AP Calculus-AB

Derivative of Inverse

Name__________________ Pd.____

Derivatives (2) Day 4

$f'(x)=5x^4+6x^2+1$

1. Let $f(x)=x^5+2x^3+x-1$ .

A. Find $f(1)$ and $f'(1)$ .

$f(1)=(1)^5+2(1)^3\cancel{+1-1}=1+2=3$

$f'(1)=5(1)^4+6(1)^2+1=12$

B. Find $f^{-1}(3)$ and $(f^{-1})'(3)$.

Because $f(1)=3$ then $f^{-1}(3)=1$

$\frac{d}{dx}[f^{-1}(x)]=\frac{1}{f'[f^{-1}(x)]}=\frac{1}{f'[f^{-1}(3)]}$

$=\frac{1}{f'[f^{-1}(3)]}=\frac{1}{f'[1]}=\boxed{\frac{1}{12}}$

$f'(x)=-\sin x+3$

2. Let $f(x)=\cos x+3x$ .

A. Find $f(0)$ and $f'(0)$ .

$f(0)=\cos(0)+3(0)=1+0=1$

$f'(0)=-\sin(0)+3=0+3=3$

B. Find $f^{-1}(1)$ and $(f^{-1})'(1)$.

Because $f(0)=1$ then $f^{-1}(1)=0$

$\frac{d}{dx}[f^{-1}(x)]=\frac{1}{f'[f^{-1}(x)]}=\frac{1}{f'[f^{-1}(1)]}$

$=\frac{1}{f'[f^{-1}(1)]}=\frac{1}{f'(0)}=\boxed{\frac{1}{3}}$

3. Let: $f(x)=2x+e^x-3$ Find: $(f^{-1})'(-2)$

$\frac{d}{dx}[f^{-1}(x)]=\frac{1}{f'[f^{-1}(-2)]}$

$\frac{1}{f'[f^{-1}(-2)]}=\frac{1}{f'[0]}=\boxed{\frac{1}{3}}$

$f^{-1}(-2)=\underline{0}$

$-2=2x+e^x-3$

let $x=0$ $-2=2(0)+e^0-3$

$-2=-2$ ✓

$f(x)=2x+e^x-3$

$f'(x)=2+e^x$

$f'(0)=2+e^0=3$

4. Let: $f(x)=x^3-3$ Find: $(f^{-1})'(-11)$

$\frac{d}{dx}[f^{-1}(x)]=\frac{1}{f'[f^{-1}(-11)]}$

$\frac{1}{f'[f^{-1}(-11)]}=\frac{1}{f'[-2]}=\boxed{\frac{1}{12}}$

$f^{-1}(-11)=\underline{-2}$

$-11=x^3-3$

let $x=-2$ $-11=(-2)^3-3$

$-11=-11$

$f(x)=x^3-3$

$f'(x)=3x^2$

$f'(-2)=3(-2)^2=12$

5. Let: $f(x)=2x+9$ Find: $(f^{-1})'(-7)$

$\frac{d}{dx}[f^{-1}(x)]=\frac{1}{f'[f^{-1}(-7)]}$

$\frac{1}{f'[f^{-1}(-7)]}=\frac{1}{f'[-8]}=\boxed{\frac{1}{2}}$

$f^{-1}(-7)=\underline{-8}$

$-7=2x+9$

$-9$ $\quad -16=2x$

$x=-8$

$f(x)=2x+9$

$f'(x)=2$

$f'(\text{Any }\#)=2$

6. Let: $f(x)=x^3+2x^2+4x+8$ Find: $(f^{-1})'(15)$

$\frac{d}{dx}[f^{-1}(x)]=\frac{1}{f'[f^{-1}(15)]}=$

$\frac{1}{f'[f^{-1}(15)]}=\frac{1}{f'(1)}=\frac{1}{11}$

$f^{-1}(15)=\underline{1}$

$15=x^3+2x^2+4x+8$

let $x=1$ $\quad 15=15$

$f(x)=x^3+2x^2+4x+8$

$f'(x)=3x^2+4x+4$

$f'(1)=3+4+4=11$

7-10: Find the derivative of y with respect to the appropriate variable.

7. $y=\cos^{-1}(x^2)$ $\frac{d}{dx}[\cos^{-1}(AT)]=\frac{-1}{\sqrt{1-(AT)^2}}\cdot\frac{d}{dx}[AT]$

$\frac{-1}{\sqrt{1-(x^2)^2}}\cdot\frac{d}{dx}[x^2]=\boxed{\frac{-2x}{\sqrt{1-x^4}}}$

common denominator

8. $y=\cos^{-1}\left(\frac{1}{x}\right)$ $\frac{d}{dx}[\cos^{-1}(AT)]=\frac{-1}{\sqrt{1-(AT)^2}}\cdot\frac{d}{dx}[AT]$

$\frac{-1}{\sqrt{\frac{x^2}{x^2}1-\left(\frac{1}{x}\right)^2}}\cdot\frac{d}{dx}[1x^{-1}]=\frac{-1}{\sqrt{\frac{x^2-1}{x^2}}}\cdot -1x^{-2}=\frac{-1}{\frac{\sqrt{x^2-1}}{x}}\cdot\frac{-1}{x^2}$

$=\frac{-x}{\sqrt{x^2-1}}\cdot\frac{-1}{x^2}=\boxed{\frac{1}{x\sqrt{x^2-1}}}$

9. $y=\sin^{-1}(\sqrt{2}t)$ $\frac{d}{dx}[\sin^{-1}(AT)]=\frac{1}{\sqrt{1-(AT)^2}}\cdot\frac{d}{dx}[AT]$

$\frac{1}{\sqrt{1-(\sqrt{2}t)^2}}\cdot\frac{d}{dt}[\sqrt{2}t]=\boxed{\frac{\sqrt{2}}{\sqrt{1-2t^2}}}$

10. $y=\sin^{-1}(1-t)$ $\frac{d}{dx}[\sin^{-1}(AT)]=\frac{1}{\sqrt{1-(AT)^2}}\cdot\frac{d}{dx}[AT]$

$\frac{1}{\sqrt{1-(1-t)^2}}\cdot\frac{d}{dt}[1-t]=\frac{1}{\sqrt{1-(1-2t+t^2)}}\cdot(-1)$

$=\frac{-1}{\sqrt{\cancel{1-1}+2t-t^2}}=\boxed{\frac{-1}{\sqrt{2t-t^2}}}$

11-16: Find the derivative of y with respect to the appropriate variable. $\frac{d}{dx}[\cot(AT)] = \frac{-1}{(AT)^2+1}\cdot\frac{d}{dx}[AT]$

▧ common denominator

11. $y = \sin^{-1}\left(\frac{3}{x^2}\right)$ $\frac{d}{dx}[\sin^{-1}(AT)] = \frac{1}{\sqrt{1-(AT)^2}}\cdot\frac{d}{dx}[AT]$

$\frac{1}{\sqrt{1\cdot\frac{x^4}{x^4}-\frac{9}{x^4}}}\cdot\frac{d}{dx}[3x^{-2}] = \frac{1}{\sqrt{\frac{x^4}{x^4}-\frac{9}{x^4}}}\cdot(-6x^{-3}) = \frac{1}{\sqrt{\frac{x^4-9}{x^4}}}\cdot\frac{-6}{x^3}$

$\frac{1}{\frac{\sqrt{x^4-9}}{x^2}}\cdot\frac{-6}{x^3} = \frac{x^2}{\sqrt{x^4-9}}\cdot\frac{-6}{x^3} = \boxed{\frac{-6}{x\sqrt{x^4-9}}}$

12. $y = \cot^{-1}(\sqrt{t})$

$\frac{-1}{(\sqrt{t})^2+1}\cdot\frac{d}{dt}[t^{1/2}] = \frac{-1}{t+1}\cdot\frac{1}{2}t^{-1/2}$

$= \boxed{\frac{-1}{2\sqrt{t}(t+1)}}$

product

13. $y = x\sqrt{1-x^2} + \cos^{-1}x$ $\frac{d}{dx}[\cos^{-1}(AT)] = \frac{-1}{\sqrt{1-(AT)^2}}\cdot\frac{d}{dx}[AT]$

$\underbrace{x\cdot\frac{d}{dx}[(1-x^2)^{1/2}] + \sqrt{1-x^2}\cdot\frac{d}{dx}[x]}_{} + \frac{-1}{\sqrt{1-x^2}}(1)$

$x\cdot\frac{1}{\cancel{2}}(1-x^2)^{-1/2}\cdot(-\cancel{2}x) + \sqrt{1-x^2}(1) - \frac{1}{\sqrt{1-x^2}}$

$\frac{-x^2}{\sqrt{1-x^2}} + \frac{\sqrt{1-x^2}}{1}\cdot\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} = \frac{-x^2+\cancel{1}-x^2-\cancel{1}}{\sqrt{1-x^2}}$

▧ common denominator

$= \boxed{\frac{-2x^2}{\sqrt{1-x^2}}}$

14. $y = \frac{1}{\sin^{-1}(2x)}$ $\frac{d}{dx}[\sin^{-1}(AT)] = \frac{1}{\sqrt{1-(AT)^2}}\cdot\frac{d}{dx}[AT]$

$y = [\sin^{-1}(2x)]^{-1}$

$y' = -1[\sin^{-1}(2x)]^{-2}\cdot\frac{d}{dx}[\sin^{-1}(2x)]$

$y' = -1[\sin^{-1}(2x)]^{-2}\cdot\frac{1}{\sqrt{1-4x^2}}\cdot(2)$

$y' = \boxed{\frac{-2}{[\sin^{-1}(2x)]^2\sqrt{1-4x^2}}}$

15. $y = \cot^{-1}\sqrt{x-1}$ $\frac{d}{dx}[\cot^{-1}(AT)] = \frac{-1}{(AT)^2+1}\cdot\frac{d}{dx}[AT]$

$\frac{dy}{dx} = \frac{-1}{(\sqrt{x-1})^2+1}\cdot\frac{d}{dx}[(x-1)^{1/2}]$

$\frac{dy}{dx} = \frac{-1}{(x\cancel{-1})\cancel{+1}}\cdot\frac{1}{2}(x-1)^{-1/2}(1) = \boxed{\frac{-1}{2x\sqrt{x-1}}}$

16. $y = \sec^{-1}(5m)$ $\frac{d}{dx}[\sec(AT)] = \frac{1}{|AT|\sqrt{(AT)^2-1}}\cdot\frac{d}{dx}[AT]$

$\frac{dy}{dm} = \frac{1}{|5m|\sqrt{25m^2-1}}\cdot\frac{d}{dm}[5m]$

$\frac{dy}{dm} = \frac{\cancel{5}}{\cancel{5}|m|\sqrt{25m^2-1}} = \boxed{\frac{1}{|m|\sqrt{25m^2-1}}}$

17-20: Find an equation for the tangent to the graph of y at the indicated point.

17. $y = \sec^{-1}x$, at $x = 2$ $\frac{d}{dx}[\sec(AT)] = \frac{1}{|AT|\sqrt{(AT)^2-1}}\cdot\frac{d}{dx}[AT]$

$y(2) = \sec^{-1}(2) = \frac{1}{\cos^{-1}(2)} \approx -2.403$

Point $= (2, \sec^{-1}(2))$ OR $(2, -2.403)$

$y' = \frac{1}{|x|\sqrt{x^2-1}}(1)$

$m = \frac{1}{2\sqrt{3}}$

$y'(2) = \frac{1}{|2|\sqrt{4-1}} = \frac{1}{2\sqrt{3}}$

$\boxed{y-\sec^{-1}(2) = \frac{1}{2\sqrt{3}}(x-2) \text{ OR } y+2.403 = \frac{1}{2\sqrt{3}}(x-2)}$

18. $y = \tan^{-1}x$, at $x = 2$ $\frac{d}{dx}[\tan(AT)] = \frac{1}{(AT)^2+1}\cdot\frac{d}{dx}[AT]$

$y(2) = \tan^{-1}(2) \approx 1.10715$

Point $= (2, \tan^{-1}2)$

$m = 1/5$

$y' = \frac{1}{x^2+1}(1)$

$y'(2) = \frac{1}{(2)^2+1} = \frac{1}{5}$

$\boxed{y-\tan^{-1}(2) = \frac{1}{5}(x-2) \text{ OR } y-1.107 = \frac{1}{5}(x-2)}$

19. $y = \sin^{-1}\left(\frac{x}{4}\right)$, at $x = 3$ $\frac{d}{dx}[\sin^{-1}(AT)] = \frac{1}{\sqrt{1-(AT)^2}}\cdot\frac{d}{dx}[AT]$

$y = \sin^{-1}\left(\frac{x}{4}\right)$ $y(3) = \sin^{-1}\left(\frac{3}{4}\right)$

$y(3) = .848$

Point $(3, .848)$

Slope $= \frac{1}{\sqrt{7}}$

$y' = \frac{1}{\sqrt{1-\frac{x^2}{16}}}\cdot\frac{d}{dx}\left[\frac{1}{4}x\right] = \frac{1}{\sqrt{1-\frac{x^2}{16}}}\cdot\frac{1}{4}$

$y'(3) = \frac{1}{\sqrt{1-\frac{9}{16}}}\cdot\frac{1}{4} = \frac{1}{\sqrt{\frac{16}{16}-\frac{9}{16}}}\cdot\frac{1}{4} = \frac{1}{\sqrt{\frac{7}{16}}}\cdot\frac{1}{4} = \frac{1}{\frac{\sqrt{7}}{4}}\cdot\frac{1}{4} = \frac{\cancel{4}}{\sqrt{7}}\cdot\frac{1}{\cancel{4}} = \frac{1}{\sqrt{7}}$

$\boxed{y-\sin^{-1}(3/4) = \frac{1}{\sqrt{7}}(x-3)}$ OR $\boxed{y-.848 = \frac{1}{\sqrt{7}}(x-3)}$

20. $y = \tan^{-1}(x^2)$, at $x = 2$ $\frac{d}{dx}[\tan^{-1}(AT)] = \frac{1}{(AT)^2+1}\cdot\frac{d}{dx}[AT]$

$y(2) = \tan^{-1}(4)$

Point $(2, \tan^{-1}4)$ OR $(2, 1.157)$

$y' = \frac{1}{(x^2)^2+1}\cdot\frac{d}{dx}[x^2]$

$y' = \frac{2x}{x^4+1}$ $y'(2) = \frac{2(2)}{(2)^4+1} = \frac{4}{17}$ $m = \frac{4}{17}$

$\boxed{y-\tan^{-1}(4) = \frac{4}{17}(x-2) \text{ OR } y-1.157 = \frac{4}{17}(x-2)}$