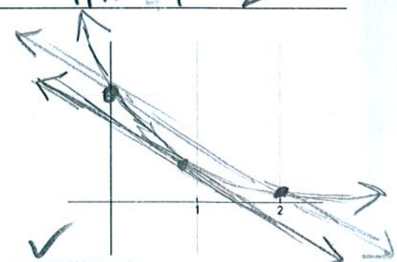

$m = \frac{2}{4} = \frac{1}{2}$

13. $f(x) = e^{-x}$, $[0,2]$

$f'(x) = e^{-x}(-1)$

$\frac{f(2)-f(0)}{2-0} = \frac{e^{-2}-1}{2}$

$-e^{-x} = \frac{e^{-2}-1}{2}$ $\ln(e^{-x}) = \ln\left(\frac{1-e^{-2}}{2}\right)$ $-x = \ln\left(\frac{1-e^{-2}}{2}\right)$




✓

$x = -\ln(1-e^{-2}/2) \approx \boxed{.839}$

✓14. Let $f(x) = (x-3)^{-2}$. Show that there is no value of c in (1,4) such that $f(4) - f(1) = f'(c)(4-1)$. Why does this not contradict the Mean Value Theorem?

$1 - \frac{1}{4} = f'(c)(3)$

$\left(\frac{1}{3}\right)\frac{3}{4} = f'(c)3\left(\frac{1}{3}\right)$

$f'(c) = \frac{1}{4}$

$f'(x) = -2(x-3)^{-3} = \frac{-2}{(x-3)^3} = \frac{1}{4}$ $-8 = (x-3)^3$

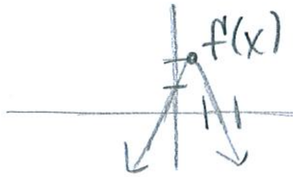
$-2 = x - 3$

$x = 1$

$f'(3) =$ dne

∴ not differentiable on (1, 4)

✓15. Let $f(x) = 2 - |2x - 1|$. Show that there is no value c such that $f(3) - f(0) = f'(c)(3-0)$. Why does this not contradict the Mean Value Theorem?



f(x) is not differentiable at $x = \frac{1}{2}$.