AP Calculus

Average Value

Name________________________

Application of Integration Day 1

1-6: Find the average value of the function on the given interval.

1. $f(x) = 4x - x^2$, $[0,4]$

$\frac{1}{4-0}\int_0^4 4x - x^2 dx$

$\frac{1}{4}\left[\frac{4x^2}{2} - \frac{x^3}{3}\right|_0^4 \quad \frac{1}{4}\left[2x^2 - \frac{x^3}{3}\right|_0^4 \quad \frac{1}{2}x^2 - \frac{1}{12}x^3\Big|_0^4$

$\frac{1}{2}(16) - \frac{1}{12}(64) - 0 + 0 = 8 - \frac{16}{3} = \frac{24}{3} - \frac{16}{3} = \boxed{\frac{8}{3}}$ ✓

2. $f(x) = \sin(4x)$, $[-\pi, \pi]$

$\frac{1}{\pi - -\pi}\int_{-\pi}^{\pi} \sin(4x)dx$

$\frac{1}{2\pi}\left[\frac{-\cos(4x)}{4}\right|_{-\pi}^{\pi} \quad -\frac{1}{8\pi}\cos(4x)\Big|_{-\pi}^{\pi}$

$\frac{-1}{8\pi}\cos(4\pi) + \frac{1}{8\pi}\cos(-4\pi) = \frac{-1}{8\pi} + \frac{1}{8\pi} = \boxed{0}$ ✓

3. $g(x) = \sqrt[3]{x}$, $[1,8]$

$\frac{1}{8-1}\int_1^8 x^{1/3} dx$

$\frac{1}{7}\left[\frac{3}{4}x^{4/3}\right|_1^8 = \frac{3}{28}(\sqrt[3]{8})^4 - \frac{3}{28}(\sqrt[3]{1})^4$

$\frac{48}{28} - \frac{3}{28} = \boxed{\frac{45}{28}}$ ✓

4. $f(t) = e^{\sin t}\cos t$, $\left[0, \frac{\pi}{2}\right]$

$u = \sin t$

$du = \cos t\, dt$

$u(0) = \sin(0) = 0$

$u(\frac{\pi}{2}) = \sin(\frac{\pi}{2}) = 1$

$\frac{1}{\frac{\pi}{2} - 0}\int_0^{\pi/2} e^{\sin t}\cos t\, dt$

$\frac{2}{\pi}\int_0^1 e^u du$

$\frac{2}{\pi}\left[e^1 - e^0\right] = \boxed{\frac{2}{\pi}[e-1]}$ ✓

5. $h(x) = \cos^4 x \sin x$, $[0,\pi]$

$u = \cos x$

$du = -\sin x dx$

$u(0) = \cos(0) = 1$

$u(\pi) = \cos(\pi) = -1$

$\frac{-1}{\pi - 0}\int_0^{\pi} -\cos^4 x \sin x dx$

$\frac{-1}{\pi}\int_1^{-1} u^4 du = \frac{+1}{\pi}\frac{u^5}{5}\Big|_{-1}^{1}$

$\frac{1}{5\pi}(1)^5 - \frac{1}{5\pi}(-1) = \frac{1}{5\pi} + \frac{1}{5\pi} = \boxed{\frac{2}{5\pi}}$ ✓

6. $h(u) = (3 - 2u)^{-1}$, $[-1,1]$

$u = 3 - 2u$

$du = -2du$

$u(-1) = 3 + 2 = 5$

$u(1) = 3 - 2 = 1$

$\frac{-1}{2}\frac{1}{1 - -1}\int_{-1}^{1} -2(3-2u)^{-1}du$

$\frac{-1}{2}\cdot\frac{1}{2}\int_5^1 \frac{1}{u}du$

$+\frac{1}{4}\ln|u|\Big]_1^5 = \frac{1}{4}\ln(5) - \cancel{\frac{1}{4}\ln(1)}^{\,0} = \boxed{\frac{1}{4}\ln 5}$ ✓

7-8: A.) Find the average Value of f on the given interval. B.) find c such that $f_{avg} = f(c)$. C.) Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f.

7. $f(x) = (x-3)^2$, $[2,5]$

$u = x - 3$

$du = dx$

$u(2) = 2 - 3 = -1$

$u(5) = 5 - 3 = 2$

$\frac{1}{5-2}\int_2^5 (x-3)^2 dx$

$\frac{1}{3}\int_{-1}^{2} u^2 du$

$\frac{1}{3}\frac{u^3}{3}\Big|_{-1}^{2} \quad \frac{1}{9}u^3\Big|_{-1}^{2}$

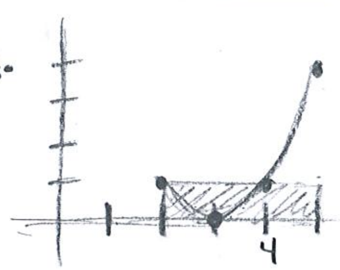
$\frac{1}{9}(8) - \frac{1}{9}(-1)$ ✓

$\frac{8}{9} + \frac{1}{9} = \boxed{1}$ A.

B. $1 = (x-3)^2$

$1 = x - 3$

$\boxed{x = 4}$ ✓



8. $f(x) = \frac{1}{x}$, $[1,3]$

$\frac{1}{3-1}\int_1^3 \frac{1}{x}dx$

$\frac{1}{2}\left[\ln x\Big|_1^3\right]$

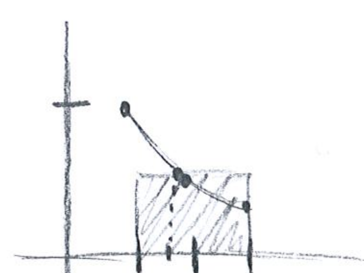
$\frac{1}{2}\ln(3) - \frac{1}{2}\ln(1)$

$\boxed{\frac{1}{2}\ln(3)}$ A.

B. $\frac{1}{2}\ln(3) = \frac{1}{x}$

$x\ln(3) = 2$

$\boxed{x = \frac{2}{\ln(3)}}$ B. ≈ 1.8



AP Calculus

Average Value

Name________________________

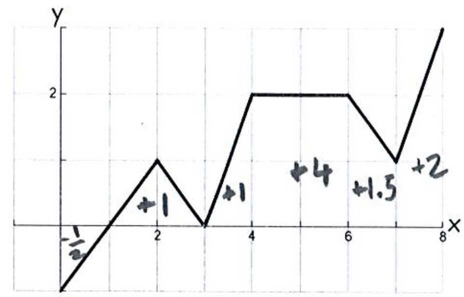
Application of Integration Day 1

9. Find the numbers b, such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0,b]$ is equal to 3. $\frac{1}{b-0}\int_0^b 2+6x-3x^2 = 3$ $\frac{1}{b}\left[2x+\frac{6x^2}{2}-\frac{3x^3}{3}\right]_0^b = 3$

$\frac{1}{b}\left[2b+3b^2-b^3-0-0+0\right]=3$ $2+3b-b^2=3$ $b^2-3b+1=0$

$\frac{3\pm\sqrt{9-4(1)(1)}}{2(1)} = \frac{3\pm\sqrt{5}}{2}$

10. Find the average value of f on $[0,8]$



$\frac{1}{8-0}\int_0^8 f$

$\frac{1}{8}\left[-\frac{1}{2}+1+5+3.5\right]$

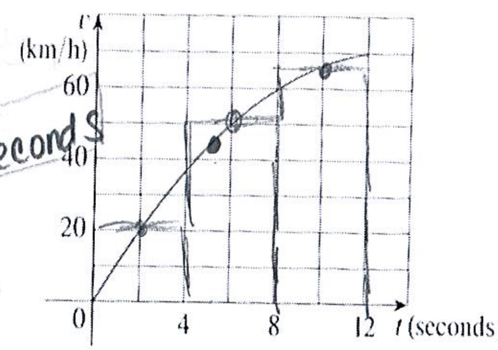
$\frac{1}{8}[9] = \boxed{\frac{9}{8}}$ ✓

11. The velocity graph of an accelerating car is shown.

A.) Use the Midpoint rule to estimate the average velocity of the car during the first 12 seconds.

B.) At what time was the instantaneous velocity equal to the average velocity? $V\left(\frac{5}{60}\right) = 45\ \frac{km}{hR}$

B. at $\boxed{5}$ seconds OR $\frac{1}{12}$ of hour



$\frac{1}{\frac{1}{5}hR-0}\int_0^{1/5} v(t)\,dt = \frac{km}{hR}$

$\frac{5}{hR}\left[\text{width}\cdot\text{length}\right] = 5\left[\frac{1}{15}(20+50+65)\right]$

$= \frac{1}{3}(135) = \boxed{45\ \frac{km}{hR}}$ A.

$\frac{12}{60} = \frac{1}{5}\ hR$

$\frac{4}{60} = \frac{1}{15}\ hR = \text{width}$

12. In a certain city the temperature (in ^oF) t hours after 9 AM was modeled by the function

$$T(t) = 50 + 14\sin\left(\frac{\pi t}{12}\right)$$

Find the average temperature during the period from 9AM to 9PM.

t=0 9AM
t=1 10AM
t=2 11AM
t=3 12PM
t=4 1PM
t=5 2PM
t=6 3PM
t=7 4PM
t=8 5PM
t=9 6PM
t=10 7PM
t=11 8PM
t=12 9PM

$\frac{1}{12-0}\int_0^{12} 50+14\sin\left(\frac{\pi}{12}t\right)dt$

$\frac{1}{12}\left[\frac{336}{\pi}+600\right] = \boxed{\frac{28}{\pi}+50}$