Name ______________

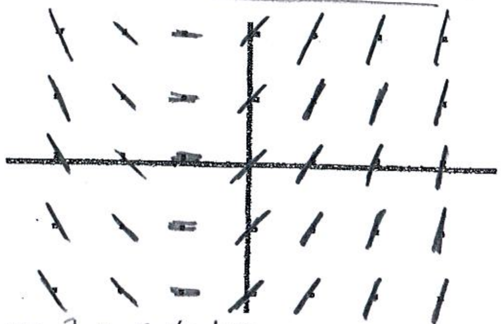
App. of Integration Day 10

Slope Fields

Draw a slope field for each of the following differential equations. Each tick mark is one unit.

$dy = x + 1\,dx$

1. $\cancel{dx}\frac{dy}{\cancel{dx}} = x + 1\,dx$ $\boxed{y = \frac{x^2}{2} + x + C}$



$x=-3$ $x=-2$ $x=-1$ $x=0$

2. $\frac{dy}{dx} = 2y$

$\cancel{dx}\frac{dy}{\cancel{dx}} = 2y\,dx$

$\frac{1}{y}dy = 2\cancel{y}\,dx\,\frac{1}{\cancel{y}}$

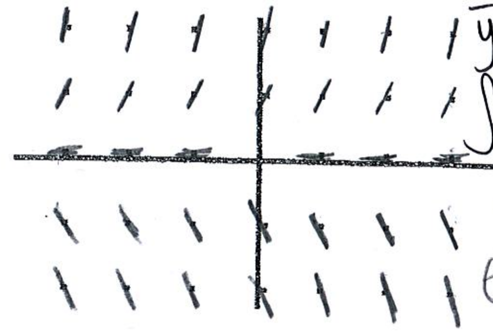
$\int \frac{1}{y}dy = \int 2\,dx$

$\ln|y| = 2x + C$

$e \quad e$

$|y| = e^{2x}e^{C}$

$\boxed{y = Ce^{2x}}$

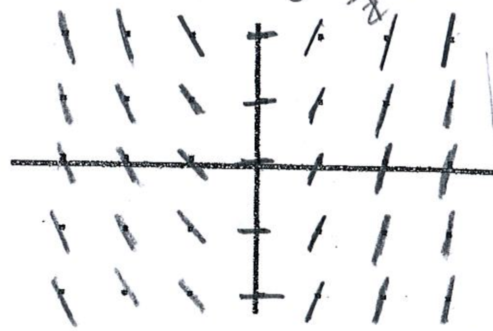


3. $\frac{dy}{dx} = x + y$ (do not solve)

4. $\cancel{dx}\frac{dy}{\cancel{dx}} = 2x\,dx$ $\int dy = \int 2x\,dx$

$y = \frac{\cancel{2}x^2}{\cancel{2}} + C$

$\boxed{y = x^2 + C}$



$\frac{1}{y-1}dy = \cancel{y-1}\,dx\left(\frac{1}{\cancel{y-1}}\right)$

$\frac{1}{y-1}dy = dx$

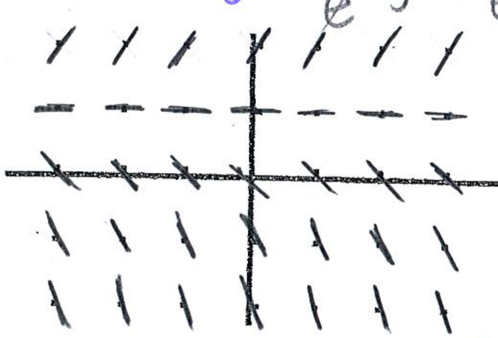
5. $\frac{1}{y-1}\cancel{dx}\frac{dy}{\cancel{dx}} = \cancel{y-1}\cdot dx\left(\frac{1}{y-1}\right)$

$\ln|y-1| = x + C$

$e \quad e$

$y=2$

$y=1$



$y - 1 = Ce^{x}$

$\boxed{y = Ce^{x} + 1}$

6. $\frac{dy}{dx} = -\frac{y}{x}$

$\cancel{dx}\frac{dy}{\cancel{dx}} = -\frac{y}{x}dx$

$\frac{1}{y}dy = -\frac{\cancel{y}}{x}dx\frac{1}{\cancel{y}}$

$\int \frac{1}{y}dy = \int -\frac{1}{x}dx$

$\ln|y| = -\ln|x| + C$

$e \quad e$

$|y| = e^{\ln|x^{-1}|}e^{C}$

$\boxed{y = C\frac{1}{x}}$

AP Calculus

Slope Fields

Name____________________________

Application of Integration Day 11

Match the slope fields with their differential equations.

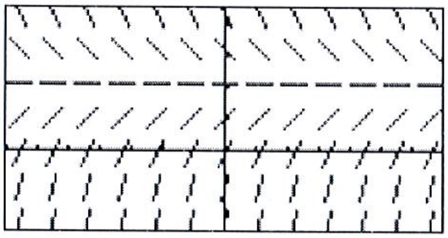
C 7. $\frac{dy}{dx} = \sin x$

D 8. $\frac{dy}{dx} = x - y$

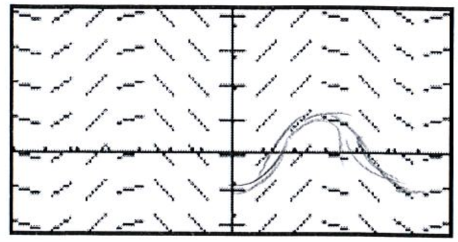
A 9. $\frac{dy}{dx} = 2 - y$

B 10. $\frac{dy}{dx} = x$

A.
only in terms of y

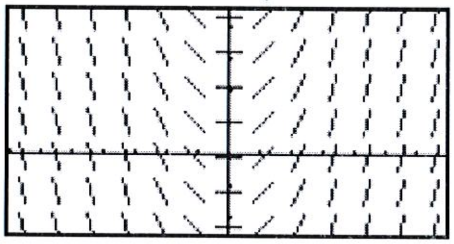


C.

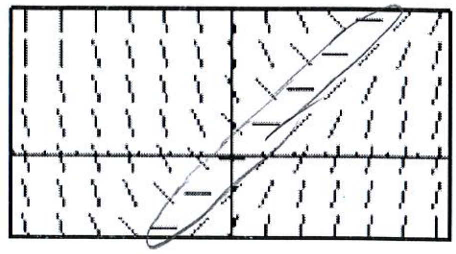


$\frac{d}{dx}[-\cos x] = \sin x$

B.
only in terms of x



D.



when x=y the slope=0

Match the slope fields with their differential equations.

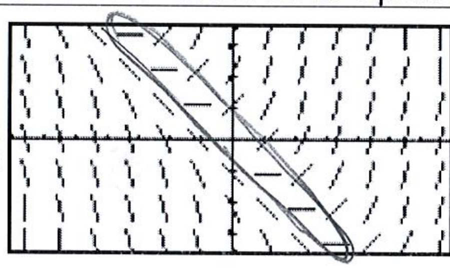
C 11. $\frac{dy}{dx} = 0.5x - 1$

B 12. $\frac{dy}{dx} = 0.5y$

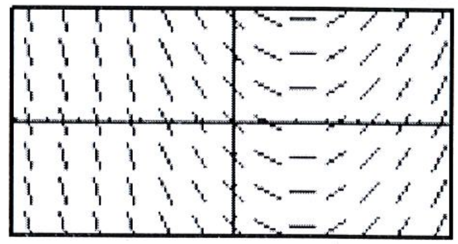
D 13. $\frac{dy}{dx} = -\frac{x}{y}$

A 14. $\frac{dy}{dx} = x + y$

A.
x=-y then slope= 0

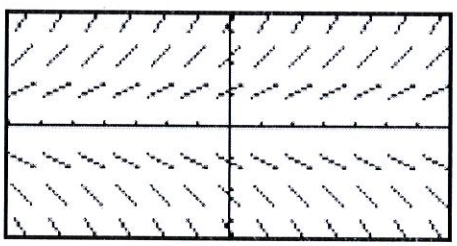


C.

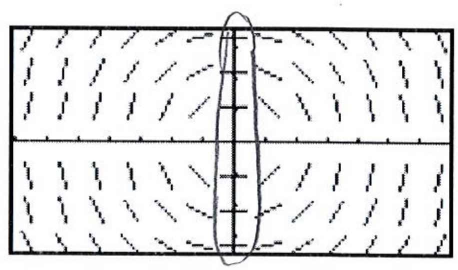


Only in terms of x

B.
only in terms of y



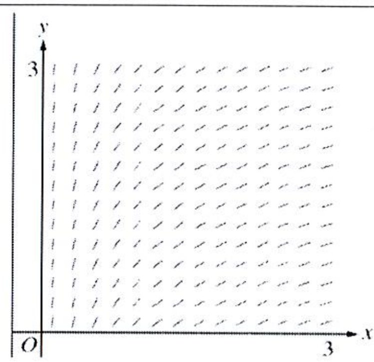
D.



when x=0 then slope=0

The slope field from a certain differential equation is shown. Which of the following could be a specific solution to the differential equation?

15.



N A.) $y = x^2$

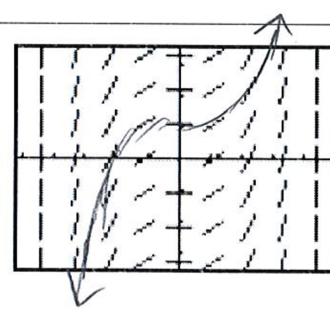
N B.) $y = e^x$

N C.) $y = e^{-x}$

N D.) $y = \cos x$

E.) $y = \ln x$

16.



A.) $y = \sin x$

B.) $y = \cos x$

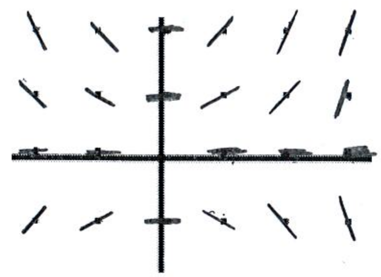
C.) $y = x^2$

D.) $y = \frac{1}{6}x^3$

E.) $y = \ln x$

17. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

A.) On the axes provided, sketch a slope field for the given differential equation.



B.) Let f be the function that satisfies the given differential equation. Write an equation for the tangent line to the curve of y=f(x) through the point (1,1). Then use your tangent line equation to estimate the value of f(1.2).

point (1,1)

slope $= \frac{dy}{dx} = \frac{(1)(1)}{2} = \frac{1}{2}$

$y - 1 = \frac{1}{2}(x-1)$

$y = \frac{1}{2}x - \frac{1}{2} + 1$

$y = \frac{1}{2}x + \frac{1}{2}$

$y(1.2) = \frac{1}{2}(1.2) + \frac{1}{2} = .6 + .5 = \boxed{1.1}$

C.) Find the particular solution y=f(x) to the differential equation with the initial condition f(1)=1. Use your solution to find f(1.2).

$\cancel{dx}\frac{dy}{\cancel{dx}} = \frac{xy}{2}dx$

$\frac{1}{y}dy = \frac{x\cancel{y}}{2}dx \cdot \frac{1}{\cancel{y}}$

$\int \frac{1}{y}dy = \int \frac{1}{2}x\,dx$

$\ln|y| = \frac{1}{2}\frac{x^2}{2} + C$

$\ln|y| = \frac{1}{4}x^2 + C$

$\ln|1| = \frac{1}{4}(1)^2 + C$

$C = -\frac{1}{4}$

$\ln|y| = \frac{1}{4}x^2 - \frac{1}{4}$

$e^{\ln|y|} = e^{\frac{1}{4}x^2 - \frac{1}{4}}$

$y = \pm e^{1/4x^2 - 1/4}$

$y = e^{1/4x^2 - 1/4}$

$y(1.2) = e^{\frac{1}{4}(1.2)^2 - \frac{1}{4}}$

$y(1.2) = \boxed{1.116}$

D.) Compare your estimate of f(1.2) found in part B to the actual value of f(1.2) found in part C.

Estimate in B.) is smaller than actual in C.).

E.) Was your estimate in part b and underestimate or an overestimate? Use your slope field to explain why. Graph is concave up at x=1.2 so the estimate is an underestimate.

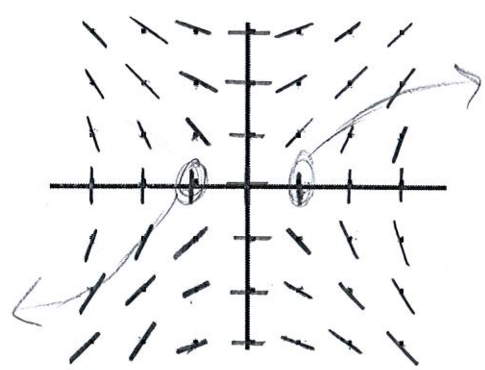
18. Consider the differential equation given by $\frac{dy}{dx} = \frac{x}{y}$.

A.) On the axes provided, sketch a slope field for the given differential equation.

$\cancel{dx}\frac{dy}{\cancel{dx}} = \frac{x}{y}dx$

$y\,dy = \frac{x}{\cancel{y}}dx\cancel{y}$

$\int y\,dy = \int x\,dx \quad \frac{y^2}{2} = \frac{x^2}{2} + C$



B.) Sketch a solution curve that passes through the point (0,1) on your slope field.

C.) Find the particular solution y=f(x) to the differential equation with the initial condition f(0)=1.

$\frac{(1)^2}{2} = \frac{(0)^2}{2} + C$

$C = \frac{1}{2}$

$\frac{y^2}{2} = \frac{x^2}{2} + \frac{1}{2}$

$\sqrt{y^2} = \sqrt{x^2+1}$

$y = \pm\sqrt{x^2+1}$

$y = \sqrt{x^2+1}$

D.) Sketch a solution curve that passes through the point (0,-1) on your slope field.

E.) Find the particular solution y=f(x) to the differential equation with the initial condition f(0)=-1.

$\frac{(-1)^2}{2} = \frac{(0)^2}{2} + C$

$C = \frac{1}{2}$

$\frac{y^2}{2} = \frac{x^2}{2} + \frac{1}{2}$

$\sqrt{y^2} = \sqrt{x^2+1}$

$y = \pm\sqrt{x^2+1}$

$y = -\sqrt{x^2+1}$