AP Calculus

Integration by Substitution

Evaluate the indefinite integral

Name____________________

Integration Day 10

1. $\int \frac{dx}{5-3x}$

$u = 5-3x$

$du = -3\,dx$

$\frac{du}{-3} = dx$

$-\frac{1}{3}\int \frac{1}{u}\,du$

$= -\frac{1}{3}\ln|u| + C$

$= \boxed{-\frac{1}{3}\ln|5-3x| + C}$

2. $\int \frac{e^u}{(1-e^u)^2}\,du$

$u = 1-e^u$

$du = -e^u\,du$

$-du = e^u\,du$

$-\int \frac{1}{u^2}\,du$

$= -\int u^{-2}\,du = u^{-1} + C = \boxed{\frac{1}{1-e^u} + C}$

3. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

$u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}dx$

$2du = \frac{1}{\sqrt{x}}\,dx$

$2\int \sin u\,du$

$= -2\cos u + C$

$= \boxed{-2\cos\sqrt{x} + C}$

4. $\int \frac{a+bx^2}{\sqrt{3ax+bx^3}}\,dx$

$u = 3ax + bx^3$

$du = (3a + 3bx^2)\,dx$

$\frac{du}{3} = (a + bx^2)\,dx$

$\frac{1}{3}\int \frac{1}{\sqrt{u}}\,du$

$= \frac{1}{3}\int u^{-1/2}\,du = \frac{2}{3}u^{1/2} + C = \boxed{\frac{2}{3}(3ax+bx^3)^{1/2} + C}$

5. $\int \frac{z^2}{z^3+1}\,dz$

$u = z^3+1$

$du = 3z^2\,dz$

$\frac{du}{3} = z^2\,dz$

$\frac{1}{3}\int \frac{1}{u}\,du$

$= \frac{1}{3}\ln|u| + C = \boxed{\frac{1}{3}\ln|z^3+1| + C}$

6. $\int \frac{(\ln x)^2}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int u^2\,du = \frac{u^3}{3} + C$

$= \boxed{\frac{(\ln x)^3}{3} + C}$

7. $\int \frac{dx}{ax+b}$

$u = ax+b$

$du = a\,dx$

$\frac{du}{a} = dx$

$\frac{1}{a}\int \frac{1}{u}\,du$

$= \frac{1}{a}\ln|u| + C$

$= \boxed{\frac{1}{a}\ln|ax+b| + C}$

8. $\int e^{\cos t}\sin t\,dt$

$u = \cos t$

$du = -\sin t\,dt$

$-du = \sin t\,dt$

$= -\int e^u\,du$

$= -e^u + C$

$= \boxed{-e^{\cos t} + C}$

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9. $\int \frac{\tan^{-1} x}{1+x^2}\,dx$

$u = \tan^{-1} x$

$du = \frac{1}{1+x^2}\,dx$

$\int u\,du = \frac{u^2}{2} + C$

$= \boxed{\frac{(\tan^{-1} x)^2}{2} + C}$

10. $\int \frac{\sin(\ln x)}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$= \int \sin u\,du$

$= -\cos u + C = \boxed{-\cos(\ln x) + C}$

11. $\int \frac{\cos\left(\frac{\pi}{x}\right)}{x^2}\,dx$

$u = \frac{\pi}{x} = \pi x^{-1}$

$du = -\pi x^{-2}\,dx = \frac{-\pi}{x^2}\,dx$

$\frac{du}{-\pi} = \frac{1}{x^2}\,dx$

$-\frac{1}{\pi}\int \cos u \cdot du$

$= -\frac{1}{\pi}\sin u + C = \boxed{-\frac{1}{\pi}\sin\left(\frac{\pi}{x}\right) + C}$

12. $\int \frac{2^t}{2^t+3}\,dt$

$u = 2^t + 3$

$du = 2^t \ln 2\,dt$

$\frac{du}{\ln 2} = 2^t\,dt$

$\frac{1}{\ln 2}\int \frac{1}{u}\,du$

$= \frac{1}{\ln 2}\ln|u| + C = \boxed{\frac{1}{\ln 2}\ln|2^t+3| + C}$

13. $\int \frac{dt}{\cos^2 t\sqrt{1+\tan t}}$

$u = 1 + \tan t$

$du = \sec^2 t\,dt$

$du = \frac{dt}{\cos^2 t}$

$\int \frac{1}{\sqrt{u}}\,du = \int u^{-1/2}\,du$

$= 2u^{1/2} + C = \boxed{2\sqrt{1+\tan t} + C}$

14. $\int \frac{\sin x}{1+\cos^2 x}\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$-\int \frac{1}{1+u^2}\,du$

$-\tan^{-1} u + C = \boxed{-\tan^{-1}(\cos x) + C}$

15. $\int \frac{dx}{\sqrt{1-x^2}\,\sin^{-1} x}$

$u = \sin^{-1} x$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$

$\int \frac{1}{u}\,du$

$= \ln|u| + C = \boxed{\ln|\sin^{-1} x| + C}$

16. $\int \frac{x}{1+x^4}\,dx$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\int \frac{x}{1+(x^2)^2}\,dx$

$\frac{1}{2}\int \frac{1}{1+u^2}\,du = \frac{1}{2}\tan^{-1} u + C$

$= \boxed{\frac{1}{2}\tan^{-1}(x^2) + C}$

17. $\int \frac{1+x}{1+x^2}\,dx = \int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\int \frac{1}{1+x^2}\,dx + \frac{1}{2}\int \frac{1}{u}\,du$

$= \tan^{-1} x + \frac{1}{2}\ln|u| + C$

$= \boxed{\tan^{-1} x + \frac{1}{2}\ln|1+x^2| + C}$