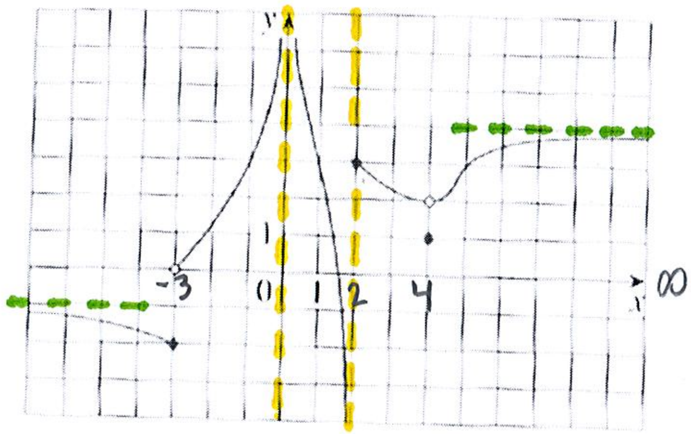
AP Calculus-AB
Review: Limits, Continuity, & R.O.C

Name________________
Day 12



1. The graph of f is given to the left.

A. Find each limit, or explain why it does not exist.

$\lim_{x\to 2^+} f(x)$ 3 $\qquad \lim_{x\to -3^+} f(x)$ 0

$\lim_{x\to -3} f(x)$ dne $\qquad \lim_{x\to 4} f(x)$ 2

$\lim_{x\to 0} f(x)$ ∞ $\qquad \lim_{x\to 2^-} f(x)$ −∞

$\lim_{x\to \infty} f(x)$ 4 $\qquad \lim_{x\to -\infty} f(x)$ −1

B. State the equations of the horizontal asymptotes. $y = 4$ & $y = -1$

C. State the equations of the vertical asymptotes. $x = 0$ & $x = 2$

D. At what numbers is f discontinuous? Explain.

$x = -3$: $\lim_{x\to -3} f(x) =$ dne
$x = 0$: $f(0) =$ undefined
$x = 2$: $\lim_{x\to 2} f(x) =$ dne
$x = 4$: $\lim_{x\to 4} f(x) \neq f(4)$

2. Sketch the graph of an example of a function f that satisfies all of the following conditions;

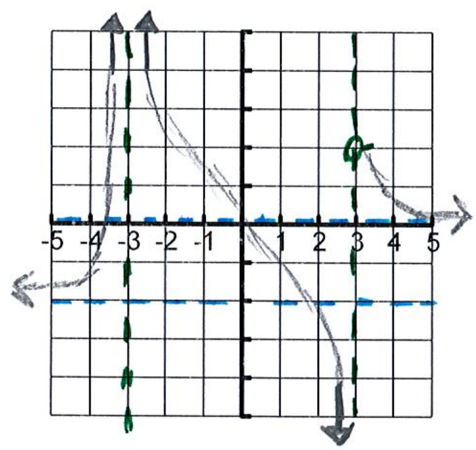
EB $\lim_{x\to -\infty} f(x) = -2$ HA: $y = -2$

EB $\lim_{x\to \infty} f(x) = 0$ HA: $y = 0$

$\lim_{x\to -3} f(x) = \infty$ VA: $x = -3$

$\lim_{x\to 3^-} f(x) = -\infty$ VA: $x = 3$

$\lim_{x\to 3^+} f(x) = 2$ open circle (3,2)



Remember: End Behavior is the last thing you graph

Find the limit:

3. $\lim_{x\to 1} e^{x^2-x} = e^{(1)^2-1} = e^0 = \boxed{1}$

4. $\lim_{x\to 3} \frac{x^2-9}{x^2+2x-3} = \lim_{x\to 3} \frac{(x-3)\cancel{(x+3)}}{(x-1)\cancel{(x+3)}} = \frac{0}{2} = \boxed{0}$

5. $\lim_{x\to -3} \frac{x^2-9}{x^2+2x-3} = \lim_{x\to -3} \frac{(x-3)\cancel{(x+3)}}{(x-1)\cancel{(x+3)}} = \frac{-6}{-4} = \boxed{\frac{3}{2}}$

6. $\lim_{x\to 1^+} \frac{x^2-9}{x^2+2x-3} = \lim_{x\to 1^+} \frac{(x-3)(x+3)}{(x-1)(x+3)} = \lim_{x\to 1^+} \frac{x-3}{x-1} = \frac{-}{+} = \boxed{-\infty}$ (1.001)

7. $\lim_{h\to 0} \frac{(h-1)^2-1}{h} = \lim_{h\to 0} \frac{h^2-2h\cancel{+1-1}}{h} = \lim_{h\to 0} \frac{\cancel{h}(h-2)}{\cancel{h}} = \boxed{-2}$

8. $\lim_{t\to 2} \frac{t^2-4}{t^3-8} = \lim_{t\to 2} \frac{\cancel{(t-2)}(t+2)}{\cancel{(t-2)}(t^2+2t+4)} = \frac{4}{4+4+4} = \boxed{\frac{1}{3}}$

9. $\lim_{R\to 9} \frac{\sqrt{R}}{(R-9)^4}$ $\lim_{R\to 9^+} = \frac{+}{+} = +\infty$ (9.001) $\lim_{R\to 9^-} = \frac{+}{+} = \boxed{+\infty}$ (8.999)

10. $\lim_{x\to \infty} \frac{1-2x^2-x^4}{5+x-3x^4} = \frac{-1}{-3} = \boxed{\frac{1}{3}}$ EB (Marilyn)

11. $\lim_{w\to 0} \frac{\frac{1}{6+w} - \frac{1}{6}}{w}$ $\frac{6(1)}{6(6+w)} - \frac{1(6+w)}{6(6+w)}$ $\lim_{w\to 0} \frac{\frac{\cancel{6-6}-w}{6(6+w)}}{w}$

$\lim_{w\to 0} \frac{\frac{-w}{6(6+w)} \cdot \frac{1}{w}}{\frac{w}{1} \cdot \frac{1}{w}} = \frac{-1}{6(6+0)} = \boxed{\frac{-1}{36}}$

12. $\lim_{4\theta\to 0} \frac{4\sin 4\theta}{4\theta} = \lim_{4\theta\to 0} \frac{4\sin 4\theta}{4\theta} = 4\lim_{4\theta\to 0} \frac{\sin 4\theta}{4\theta} = 4(1) = \boxed{4}$

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Find the limit:

13. $\lim_{x\to-\infty} \frac{x-2}{\sqrt{3x^2+1}} = \frac{-}{+} = \boxed{-\frac{1}{\sqrt{3}}}$ EB

Marilyn (only one √ sign)

14. EB $\lim_{t\to\infty} \frac{x^2-6}{1+x} = \frac{+}{+} = \boxed{+\infty}$

Dolly

15. $\lim_{v\to0^+} \frac{6-v^2}{v^3-v} = \lim_{v\to0^+} \frac{6-v^2}{v(v-1)(v+1)} = \frac{+}{(+)(-)(+)}$

$= \boxed{-\infty}$

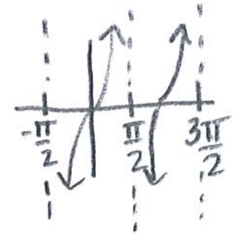
16. EB $\lim_{x\to\infty} \frac{x^2+2x-3}{x^3+2x^2} = \boxed{0}$

J-Lo

17. $\lim_{x\to\frac{\pi}{6}} tanx = \tan\left(\frac{\pi}{6}\right) = \boxed{\sqrt{3}}$

$\frac{\pi}{6}$ $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ $t = \sqrt{3}$

18. $\lim_{x\to\frac{\pi}{2}^+} tanx = \boxed{-\infty}$



19. $\lim_{x\to1} \cos\pi x = \cos(\pi\cdot1) = \boxed{-1}$

$= \cos\pi$

20. $\lim_{x\to0} \frac{(\sqrt{3+x}-\sqrt{3})(\sqrt{3+x}+\sqrt{3})}{(x)(\sqrt{3+x}+\sqrt{3})} = \lim_{x\to0} \frac{\cancel{3}+x-\cancel{3}}{x(\sqrt{3+x}+\sqrt{3})}$

$\lim_{x\to0} \frac{\cancel{x}}{\cancel{x}(\sqrt{3+x}+\sqrt{3})} = \frac{1}{\sqrt{3}+\sqrt{3}} = \boxed{\frac{1}{2\sqrt{3}}}$

x	−1.1	−1.003	−1.0001	−0.9999	−0.8762	−0.6522
	left		→	←	Right	
h(x)	-89	-677	-5009 $-\infty$	-5.003 -5	-5.088	-5.113
p(x)	9.222	9.111	9.002 9	9 8.999	8.802	8.777
r(x)	−99	−999	−9999 $-\infty$	−8853 $-\infty$	−871	−86

21.

$\lim_{x\to-1^+} h(x) = -5$

$\lim_{x\to-1^-} h(x) = -\infty$

$\lim_{x\to-1} h(x) =$ dne

$\lim_{x\to-1} p(x) = 9$

$\lim_{x\to-1} r(x) = -\infty$

22. $f(x) = \begin{cases} \sqrt{-x} & if \quad x<0 \\ 3-x & if \quad 0\le x\le3 \\ (x-3)^2 & if \quad x>3 \end{cases}$

left (0)

Right (0) left (3)

Right (3)

Evaluate each limit, if it exists.

A. $\lim_{x\to0^+} f(x) = 3-0=3$

B. $\lim_{x\to0^-} f(x) = \sqrt{-0}=0$

C. $\lim_{x\to0} f(x) =$ dne

D. $\lim_{x\to3^+} f(x) = (3-3)^2=0$

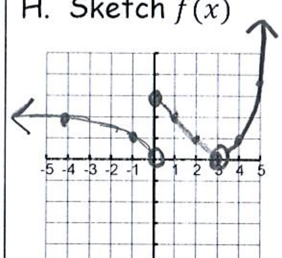
E. $\lim_{x\to3^-} f(x) = 3-3=0$

F. $\lim_{x\to3} f(x) = 0$

G. Where is f discontinuous? Justify your answer.

X=0 discontinuous because $\lim_{x\to0} f(x) =$ d.n.e.

H. Sketch $f(x)$



23. Use the Intermediate Value Theorem to show that there is a root of the equation $f(x) = x^5 - x^3 + 3x - 5$ on the interval $(1,2)$.

f(x) is continuous $(-\infty, \infty)$ so continuous [1,2]

$f(1) = 1-1+3-5 = -2$

$f(2) = 32-8+6-5 = 25$

Then by I.V.T. $f(x)=0$ on $(1,2)$.

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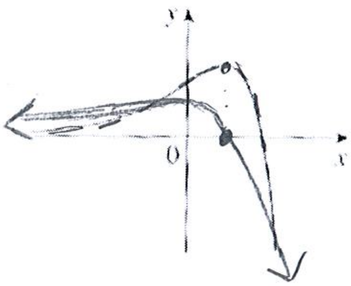
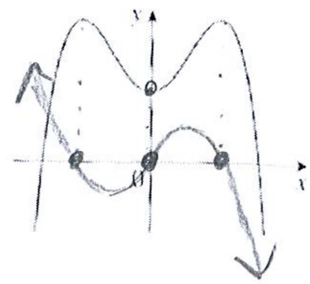
Sketch the graph of $f'(x)$ directly onto the graph of $f(x)$.

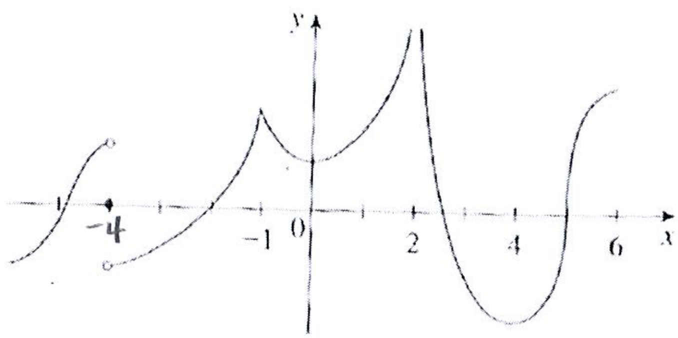
24.



25.



26. The graph of f is shown. State, with reasons, the numbers at which f is not differentiable.



x=-4 discontinuous

x=-1 corner

x=2 discontinuous

x=5 vertical tangent

27. Let T(t) be the temperature (in $^{\circ}F$) in Phoenix t hours after midnight on September 10, 2008. The table shows values of the function recorded every two hours.

x t	0	2	4	6	8	10	12	14
y T	82	75	74	75	84	90	93	94

What is the meaning of T'(8)? Estimate its value.

$T'(8) = \frac{\Delta y}{\Delta x} = \frac{\text{temperature } ^{\circ}F}{\text{hours}} = \frac{90^{\circ}F - 75^{\circ}F}{10-6} = \frac{15}{4} = 3.75^{\circ}F/hr$

Temp. is increasing 3.75°F/hr at 8:00.

28. The cost of producing x ounces of gold from a new gold mine is C=f(x) dollars.

a.) What is the meaning of the derivative f'(x)? What are its units?

f'(x) = Rate of change in cost \$ / ounce gold.

b.) What does the statement f'(800)=17 mean?

Cost is increasing 17\$/ounce at 800 ounces of gold.

c.) Do you think the values of f'(x) will increase or decrease in the short term? What about the long term? Explain. Short term ? not sure

Long term should increase because as supply decreases cost ↑

Use the given table to approximate the expressions to the right.

x	-2	-1	0	1	2
$f(x)$	-4	0	2	9	10
$g(x)$	30	16	8	1	.5

29. $f'(0) = \frac{f(1)-f(-1)}{1--1} = \frac{9-0}{2} = 4.5$

30. $2g'(-1) + f'(2) =$

$2\left[\frac{g(0)-g(-2)}{0--2}\right] + \left[\frac{f(2)-f(1)}{2-1}\right]$

$2\left[\frac{8-30}{2}\right] + \left[\frac{10-9}{1}\right] = -22+1 = \boxed{-21}$

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31. If f is continuous on [2,6], with $f(2) = 20$ and $f(6) = 10$, then the Intermediate Value Theorem says which of the following is true?

I. $f(x) = 25$ does not have a solution on [2,6]. ✓

II. $f(x) = 17$ has a solution on [2,6]. ✓

~~III.~~ $f(x) = 0$ has a solution on [2,6].

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III

32. Using the table of values of $f(x)$, the average rate of change of f on the interval $[-2,4]$ is

x	-6	-4	-2	0	2	4	6
$f(x)$	9	3	1	5	8	15	31

(A) $\frac{1}{6}$
(B) 1
(C) $\frac{3}{2}$
(D) $\frac{7}{3}$
(E) 12

$\frac{f(4)-f(-2)}{4--2} = \frac{15-1}{6} = \frac{14}{6} = \frac{7}{3}$

33. $\lim_{x\to\infty} \frac{3x+2}{\sqrt{x^2+4}}$ is Marilyn $= \frac{+}{+} = \frac{3}{1} = 3$

(A) $-\infty$
(B) -3
(C) 0
(D) 3
(E) ∞

34. $\lim_{x\to 1} \frac{x^2+2x-3}{x^2-1} = \frac{(x+3)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \frac{1+3}{1+1} = \frac{4}{2}$

(A) -2
(B) -1
(C) 10
(D) 1
(E) 2

35. $\lim_{h\to 25} \frac{\sqrt{h}-5}{h-25}$ is $\lim_{h\to 25} \frac{\cancel{\sqrt{h}-5}}{\cancel{(\sqrt{h}-5)}(\sqrt{h}+5)}$

$= \frac{1}{\sqrt{25}+5} = \frac{1}{10}$

(A) 0
(B) $\frac{1}{25}$
(C) $\frac{1}{10}$
(D) $\frac{1}{5}$
(E) nonexistent

36. $\lim_{x\to 4^-} \frac{x+6}{x^2-6x+8}$ is $\lim_{x\to 4^-} \frac{x+6}{(x-2)(x-4)}$ $= \frac{+}{(+)(-)}$

3.999

$= -\infty$

(A) 0
(B) $\frac{1}{24}$
(C) $\frac{3}{4}$
(D) ∞
(E) $-\infty$

37. Let $f(x) = \begin{cases} \frac{x^2-9}{\cancel{x-3}}, & x \neq 3 \\ 6, & x = 3 \end{cases}$. $(x+3)(\cancel{x-3})$ $f(x) = \begin{cases} x+3, & x \neq 3 \\ 6, & x = 3 \end{cases}$

Which of the following is true?

I. $\lim_{x\to 3} f(x)$ does not exist.
II. f is continuous at $x = 3$.
III. The line $x = 3$ is a vertical asymptote

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only

$\lim_{x\to 3} f(x) = 3+3 = 6$

$f(3) = 6$

38. What is $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$? $= \lim_{x\to 1} \frac{\cancel{\sqrt{x}-1}}{\cancel{(\sqrt{x}-1)}(\sqrt{x}+1)}$

$\frac{1}{1+1} = \frac{1}{2}$

(A) 0
(B) $\frac{1}{2}$
(C) 1
(D) $\frac{3}{2}$
(E) nonexistent

39. What is $\lim_{x\to\infty} \frac{x^2-6}{2+x-3x^2}$ (Marilyn) $= \frac{1}{-3}$

EB

(A) -3 (B) $-\frac{1}{3}$ (C) $\frac{1}{3}$ (D) 2 (E) the lim it does not exist

40. The function f is continuous at $x = 1$. If $f(x) = \begin{cases} \frac{\sqrt{x+3}-\sqrt{3x+1}}{x-1} & \text{for } x \neq 1 \\ k = -\frac{1}{2} & \text{for } x = 1 \end{cases}$ then $k =$

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$ (E) none of the above

$\lim_{x\to 1} \frac{(\sqrt{x+3}-\sqrt{3x+1})(\sqrt{x+3}+\sqrt{3x+1})}{(x-1)(\sqrt{x+3}+\sqrt{3x+1})} = \lim_{x\to 1} \frac{x+3-(3x+1)}{(x-1)(\sqrt{x+3}+\sqrt{3x+1})} = \lim_{x\to 1} \frac{-2x+2}{(x-1)(\sqrt{x+3}+\sqrt{3x+1})}$

$\lim_{x\to 1} \frac{-2\cancel{(x-1)}}{\cancel{(x-1)}(\sqrt{x+3}+\sqrt{3x+1})}$ $\lim_{x\to 1} = \frac{-2}{\sqrt{4}+\sqrt{4}} = \frac{-2}{4} = \boxed{-\frac{1}{2} = \lim}$