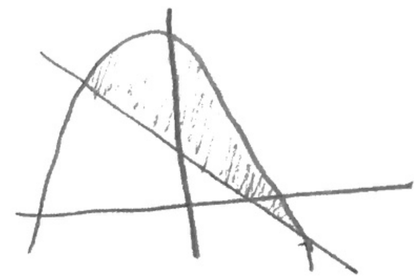
Find the area of the region bounded by the curves $y = 16 - 4x - x^2$ and $y = 4 - 3x$.

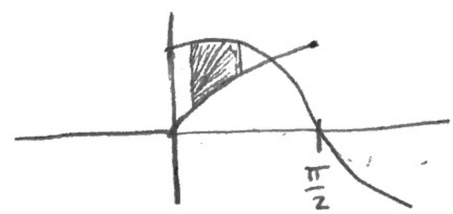
$$\int_{-4}^{3} (16-4x-x^2)-(4-3x)\,dx = \frac{343}{6}$$

$$= 57.167$$

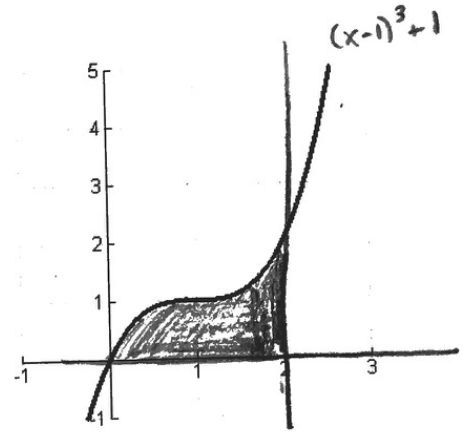


. Find the area of the region bounded by the curves $f(x) = \sin x$ and $g(x) = 4\cos x$ for $\frac{\pi}{6} < x < \frac{\pi}{3}$. Include a ketch of the graphs and the bounded area.

$$\int_{\pi/6}^{\pi/3} 4\cos x - \sin x\,dx = 1.098$$



. Let R be the region bounded by the graph of $y = (x-1)^3 + 1$, $x = 2$, and $y = 0$.



$$y = (x-1)^3 + 1$$

$$y - 1 = (x-1)^3$$

$$1 + \sqrt[3]{y-1} = x$$

a. Find the volume of the solid obtained by rotating R about the x-axis.

$$\pi \int_0^2 \left((x-1)^3+1\right)^2 dx = \frac{16\pi}{7}$$

b. Find the volume of the line obtained by rotating R about the y-axis.

$$\pi \int_0^2 2^2 - \left(1+\sqrt[3]{y-1}\right)^2 dy = \frac{24\pi}{5}$$

4. *Multiple Choice* Let S be the region enclosed by the graphs of $y = 2x$ and $y = 2x^2$ for $0 \le x \le 1$. What is the volume of the solid generated when S is revolved about the line $y = 3$?

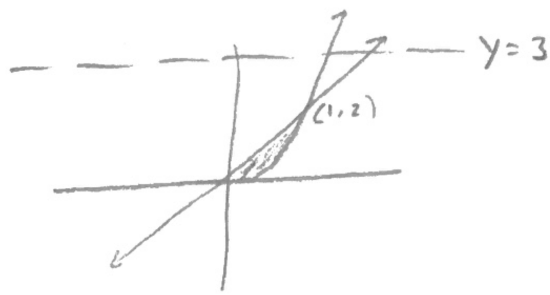
(A) $\pi\int_0^1 \left[(3-2x^2)^2-(3-2x)^2\right]dx$

(B) $\pi\int_0^1 \left[(3-2x)^2-(3-2x^2)^2\right]dx$

(C) $\pi\int_0^2 (4x^2-4x^4)\,dx$

(D) $\pi\int_0^2 \left[\left(3-\frac{y}{2}\right)^2-\left(3-\sqrt{\frac{y}{2}}\right)^2\right]dy$

(E) $\pi\int_0^2 \left[\left(3-\sqrt{\frac{y}{2}}\right)^2-\left(3-\frac{y}{2}\right)^2\right]dy$

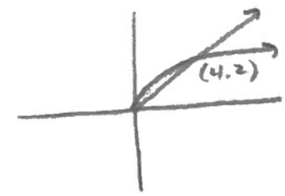


$$\pi\int_0^1 (3-2x^2)^2-(3-2x)^2\,dx$$

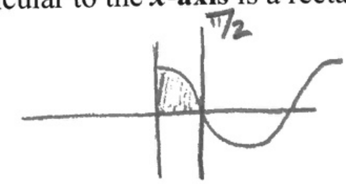
$x=y^2$ $\quad x=2y$

5. Let R be the region bounded by the graphs of $y=\sqrt{x}$ and $y=\frac{x}{2}$. The region R is the base of a solid. For this solid, each cross section perpendicular to the ***y*-axis** are squares. Find the volume of the solid. $\underline{dy}$

$$\int_0^2 (2y-y^2)^2\,dy = \frac{16}{15}$$

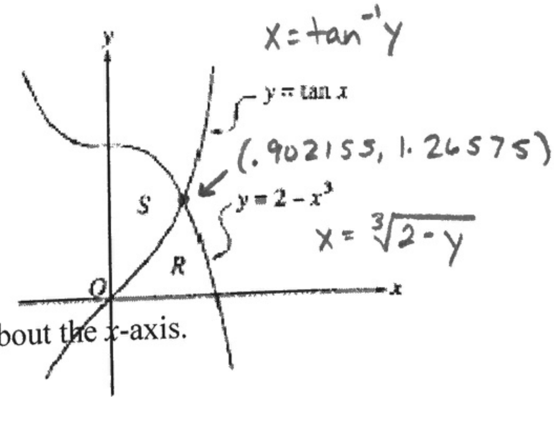


6. Let $\boldsymbol{R}$ be the region bounded by the x-axis, the y-axis, the graph of $y=\cos x$ and the line $x=\frac{\pi}{2}$. The region $\boldsymbol{R}$ is the base of a solid. For this solid, each cross section perpendicular to the ***x*-axis** is a rectangle whose height is $2-x$. Find the volume of the solid.



$$\int_0^{\pi/2} (2-x)\cos x\,dx = 1.429$$

8. Let R and S be the regions in the first quadrant shown in the figure. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.



a. Find the area of R.

$$\int_0^{1.26575} \sqrt[3]{2-y}-\tan^{-1}y\,dy = .729$$

b. Find the area of S.

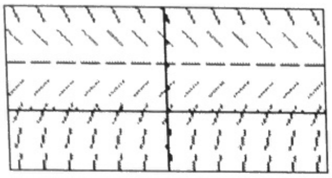
$$\int_0^{.902155} 2-x^3-\tan x\,dx = 1.161$$

c. Find the volume of the solid generated when S is revolved about the x-axis.

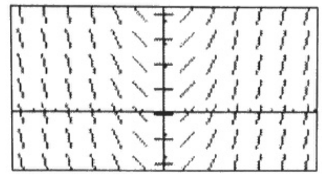
$$\pi\int_0^{.902155} (2-x^3)^2-(\tan x)^2\,dx = 2.652\pi$$

Match the slope fields with their differential equations.

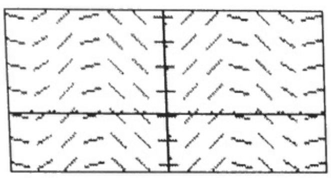
(A)



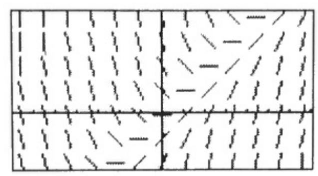
(B)



(C)



(D)



9. $\frac{dy}{dx} = \sin x$ C

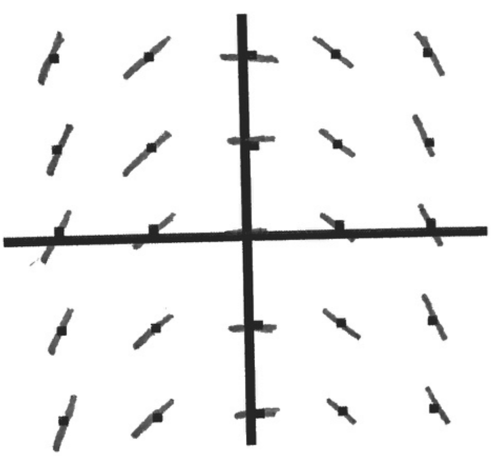
10. $\frac{dy}{dx} = x - y$ D

11. $\frac{dy}{dx} = 2 - y$ A

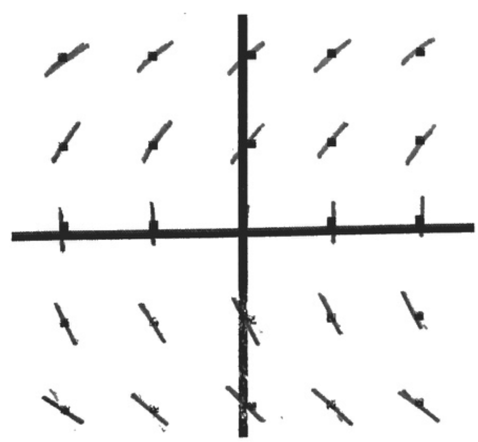
12. $\frac{dy}{dx} = x$ B

13. Construct the slope field for each of the following equations.

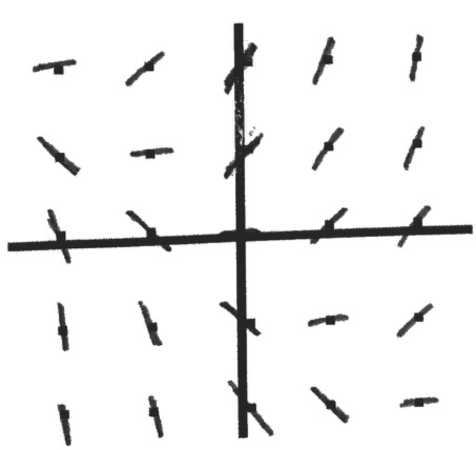
a. $\frac{dy}{dx} = -x$



b. $\frac{dy}{dx} = \frac{2}{y}$



c. $\frac{dy}{dx} = x + y$



Solve each differential equation.

14. $\frac{dy}{dx} = -\frac{4x}{y}$

$\int y\,dy = \int -4x\,dx$

$\frac{y^2}{2} = -2x^2 + C$

$y^2 = -4x^2 + C$

$\boxed{y = \pm\sqrt{-4x^2 + C}}$

15. $\frac{dy}{dx} = \frac{x + \sin x}{3y^2}$

$\int 3y^2\,dy = \int (x + \sin x)\,dx$

$y^3 = \frac{x^2}{2} - \cos x + C$

$\boxed{y = \sqrt[3]{\frac{x^2}{2} - \cos x + C}}$

16. $\frac{dy}{dx} = xy + 2y$

$\frac{dy}{dx} = y(x+2)$

$\int \frac{dy}{y} = \int (x+2)\,dx$

$\ln|y| = \frac{x^2}{2} + 2x + C$

$|y| = e^{x^2/2 + 2x + C}$

$\boxed{y = \pm A e^{x^2/2 + 2x}}$

17. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = \frac{x}{y^2}$ with initial condition $f(0) = 1$. Then $f(x) =$

(A). $2x$

(B). $\sqrt[3]{3x^2 + \frac{1}{3}}$

(C). $\sqrt[3]{3x + 1}$

(D). $\sqrt[3]{3x^2 + 1}$

(E). $\sqrt[3]{\frac{3}{2}x^2 + 1}$

$\int y^2\,dy = \int x\,dx$

$\frac{y^3}{3} = \frac{x^2}{2} + C$

$y^3 = \frac{3x^2}{2} + C$

$(0, 1)$

$y = \sqrt[3]{\frac{3x^2}{2} + C}$

$1 = \sqrt[3]{0 + C}$

$C = 1^3 = 1$

18. Find the average value of $f(x) = x^3 - x$ on the interval [1, 3] (no calculator).

$\frac{1}{3-1}\int_1^3 x^3 - x\,dx = \frac{1}{2}\left(\frac{x^4}{4} - \frac{x^2}{2}\right)\Big|_1^3 = \frac{1}{2}\left(\frac{81}{4} - \frac{9}{2} - \left(\frac{1}{4} - \frac{1}{2}\right)\right)$

$= \frac{1}{2}\left(\frac{63}{4} + \frac{1}{4}\right) = \frac{1}{2}\left(\frac{64}{4}\right) = \frac{64}{8} = \boxed{8}$

19. If the average value of the function f on the interval [a, b] is 10, then $\int_a^b f(x)dx =$

A. $\frac{10}{b-a}$

B. $\frac{f(a)+f(b)}{10}$

C. $10b - 10a$

D. $\frac{b-a}{10}$

E. $\frac{f(b)+f(a)}{20}$