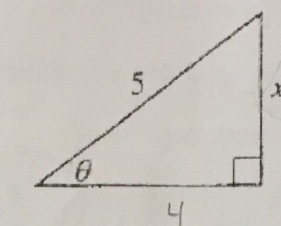
AB Calculus AP Practice – Applications of Derivatives

No Calculator for #1-11

✓ $\cos\theta = \frac{4}{5}$




Know: $\frac{d\theta}{dt} = 3 \text{ rad/min}$

find $\frac{dx}{dt}$ when $x = 3$

1. In the triangle shown above, if $\theta$ increases at a constant rate of 3 radians per minute, at what rate is $x$ increasing in units per minute when x equals 3 units?

$\frac{d}{dt}\left(\sin\theta = \frac{x}{5}\right)$

A. 3 B. $\frac{15}{4}$ C. 4 D. 9 E. 12

$\cos\theta \frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt}$ $\quad \frac{dx}{dt} = 12$

$\frac{4}{5} \cdot 3 = \frac{1}{5}\frac{dx}{dt}$

2. The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$, obtained from the tangent to the graph of $x = 0$, is

A. 2.00 B. 2.03 C. 2.06 D. 2.12 E. 2.24

$(0, \sqrt{4+\sin 0}) = (0, 2)$

$y' = \frac{1}{2}(4+\sin x)^{-1/2}(\cos x)\Big|_{x=0} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$y - 2 = \frac{1}{4}(x - 0)$ $\quad y - 2 = .03$

$y - 2 = \frac{1}{4}(.12)$ $\quad y = 2.03$

3. If $y = 2x - 8$, what is the minimum value of the product $xy$?

A. -16 B. -8 C. -4 D. 0 E. 2

$P = xy$

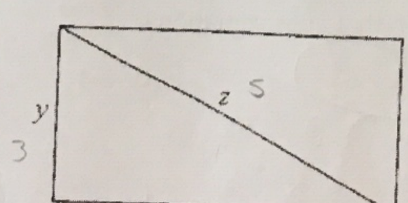
$P = x(2x - 8)$

$P = 2x^2 - 8x$ $\quad x = 2$

$P' = 4x - 8$ $\quad y = 2(2) - 8$

$0 = 4x - 8$ $\quad = 4 - 8 = -4$

$2(-4)$




$\frac{dy}{dt} = \frac{1}{3}\frac{dx}{dt}$

4. The sides of the rectangle above increasing in such a way that $\frac{dz}{dt} = 1$ and $\frac{dx}{dt} = 3\frac{dy}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?

Know $\frac{dz}{dt} = 1$, $\frac{dx}{dt} = 3\frac{dy}{dt}$

find $\frac{dx}{dt}$ when $x = 4$, $y = 3$

A. $\frac{1}{3}$ B. 1 C. 2 D. $\sqrt{5}$ E. 5

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(4) \cdot \frac{dx}{dt} + 2(3) \cdot \frac{1}{3}\frac{dx}{dt} = 2z \cdot 1$

$8\frac{dx}{dt} + 2\frac{dx}{dt} = 2 \cdot 5$

$10\frac{dx}{dt} = 10$ $\quad \frac{dx}{dt} = 1$

$16\pi = \pi r^2 h \quad h = \frac{16\pi}{\pi r^2} = \frac{16}{r^2} \quad SA = 2\pi r^2 + 2\pi r h$

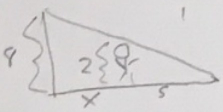
5. The volume of a cylindrical tin can with a top and a bottom is to be $16\pi$ cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can?

$SA = 2\pi r^2 + 2\pi r \cdot \frac{16}{r^2}$
$= 2\pi r^2 + \frac{32\pi}{r}$
$SA' = 4\pi r - 32\pi r^{-2}$
$32\pi = 4\pi r^3$
$8 = r^3 \quad r = 2$

A. $2\sqrt[3]{2}$ B. $2\sqrt{2}$ C. $2\sqrt[3]{4}$ D. 4 E. 8

$16\pi = \pi(2)^2 h$
$16\pi = 4\pi h \quad h = 4$

6. A person 2 meters tall walks directly away from a streetlight that is 8 meters above the ground. If the person is walking at a constant rate and the person's shadow is lengthening at the rate of $\frac{4}{9}$ meters per second, at what rate, in meters per second, is the person walking?



A. $\frac{4}{27}$ B. $\frac{4}{9}$ C. $\frac{3}{4}$ D. $\frac{4}{3}$ E. $\frac{16}{9}$

$\frac{8}{x+s} = \frac{2}{s} \quad 8s = 2x + 2s \quad 6s = 2x \quad 6\frac{ds}{dt} = \frac{2dx}{dt}$

$6\left(\frac{4}{9}\right) = 2\frac{dx}{dt} \quad \frac{24}{9} = 2\frac{dx}{dt} \quad \frac{12}{9} = \frac{dx}{dt} = \frac{4}{3}$

Know: $\frac{ds}{dt} = \frac{4}{9}$ m/sec
find $\frac{dx}{dt}$

7. The point on the curve $2y = x^2$ nearest to (4, 1) is

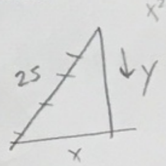
A. (0, 0) B. (2, 2) C. $(\sqrt{2}, 1)$ D. $(2\sqrt{2}, 4)$ E. (4, 8)

$D = \sqrt{(x-4)^2 + \left(\frac{x^2}{2} - 1\right)^2} = \sqrt{x^2 - 8x + 16 + \frac{x^4}{4} - x^2 + 1} = \sqrt{\frac{x^4}{4} - 8x + 17}$

$D' = \frac{1}{2}\left(\frac{x^4}{4} - 8x + 17\right)^{-1/2}(x^3 - 8)$
$x^3 - 8 = 0 \quad x = 2$

8. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

$x^2 + 7^2 = 25^2$
$x^2 + 49 = 625$
$x^2 = 576$
$x = 24$



A. $-\frac{7}{8}$ feet per minute B. $-\frac{7}{24}$ feet per minute C. $\frac{7}{24}$ feet per minute

D. $\frac{7}{8}$ feet per minute E. $\frac{21}{25}$ feet per minute

$x^2 + y^2 = z^2$
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(24)\frac{dx}{dt} + 2(7)(-3) = 0$
$48\frac{dx}{dt} = 42 \quad \frac{dx}{dt} = \frac{42}{48} = \frac{7}{4}$

Know: $\frac{dy}{dt} = -3$ ft/min
find: $dx/dt$ when $y = 7$

9. Let $f$ be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of $f$ at $x = 3$ is used to find an approximation to a zero of $f$, that approximation is

A. 0.4 B. 0.5 C. 2.6 D. 3.4 E. 5.5

$y - 2 = 5(x - 3)$
$y = 5x - 15 + 2$
$0 = 5x - 13$
$\frac{13}{5} = x$

10. The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

A. $\frac{1}{2}\pi$ B. $10\pi$ C. $24\pi$ D. $54\pi$ E. $108\pi$

Know: $\frac{dr}{dt} = \frac{1}{2}$ cm/sec, $\frac{dh}{dt} = \frac{1}{2}$ cm/sec

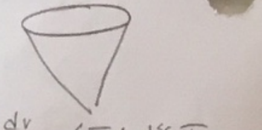
find: $\frac{dv}{dt}$

when $h = 9$cm and $r = 6$cm

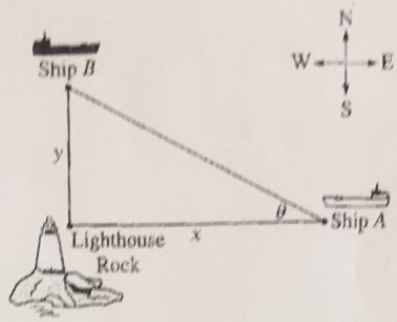
$V = \frac{1}{3}\pi r^2 h$

$\frac{dv}{dt} = \frac{1}{3}\pi r^2 \frac{dh}{dt} + \frac{2}{3}\pi r h \frac{dr}{dt}$

$\frac{dv}{dt} = \frac{1}{3}\pi(6)^2 \cdot \frac{1}{2} + \frac{2}{3}\pi \cdot 6 \cdot 9 \cdot \frac{1}{2}$



$\frac{dv}{dt} = 6\pi + 18\pi$
$= 24\pi$




11. Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour. Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure above.

a. Find the distance, in kilometers, between Ship A and Ship B when x = 4 km and y = 3 km.

$\sqrt{3^2+4^2} = \boxed{5 \text{ Km}}$

b. Find the rate of change, in km/hr, of the distance between the two ships when x = 4 km and y = 3 km.

know: $dx/dt = -15$ km/hr, $\frac{dy}{dt} = 10$ km/hr

find $\frac{dz}{dt}$

when $x=4, y=3$

$x^2+y^2=z^2$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$

$2(4)(-15)+2(3)(10)=2(5)\frac{dz}{dt}$

$-120+60=10\frac{dz}{dt}$

$-60=10\frac{dz}{dt}$

$\boxed{\frac{dz}{dt}=-6 \text{ km/hr}}$

c. Let $\theta$ be the angle shown in the figure. Find the rate of change of $\theta$, in radians per hour, when x = 4 km and y = 3 km.

find: $\frac{d\theta}{dt}$

when $x=4, y=3$

$\tan\theta = \frac{y}{x}$

$x\tan\theta = y$

$x\sec^2\theta\frac{d\theta}{dt}+\tan\theta\frac{dx}{dt}=\frac{dy}{dt}$

$\frac{x}{(\cos\theta)^2}\frac{d\theta}{dt}+\tan\theta\frac{dx}{dt}=\frac{dy}{dt}$

$\frac{4}{(\frac{4}{5})^2}\frac{d\theta}{dt}+\frac{3}{4}(-15)=10$

$\frac{25}{4}\frac{d\theta}{dt}-\frac{45}{4}=10$

$\frac{25}{4}\frac{d\theta}{dt}=\frac{85}{4}$

$\frac{d\theta}{dt}=\frac{85}{25}=\boxed{\frac{17}{5} \text{ rad/hr}}$

12. ***Calculator*** The number of gallons, P(t), of a pollutant in a lake changes at the rate $P'(t)=1-3e^{-0.2\sqrt{t}}$ gallons per day, where *t* is measured in days. There are 50 gallons of the pollutant in the lake at time *t* = 0. The lake is considered to be safe when it contains 40 gallons or less of a pollutant.

a. Is the amount of pollutant increasing at time *t* = 9? Why or why not?

$P'(9)=1-3e^{-.6}=-.646$

$P'(9)<0$ ∴ the amount is not increasing

b. For what value of *t* will the number of gallons of pollutant be at its minimum? Justify your answer.

$P'(t)=1-3e^{-.2\sqrt{t}}$

$0=1-3e^{-.2\sqrt{t}}$

$t=30.174$

− + (30.174)

There is a minimum at $t=30.174$ b/c $P'(t)$ changes from negative to positive

c. An investigator uses the tangent line approximation to *P*(*t*) at *t* = 0 as a model for the amount of pollutant in the lake. At what time *t* does this model predict that the lake becomes safe?

$P'(0)=1-3e^0=-2$

$y-50=-2(t-0)$

$y=-2t+50$

Safe when $y=40$

$40=-2t+50$

$-10=-2t$

$t=5$