AP Calculus

U-Substitution

Evaluate the definite integral.

Name______________________________

Integration Day 11

1. $\int_0^3 \frac{dx}{5x+1}$

$u = 5x+1$

$du = 5\,dx$

$\frac{du}{5} = dx$

$\int_{u=1}^{u=16} \frac{1}{u}\cdot\frac{du}{5} = \frac{1}{5}\int_1^{16} \frac{1}{u}\,du$

$= \frac{1}{5}\ln|u|\Big|_1^{16}$

$= \frac{1}{5}\ln|16| - \cancel{\frac{1}{5}\ln|1|}$

$= \boxed{\frac{1}{5}\ln|16|}$

2. $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}\,dx$

$u = \frac{1}{x} = x^{-1}$

$du = -x^{-2}\,dx$

$-du = x^{-2}\,dx$

$\int_{u=1}^{u=1/2} e^u(-du)$

$= -\int_1^{1/2} e^u\,du = -e^u\Big|_1^{1/2} = \boxed{-e^{1/2}+e}$

3. $\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}}$

$u = 1+2x$

$du = 2\,dx$

$\frac{du}{2} = dx$

$\frac{1}{2}\int \frac{1}{\sqrt[3]{u^2}}\,du = \frac{1}{2}\int_{u=1}^{u=27} u^{-2/3}\,du$

$= \frac{1}{2}\cdot 3u^{1/3}\Big|_1^{27} = \frac{3}{2}u^{1/3}\Big|_1^{27}$

$= \frac{3}{2}(27^{1/3}) - \frac{3}{2}(1)^{1/3}$

$= \frac{9}{2} - \frac{3}{2} = \frac{6}{2} = \boxed{3}$

4. $\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx$

$u = 1+2x$

$du = 2\,dx$

$\frac{du}{2} = dx$

$x = \frac{u-1}{2}$

$\int \frac{\frac{u-1}{2}}{\sqrt{u}}\cdot\frac{du}{2}$

$= \frac{1}{4}\int_{u=1}^{u=9} \frac{u-1}{\sqrt{u}}\,du = \frac{1}{4}\int_1^9 (u-1)u^{-1/2}\,du$

$= \frac{1}{4}\int_1^9 u^{1/2} - u^{-1/2}\,du = \frac{1}{4}\left(\frac{2}{3}u^{3/2} - 2u^{1/2}\right)\Big|_1^9$

$= \frac{1}{6}u^{3/2} - \frac{1}{2}u^{1/2}\Big|_1^9 = \frac{1}{6}(9)^{3/2} - \frac{1}{2}(9)^{1/2} - \left(\frac{1}{6}-\frac{1}{2}\right)$

$\frac{27}{6} - \frac{3}{2} - \frac{1}{6} + \frac{1}{2} = \frac{26}{6} - 1 = \boxed{\frac{20}{6}}$

5. $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$u(e) = \ln(e) = 1$

$u(e^4) = \ln e^4 = 4$

$\int_{u=1}^{u=4} \frac{1}{\sqrt{u}}\,du = \int_1^4 u^{-1/2}\,du = 2u^{1/2}\Big|_1^4$

$= 2\sqrt{4} - 2\sqrt{1}$

$= 4 - 2 = \boxed{2}$

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6. $\int_0^{\frac{1}{2}} \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx$

$u = \sin^{-1} x$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$

$u(0) = \sin^{-1}(0) = 0$

$u(\tfrac{1}{2}) = \sin^{-1}(\tfrac{1}{2}) = \frac{\pi}{6}$

$\int_{u=0}^{u=\pi/6} u\,du$

$\frac{u^2}{2}\Big|_0^{\pi/6} = \frac{(\frac{\pi}{6})^2}{2} - 0 = \frac{\pi^2}{2(36)}$

$= \boxed{\frac{\pi^2}{72}}$

7. $\int_0^1 \frac{e^z+1}{e^z+z}\,dz$

$u = e^z + z$

$du = (e^z+1)\,dz$

$u(0) = e^0 + 0 = 1$

$u(1) = e^1 + 1 = e+1$

$\int_{u=1}^{u=e+1} \frac{1}{u}\,du$

$= \ln|u| \Big|_1^{e+1} = \ln|e+1| - \ln|1|$

$= \boxed{\ln|e+1|}$

8. $\int_0^{\frac{T}{2}} \sin\left(\frac{2\pi t}{T} - \alpha\right)dt$

* You can omit this *

:)

$u = \frac{2\pi t}{T} - \alpha$

$du = \frac{2\pi}{T}\,dt$

$\frac{T}{2\pi}\,du = dt$

$u(0) = -\alpha$

$u(\tfrac{T}{2}) = \pi - \alpha$

$\int_{u=-\alpha}^{\pi-\alpha} \sin u \cdot \frac{T}{2\pi}\,du$

$\frac{T}{2\pi}\int_{-\alpha}^{\pi-\alpha} \sin u\,du$

$-\frac{T}{2\pi}\cos u\Big|_{-\alpha}^{\pi-\alpha} = -\frac{T}{2\pi}\cos(\pi-\alpha) + \frac{T}{2\pi}\cos(-\alpha)$

$\cos(\pi-\alpha) = -\cos\alpha$

* trig identity

$= \frac{T}{2\pi}\cos\alpha + \frac{T}{2\pi}\cos(-\alpha)$

* $\cos(-\alpha) = \cos(\alpha)$ b/c even function

$= \boxed{\frac{T}{\pi}\cos\alpha}$

9. $\int_0^1 \frac{dx}{(1+\sqrt{x})^4}$

$u = 1+\sqrt{x}$

$du = \frac{1}{2}x^{-1/2}\,dx$

$2\sqrt{x}\,du = dx$

hmm...

$\int \frac{2\sqrt{x}\,du}{u^4} \rightarrow \sqrt{x} = u-1$

$\int \frac{2(u-1)}{u^4}\,du = 2\int \frac{u-1}{u^4}\,du$

$= 2\int_{u=1}^{u=2} u^{-3} - u^{-4}\,du$

$= 2\left(-\frac{u^{-2}}{2} + \frac{u^{-3}}{3}\right)\Big|_1^2$

$= -u^{-2} + \frac{2}{3}u^{-3}\Big|_1^2$

$= -\frac{1}{2^2} + \frac{2}{3(2)^3} - \left(-1 + \frac{2}{3}\right)$

$= -\frac{1}{4} + \frac{2}{24} + 1 - \frac{2}{3} = \boxed{\frac{1}{6}}$