

1-6: Suppose the functions f and g and their first derivatives have the values given in the table to the right at $x = -1$ and $x = 0$. Find the first derivative of each of the following at the given value of x .

x	$f(x)$	$f'(x)$
0	9	-2
1	-3	$\frac{1}{5}$

(first) · d/dx(second) + (second) · d/dx(first)

1. $\sqrt{x}f(x)$, $x=1$

$$\sqrt{x} \cdot \frac{d}{dx}[f(x)] + f(x) \cdot \frac{d}{dx}[x^{1/2}]$$

$$\sqrt{x} \cdot f'(x) + f(x) \cdot \frac{1}{2}x^{-1/2}$$

$$\sqrt{1} \cdot f'(1) + f(1) \cdot \frac{1}{2} = 1(\frac{1}{5}) + \frac{(-3)}{2}$$

$$\frac{1}{5} - \frac{3}{2} = \frac{(2) \cdot \frac{1}{5} - 3(5)}{2(5)} = \frac{-13}{10}$$

d/dx[(AT)^{1/2}] = 1/2(AT)^{-1/2} · d/dx(AT)

2. $\sqrt{f(x)}$, $x=0$

$$(f(x))^{1/2}$$

$$\frac{1}{2}(f(x))^{-1/2} \cdot \frac{d}{dx}[f(x)]$$

$$\frac{1}{2} \cdot \frac{1}{\sqrt{f(x)}} \cdot f'(x) = \frac{f'(0)}{2\sqrt{f(0)}} = \frac{-2}{2\sqrt{9}} = \frac{-2}{6} = -\frac{1}{3}$$

d/dx[f(AT)] = f'(AT) · d/dx(AT)

3. $f(\sqrt{x})$, $x=1$

$$f'(\sqrt{x}) \cdot \frac{d}{dx}[x^{1/2}]$$

$$f'(\sqrt{x}) \cdot \frac{1}{2}x^{-1/2}$$

$$\frac{f'(\sqrt{1})}{2\sqrt{1}} = \frac{f'(1)}{2(1)} = \frac{\frac{1}{5} \cdot \frac{1}{2}}{1} = \frac{1}{10}$$

d/dx[f(AT)] = f'(AT) · d/dx(AT)

4. $f(1-5\tan x)$, $x=0$

$$f'(1-5\tan x) \cdot \frac{d}{dx}[1-5\tan x]$$

$$f'(1-5\tan x) \cdot [-5 \sec^2 x]$$

$$f'(1-5\tan(0)) \cdot [-5 \sec^2(0)]$$

$$f'(1-5(0)) \cdot [-5(1)^2] = f'(1)(-5) = \frac{1}{5}(-5) = -1$$

ln d hi - hid lo

5. $\frac{f(x)}{2+\cos x}$, $x=0$

$$\frac{[2+\cos x]f'(x) - f(x)[- \sin x]}{[2+\cos x]^2}$$

$$\frac{[2+\cos(0)] \cdot f'(0) - f(0)[- \sin(0)]}{[2+\cos(0)]^2}$$

$$\frac{(2+1)(-2) - 9(0)/[2+1]^2}{[2+1]^2} = \frac{-2}{9}$$

Product & Chain

6. $10 \sin\left(\frac{\pi x}{2}\right) f^2(x)$, $x=1$

$$\frac{d}{dx}[\sin(AT)] = \cos AT \cdot \frac{d}{dx}[AT]$$

separate paper :)

7-15: Suppose the functions f and g and their 1st derivatives have the values given in the table to the right at $x = -1$ and $x = 0$. Find the first derivative of each of the following at the given value of x .

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	0	-1	2	1
0	-1	-3	-2	4

Quotient & Chain

7. $\frac{f(2x)}{x-1}$, $x=0$

$$\frac{(x-1)f'(2x) \cdot 2 - [f(2x)](1)}{(x-1)^2}$$

$$\frac{(-1)f'(0)(2) - [f(0)]}{(-1)^2}$$

$$\frac{(-1)(-2)(2) - (-1)}{1} = \frac{4+1}{1} = 5$$

Product & Chain

8. $f^2(x)g^3(x)$, $x=0$

$$[f(x)]^2 \cdot [g(x)]^3$$

$$[f(x)]^2 \cdot 3[g(x)]^2 \cdot g'(x) + [g(x)]^3 \cdot 2[f(x)] \cdot f'(x)$$

$$[f(0)]^2 \cdot 3 \cdot [g(0)]^2 \cdot g'(0) + [g(0)]^3 \cdot 2 \cdot f(0) \cdot f'(0)$$

$$(-1)^2(3)(-3)^2(4) + (-3)^3(2)(-1)(-2)$$

$$108 - 108 = 0$$

d/dx[2^{AT}] = 2^{AT} · ln 2 · d/dx(AT)

9. $2^{g(2x)}$, $x=0$

$$f(x) \cdot 2^{g(2x)} \cdot \ln(2) \cdot \frac{d}{dx}[g(2x)]$$

$$2^{g(0)} \cdot \ln 2 \cdot g'(2x) \cdot 2$$

$$2^{-3} \cdot \ln(2) \cdot g'(0) \cdot 2 = \frac{1}{8} \cdot \ln(2) \cdot (4)(2) = \ln 2$$

d/dx[g(AT)] = g'(AT) · d/dx(AT)

10. $g(f(x))$, $x=-1$

$$g'[f(x)] \cdot f'(x)$$

$$g'[f(-1)] \cdot f'(-1)$$

$$g'(0) \cdot (2) = 4(2) = 8$$

d/dx[f(AT)] = f'(AT) · d/dx(AT)

11. $f(g(x))$, $x=-1$

$$f'[g(x)] \cdot g'(x)$$

$$f'[g(-1)] \cdot g'(-1)$$

$$f'[-1] \cdot (1) = 2(1) = 2$$

Chain 3 times.

12. $f(g(2x-1))$, $x=0$

$$f'[g(2x-1)] \cdot \frac{d}{dx}[g(2x-1)]$$

$$f'[g(2x-1)] \cdot g'(2x-1) \cdot \frac{d}{dx}[2x-1]$$

$$f'[g(-1)] \cdot g'(-1) \cdot 2$$

$$f'[-1] \cdot (1)(2) = 2(1)(2) = 4$$

d/dx[g(AT)] = g'(AT) · d/dx(AT)

13. $g(x+f(x))$, $x=0$

$$g'[x+f(x)] \cdot \frac{d}{dx}[x+f(x)]$$

$$g'[x+f(x)] \cdot (1+f'(x))$$

$$g'[f(0)] \cdot (1+f'(0))$$

$$g'[-1] \cdot (1+2) = (1)(-1) = -1$$

d/dx[g⁻¹(x)] = 1/g'[g⁻¹(x)]

14. $g^{-1}(x)$, $x=3$

$$\frac{1}{g'[g^{-1}(3)]} = \frac{1}{g'[0]} = \frac{1}{4}$$

d/dx[g⁻¹(AT)] = 1/g'[g⁻¹(AT)] · d/dx(AT)

15. $g^{-1}(f^{-1}(x))$, $x=0$

$$\frac{1}{g'[g^{-1}(f^{-1}(0))]} \cdot \frac{1}{f'[f^{-1}(0)]}$$

$$\frac{1}{g'[g^{-1}(-1)]} \cdot \frac{1}{f'[-1]} = \frac{1}{g'[-1]} \cdot \frac{1}{2} = \frac{1}{\frac{1}{5} \cdot 2} = \frac{5}{2}$$

16-30: Find the derivative of the function.

$$\frac{d}{dx} [AT]^3 = 3 \cdot AT^2 \cdot \frac{d}{dx} [AT]$$

$$\frac{d}{dx} [3^{AT}] = 3^{AT} \cdot \ln 3 \cdot \frac{d}{dx} [AT]$$

16. $y = e^{3x-7}$
 $\frac{d}{dx} [e^{AT}] = e^{AT} \cdot \frac{d}{dx} [AT]$
 $y' = e^{3x-7} (3) = \boxed{3e^{3x-7}}$

17. $y = \sin^3 x$
 $y = (\sin x)^3$
 $y' = 3(\sin x)^2 \cdot \frac{d}{dx} [\sin x]$
 $y' = \boxed{3 \cos x \cdot \sin^2 x}$

18. $y = 3^{2x^4}$
 $y' = 3^{2x^4} \cdot \ln 3 \cdot 8x^3$
 $y' = \boxed{8x^3 \cdot 3^{2x^4} \cdot \ln 3}$

19. $b = \log_5(t-7)$
 $\frac{d}{dx} [\log_5(AT)] = \frac{1}{AT \cdot \ln 5} \cdot \frac{d}{dx} [AT]$
 $\frac{1}{(t-7) \ln 5} \cdot (1) = \boxed{\frac{1}{(t-7) \ln 5}}$

20. $y = \ln(e^{x^2})$ *What are $\ln x$ & e^x to each other?*
 $y = x^2$
 $y' = \boxed{2x}$

21. $y = \ln(\sin x)$ $\frac{d}{dx} [\ln(AT)] = \frac{1}{AT} \cdot \frac{d}{dx} [AT]$
 $y' = \frac{1}{\sin x} \cdot \frac{d}{dx} [\sin x]$
 $y' = \frac{1}{\sin x} \cdot \cos x = \boxed{\cot x}$

22. $y = x^{\ln x}$ $\frac{d}{dx} [\text{variable}^{\text{variable}}]$
 $\ln y = \ln x \cdot \ln x$
 $\frac{1}{y} \frac{dy}{dx} = \ln x \cdot \frac{1}{x} + \ln x \cdot \frac{1}{x}$
 $y \cdot \frac{1}{y} \frac{dy}{dx} = \left[\frac{\ln x}{x} + \frac{\ln x}{x} \right] \cdot y$
 $\frac{dy}{dx} = \boxed{\left[\frac{2 \ln x}{x} \right] \cdot x^{\ln x}}$

23. $\ln_y \frac{d}{dx} \frac{(2x)^{2x}}{\sqrt{x^2+1}}$ *ln both sides so easier*
 $\ln y = \ln(2x) + \ln(2^x) - \frac{1}{2} \ln(x^2+1)$
 $\frac{1}{y} \frac{dy}{dx} = \frac{1}{2x} (2) + \frac{1}{2^x} \cdot 2^x \cdot \ln 2 - \frac{1}{2} \cdot \frac{1}{x^2+1} (2x)$
 $y \cdot \frac{1}{y} \frac{dy}{dx} = \left[\frac{1}{x} + \ln 2 - \frac{x}{x^2+1} \right] y$
 $\frac{dy}{dx} = \boxed{\left[\frac{1}{x} + \ln 2 - \frac{x}{x^2+1} \right] (2x) \cdot 2^x \cdot \frac{1}{\sqrt{x^2+1}}}$

24. $y = e^{\tan^{-1} x}$ $\frac{d}{dx} [e^{AT}] = e^{AT} \cdot \frac{d}{dx} [AT]$
 $e^{\tan^{-1} x} \cdot \frac{d}{dx} [\tan^{-1} x] = \frac{1}{1+(AT)^2} \cdot \frac{d}{dx} [AT]$
 $e^{\tan^{-1} x} \cdot \frac{1}{x^2+1} (1)$
 $y' = \boxed{\frac{e^{\tan^{-1} x}}{x^2+1}}$

25. $y = \sin^{-1} \sqrt{1-u^2}$
 $\frac{d}{dx} [\sin^{-1}(AT)] = \frac{1}{\sqrt{1-(AT)^2}} \cdot \frac{d}{dx} [AT]$
 $y' = \frac{1}{\sqrt{1-(\sqrt{1-u^2})^2}} \cdot \frac{d}{dx} [(1-u^2)^{1/2}]$
 $y' = \frac{1}{\sqrt{1+u^2}} \cdot \frac{1}{2} (1-u^2)^{-1/2} (-2u)$
 $y' = \frac{1}{\sqrt{u^2}} \cdot \frac{-u}{\sqrt{1-u^2}} = \frac{1}{u} \cdot \frac{-u}{\sqrt{1-u^2}} = \boxed{\frac{-1}{\sqrt{1-u^2}}}$

26. $y = (1+t^2)^{\cot^{-1} 2t}$ *Product &*
 $\frac{d}{dx} [\cot(AT)] = \frac{-1}{(AT)^2+1} \cdot \frac{d}{dx} [AT]$
 $(1+t^2) \left(\frac{-1}{4t^2+1} \right) (2) + \cot^{-1}(2t) (2t)$
 $\boxed{\frac{-2(1+t^2)}{4t^2+1} + 2t \cot^{-1}(2t)}$

27. $y = \tan^{-1} \left(\frac{1}{x} \right)$ $\frac{d}{dx} [\tan^{-1}(AT)] = \frac{1}{1+(AT)^2} \cdot \frac{d}{dx} [AT]$
 $\frac{1}{\left(\frac{1}{x} \right)^2 + 1} \cdot \frac{d}{dx} [1 \cdot x^{-1}]$
 $\frac{1}{\frac{1}{x^2} + \frac{1 \cdot x^2}{x^2}} \cdot -1x^{-2}$
 $\frac{1}{\frac{1+x^2}{x^2}} \cdot \frac{-1}{x^2}$
 $\frac{x^2}{1+x^2} \cdot \frac{-1}{x^2} = \boxed{\frac{-1}{1+x^2}}$

28. $y = \cos(1-2t)$
 $\frac{d}{dx} [\cos(AT)] = -\sin(AT) \cdot \frac{d}{dx} [AT]$
 $y' = -\sin(1-2t) (-2t)$
 $y' = \boxed{2t \sin(1-2t)}$

29. $y = x e^{-x}$ *Product &* $\frac{d}{dx} [e^{AT}] = e^{AT} \cdot \frac{d}{dx} [AT]$
 $y' = x \cdot \frac{d}{dx} [e^{-x}] + e^{-x} \cdot \frac{d}{dx} [x]$
 $y' = x e^{-x} (-1) + e^{-x} (1)$
 $y' = \boxed{e^{-x} [-x+1]}$

30. $y = \log_3(\theta^2)$
 $\frac{d}{dx} [\log_3(AT)] = \frac{1}{AT \cdot \ln 3} \cdot \frac{d}{dx} [AT]$
 $y' = \frac{1}{\theta^2 \cdot \ln 3} \cdot \frac{d}{dx} [\theta^2]$
 $y' = \frac{1}{\theta^2 \cdot \ln 3} \cdot 2\theta = \boxed{\frac{2}{\theta \ln 3}}$

Implicit: Do your variables match?

31-34: Find dy/dx .

31. $xy + 2x + 3y = 1$ **Product**

$$\frac{d}{dx}[xy] + \frac{d}{dx}[2x] + \frac{d}{dx}[3y] = \frac{d}{dx}[1]$$

$$x \cdot \frac{d}{dx}[y] + y \frac{d}{dx}[x] + 2 + 3 \frac{dy}{dx} = 0$$

$$x \frac{dy}{dx} + y(1) + 2 + 3 \frac{dy}{dx} = 0$$

$$x \frac{dy}{dx} + 3 \frac{dy}{dx} = -2 - y$$

$$\frac{dy}{dx}(x+3) = -2-y \quad \frac{dy}{dx} = \boxed{\frac{-2-y}{x+3}}$$

33. $\sqrt{xy} = 1$ $(xy)^{1/2} = 1$

$$\frac{1}{2}(xy)^{-1/2} \cdot \frac{d}{dx}[xy] = \frac{d}{dx}[1]$$

$$\frac{1}{2\sqrt{xy}} [x \cdot \frac{dy}{dx} + y(1)] = 0 \cdot 2\sqrt{xy}$$

$$x \frac{dy}{dx} + y = 0$$

$$\frac{x \frac{dy}{dx} = -y}{\frac{x}{x}} \quad \frac{dy}{dx} = \frac{-y}{x}$$

32. $5x^{4/5} + 10y^{6/5} = 15$

$$\frac{d}{dx}[5x^{4/5}] + \frac{d}{dx}[10y^{6/5}] = \frac{d}{dx}[15]$$

$$5 \cdot \frac{4}{5} x^{-1/5} + 10 \cdot \frac{6}{5} y^{1/5} \frac{dy}{dx} = 0$$

$$4x^{-1/5} + 12y^{1/5} \frac{dy}{dx} = 0$$

$$12y^{1/5} \frac{dy}{dx} = -4x^{-1/5} \quad \frac{dy}{dx} = \boxed{\frac{-1}{4x^{1/5}y^{1/5}}}$$

34. $y^2 = \frac{x}{x+1}$ $y = \sqrt{\frac{x}{x+1}} = \frac{\sqrt{x}}{\sqrt{x+1}}$

$$2y \frac{dy}{dx} = \frac{(x+1)(1) - (x)(1)}{(x+1)^2} \quad \text{Lodhi-hidlo}$$

$$\frac{1}{2y} 2y \frac{dy}{dx} = \frac{x+1-x}{(x+1)^2} \cdot \frac{1}{2y}$$

$$\frac{dy}{dx} = \frac{1}{2(x+1)^2 \cdot y} = \frac{1}{2(x+1)^2 \sqrt{x}} = \boxed{\frac{1}{2(x+1)^{3/2} \sqrt{x}}}$$

35-38: Find the equation of the tangent line at the given value of x . $\frac{d}{dx}[\tan(\arctan)] = \sec^2(\arctan) \cdot \frac{d}{dx}[\arctan]$

35. $y = \sqrt{x^2 - 2x}$, $x = 3$ Point $(3, \sqrt{3})$

$$y(3) = \sqrt{3^2 - 2(3)} = \sqrt{9-6} = \sqrt{3} \quad \text{Slope } m = \frac{2}{\sqrt{3}}$$

$$y = (x^2 - 2x)^{1/2}$$

$$y' = \frac{1}{2}(x^2 - 2x)^{-1/2} [2x - 2] \quad \boxed{y - \sqrt{3} = \frac{2}{\sqrt{3}}(x - 3)}$$

$$y' = \frac{1}{2\sqrt{x^2 - 2x}} \cdot 2(x-1)$$

$$y' = \frac{x-1}{\sqrt{x^2 - 2x}} \quad y'(3) = \frac{3-1}{\sqrt{9-6}} = \frac{2}{\sqrt{3}}$$

36. $y = \tan 2x$, $x = \pi/3$ Point $(\frac{\pi}{3}, -\sqrt{3})$

$$y(\frac{\pi}{3}) = \tan^2(\frac{\pi}{3}) \quad \text{Slope } m = 8$$

$$y(\frac{\pi}{3}) = \tan \frac{2\pi}{3}$$

$$y(\frac{\pi}{3}) = -\sqrt{3}$$

$$y = \tan(2x)$$

$$y' = \sec^2(2x) \cdot 2$$

$$y'(\frac{\pi}{3}) = 2 [\sec \frac{2\pi}{3}]^2 = 2(-\frac{2}{1})^2 = 8$$

$$\boxed{y + \sqrt{3} = 8(x - \frac{\pi}{3})}$$

37. $x^2 + 2y^2 = 9$, $(1, 2)$ Point $(1, 2)$

$$\frac{d}{dx}[x^2] + \frac{d}{dx}[2y^2] = \frac{d}{dx}[9] \quad \text{Slope } m = -\frac{1}{4}$$

$$2x + 4y \frac{dy}{dx} = 0$$

$$\frac{4y \frac{dy}{dx} = -2x}{\frac{4y}{4y}}$$

$$\boxed{y - 2 = -\frac{1}{4}(x - 1)}$$

$$\frac{dy}{dx} = \frac{-x}{2y}$$

$$\frac{dy}{dx} \Big|_{(1,2)} = \frac{-(1)}{2(2)} = -\frac{1}{4}$$

38. $x + \sqrt{xy} = 6$, $(4, 1)$ Point $(4, 1)$

Slope $m = -\frac{5}{4}$

$$\frac{d}{dx}[x] + \frac{d}{dx}[(xy)^{1/2}] = \frac{d}{dx}[6]$$

$$1 + \frac{1}{2}[xy]^{-1/2} \cdot \frac{d}{dx}[xy] = 0 \quad \boxed{y - 1 = -\frac{5}{4}(x - 4)}$$

$$1 + \frac{1}{2\sqrt{xy}} [x \frac{d}{dx}[y] + y \frac{d}{dx}[x]] = 0$$

$$1 + \frac{1}{2\sqrt{xy}} [x \frac{dy}{dx} + y(1)] = 0 \quad \rightarrow x \frac{dy}{dx} = -2\sqrt{xy} - y$$

$$\frac{dy}{dx} = \frac{-2\sqrt{xy} - y}{x}$$

$$\frac{dy}{dx} \Big|_{(4,1)} = \frac{-2\sqrt{4 \cdot 1} - 1}{4} = \frac{-4 - 1}{4} = -\frac{5}{4}$$

39. 1. The height (in meters) of a projectile shot vertically upward from a point 10 meters above ground level with an initial velocity of 40 m/s is $h = 10 + 40t - 4.9t^2$ after t seconds.

a. Find the velocity after 2 s and after 4s.

$$h = 10 + 40t - 4.9t^2$$

$$v = h' = 40 - 9.8t$$

$$v(2) = 40 - 9.8(2) = \boxed{20.4 \frac{\text{m}}{\text{sec}}}$$

$$v(4) = 40 - 9.8(4) = \boxed{-8 \frac{\text{m}}{\text{sec}}}$$

b. When does the projectile reach its maximum height?

Reaches max height when $v(t) = 0$

$$40 - 9.8t = 0$$

$$9.8t = 40$$

$$\boxed{t = 4.082 \text{ seconds}}$$

c. What is the maximum height?

$$h(4.082) = 10 + 40(4.082) - 4.9(4.082)^2$$

$$\text{max height} = \boxed{91.633 \text{ meters}}$$

d. When does it hit the ground?

hits ground when $h = 0$

$$10 + 40t - 4.9t^2 = 0$$

$$t = \cancel{-2.43} \text{ or } t = 8.406$$

garbage

e. With what velocity does it hit the ground?

$$v(8.406) = 40 - 9.8(8.406)$$

$$= \boxed{-42.379 \frac{\text{m}}{\text{sec}}}$$

Remember

position

velocity (velocity = (position)')

acceleration (acc = (position)'')

#6. $10 \sin\left(\frac{\pi x}{2}\right) f^2(x)$ at $x=1$

$$10 \sin\left(\frac{\pi x}{2}\right) \cdot \frac{d}{dx} [f(x)^2] + [f(x)]^2 \cdot \frac{d}{dx} [10 \sin\left(\frac{\pi x}{2}\right)]$$

$$10 \sin\left(\frac{\pi x}{2}\right) \cdot 2[f(x)]' \cdot \frac{d}{dx} [f(x)] + [f(x)]^2 \cdot 10 \cos\left(\frac{\pi x}{2}\right) \frac{d}{dx} \left[\frac{\pi x}{2}\right]$$

$$20 \sin\left(\frac{\pi x}{2}\right) \cdot f(x) \cdot f'(x) + [f(x)]^2 \cdot 10 \cos\left(\frac{\pi x}{2}\right) \cdot \frac{\pi}{2}$$

$$20 \sin\left(\frac{\pi x}{2}\right) f(x) f'(x) + 5\pi [f(x)]^2 \cos\left(\frac{\pi x}{2}\right)$$

$$20 \sin\left(\frac{\pi}{2}\right) f(1) f'(1) + 5\pi [f(1)]^2 \cos\left(\frac{\pi}{2}\right) \rightarrow \cos\frac{\pi}{2} = 0$$

$$4 \cancel{20} (1) (-3) \left(\frac{1}{5}\right)$$

$$\boxed{-12}$$